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Mixed Problem for Partial Differential Equations with Quasihomogeneous Principal Part

S. G. Gindikin
L. R. Volevich

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Mixed Problem for Partial Differential Equations with Quasihomogeneous Principal Part

S. G. Gindikin
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American Mathematical Society
Providence, Rhode Island

Л. Р. Волевич, С. Г. Гиндикин

СМЕШАННАЯ ЗАДАЧА ДЛЯ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ С КВАЗИОДНОРОДНОЙ СТАРШЕЙ ЧАСТЬЮ

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Abstract. The book gives a uniform exposition of the mixed problem for hyperbolic and $2b$-parabolic differential equations with variable coefficients. The method makes it possible to consider simultaneously a nonclassical type of equations, q-hyperbolic equations.

The book is intended for researchers and graduate students working in partial differential equations.

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Preface

The prime objective of the present book is systematic presentation of the theory of mixed problem for hyperbolic differential equations with variable coefficients. In a sense, this theory completes the classical line of investigation in partial differential equations which began with studying the Cauchy problem and mixed problem for the wave equation (for related historical remarks see the survey by Gindikin and Volevich [**GV8**]). The main stages in its construction were the introduction by Petrovskii in the thirties of high-order hyperbolic equations (systems) generalizing the wave equation, Petrovskii's and Leray's investigations of the Cauchy problem for hyperbolic equations of arbitrary order with variable coefficients, and, finally, the study of mixed problem for hyperbolic equations of arbitrary order (first-order systems), which took several decades and was completed by Agmon, Kreiss, and Sakamoto. In our opinion, the most adequate approach to the proof of solvability of hyperbolic problems is the method using the "separating" operator and the related definite quadratic form. This method was suggested in the early fifties by Leray, who introduced energy quadratic forms for high-order hyperbolic equations with variable coefficients proceeding from the law of conservation of energy for a membrane (its motion is described by the wave equation). Gårding constructed an orderly theory of solvability for the Cauchy problem by combining energy estimates and methods of functional analysis. Sakamoto substantially developed the Leray-Gårding approach in application to the mixed problem. The fundamental idea of Leray is that in the case of a strictly hyperbolic operator (with simple real characteristics) the corresponding energy form is (in modern terminology) an elliptic form with parameters, and this makes it possible to derive estimates in the case of variable coefficients. The presentation of the energy method for hyperbolic equations constitutes a major part of this book.

This main investigation line is complemented with two other topics. First, it turns out that the energy method elaborated for hyperbolic equations can be extended almost automatically to parabolic equations; more precisely, to $2b$-parabolic differential equations with variable coefficients. This class of equations was also introduced by Petrovskii as a generalization of the heat equation to the case of high-order equations (systems). For these equations, the mixed problem as well as the Cauchy problem is traditionally investigated with the aid of some adequate estimates and some form of Levi's parametrix. Second, it turns out that the energy method makes it possible to investigate completely the solvability of the Cauchy problem and mixed problem for parabolic equations as well. The only natural change that should be made in all the constructions is to replace the ordinary principal homogeneous parts of operators by principal quasihomogeneous parts that are obtained if the time derivatives are taken with weight $2b$. This is a general situation in the theory of parabolic equations (for the heat equation we have $b = 1$). What has been said accounts for the term "quasihomogeneous principal part" in the title of the book. Moreover, within the framework of the energy estimate method the theories of hyperbolic and parabolic operators can be combined as a unified theory of operators with dominant principal quasihomogeneous part. The latter operators are characterized by the property that the correctness (well-posedness) conditions of the Cauchy problem and mixed problem are stated for them only in terms of principal quasihomogeneous parts of equations and boundary operators, and therefore they are stable with respect to the perturbations of the problem by arbitrary lower degree terms.

However, there is another possible extension of the class of operators in question, and the main innovation of the suggested presentation is related to it. Namely, we study a new class of the so-called $(2b+1)$-hyperbolic operators. In this case the weight $2b+1$ is assigned to the operator of time differentiation. In contrast to parabolic and hyperbolic operators, the $(2b+1)$-hyperbolic operators have no classical prototypes. Nevertheless, a simple and natural example of an equation corresponding to $b = 1$ can be given:

$$\pm\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x_1^3} + \cdots + \frac{\partial^3 u}{\partial x_n^3} = 0.$$

In the case of one spatial variable ($n = 1$) this equation coincides with the linear part of the Korteweg-de Vries (KdV) equation, and the linear parts of some multi-dimensional KdV analogs are also 3-hyperbolic operators. For $(2b+1)$-hyperbolic equations with variable coefficients it is also possible to construct an equally complete theory for the Cauchy problem and the mixed problem using the method of energy estimates. Finally, all three classes of operators under consideration can be combined to form the class of operators with dominant principal quasihomogeneous part. From the onset, the presentation could have been given for this general class of operators, which, however, would have been very cumbersome, and therefore we preferred to work with special classes and to present the unified result only in the end.

The book consists of four chapters, each starting with a detailed introduction. The first chapter compiles all the necessary material related to polynomials that in the subsequent chapters serve as symbols for all the three above-mentioned classes of operators. Properties of these polynomials (including their description which makes it possible to combine them into a unified class) that are necessary for constructing energy estimates are studied in detail. The second chapter is devoted to the Cauchy problem which is a constituent of the mixed problem. However, we consider the Cauchy problem separately because the theory of solvability for this problem is of interest by itself, and this facilitates the presentation in the last two most difficult chapters devoted to the mixed problem. It should be noted that we do not touch upon all aspects of the Cauchy problem and consider only those related to the mixed problem. For instance, we do not practically discuss the possibility of extending the correctness theory for the Cauchy problem to larger classes of operators with both variable and constant coefficients and do not investigate the exact exponential classes of correctness for the Cauchy problem either. For detailed presentation of these questions see the books and survey by Gindikin and Volevich **[GV1, 2, 8]** and **[Gi3]**.

In Chapter II we first reformulate the results of Chapter I related to polynomials in the form of estimates for the corresponding differential operators with constant coefficients and then describe the general scheme of studying the Cauchy problem for equations with variable coefficients using energy estimates and specialize this scheme in application to hyperbolic, $2b$-parabolic, and $(2b+1)$-hyperbolic equations. After that a unified theorem is presented. Chapter II is completed by presenting the theory of the Cauchy problem for three classes of systems generalizing the classes of scalar operators considered in the book. Unfortunately, neither a satisfactory theory of the mixed problem for high-order hyperbolic systems nor a similar theory for $(2b+1)$-hyperbolic systems has yet been created, and therefore the questions of solvability of the mixed problem for systems of equations are beyond the scope of the present book.

Chapters III and IV are devoted to the mixed problem proper, and the presentation of the material in them is in many respects "in parallel" with that in Chapter II. Chapter III considers systematically a more difficult case of hyperbolic equations, and the last comparatively small chapter compiles some additional facts making it possible to consider the mixed problem for $2b$-parabolic and $(2b+1)$-hyperbolic equations and to derive the general result unifying all the three above-mentioned theories. We note that the combined presentation of the three independent theories for mixed problem and the Cauchy problem makes it necessary to state a large number of "similar" assertions. As a rule, a thorough proof is carried out only once, say, for hyperbolic equations, and in the other cases we dwell only on some specific details or (if these details are not of fundamental importance) leave the proofs to the reader.

Sections in each chapter and formulas in each section are numbered autonomously; the subsections have double numbering (for example, 2.1 means 1 of §2). Within the subsections autonomous numbering of theorems, propositions, lemmas, etc. is used. If in a subsection there is only one theorem, proposition, lemma, etc., it has no number. When referring to the material in another subsection of the given section we write only the number of the subsection: for example, Proposition 3.1. If the given subsection contains several propositions, we write, for instance, Proposition 1, 2, etc. of 3.1. When mentioning formulas belonging to other sections we first indicate the number of the corresponding section: for example, (3.20) means formula (20) in §3. When referring to the material in other chapters we write the number of the chapter before the numbers of the sections, subsections, and formulas. For instance, §IV.2, IV.2.1, and (IV.2.7) mean, respectively, §2 of Chapter IV, subsection 1 in §2 of Chapter IV, and formula (7) in §2 of Chapter IV.

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