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Asymptotic Methods in the Theory of Gaussian Processes and Fields

Vladimir I. Piterbarg

American Mathematical Society

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Asymptotic Methods in the Theory of Gaussian Processes and Fields

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В. И. Питербарг

АСИМПТОТИЧЕСКИЕ МЕТОДЫ В ТЕОРИИ ГАУССОВСКИХ СЛУЧАЙНЫХ ПРОЦЕССОВ И ПОЛЕЙ

ИЗДАТЕЛЬСТВО МОСКОВСКОГО УНИВЕРСИТЕТА, 1988

Translated by V. V. Piterbarg
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ABSTRACT.
The book is devoted to systematic analysis of asymptotic behavior of distributions for various typical functionals of Gaussian random variables and fields. For a large class of Gaussian and similar processes, good approximate formulas and sharp estimates of the remainders are obtained. Special attention is paid to the development of asymptotic analysis methods allowing the reader to describe asymptotic behavior of various functionals.
The book is useful to researchers and graduate students working in probability, mathematical statistics, and random processes.

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Contents

Preface to the Russian Edition

Analysis of the limiting behavior of distributions of various functionals defined on trajectories of stochastic processes and fields is one of the central areas in the theory of stochastic processes. Asymptotic analysis of distributions of functionals of general type processes has originated in fundamental articles by Ibragimov [**35**], Rozanov [**86**], Volkonskij and Rozanov [**98**], Rozenblatt [**85**]. They were considering problems which look especially attractive for Gaussian processes. It is so because natural characteristics, such as the expected value and the covariance, can be used in formulation of variants of general limit theorems. For instance, it is possible to replace general conditions of mixing, appearing in various limit theorems, by conditions on the behavior of spectral or covariance functions which are easier to work with. As a result, the behavior of the distribution of the maximum of a Gaussian process can be analyzed in considerably more detail. The role of Gaussian distribution in the probability theory and its applications is well known, as it is also well known how naturally and simply Gaussian distributions are described. So it is no wonder that the theory of Gaussian stochastic processes has been enjoying such remarkable attention during the last few decades.

A spectacular example of the interaction between general theorems and their "Gaussian" analogs is the theory of local characteristics of trajectories of stochastic processes. It all began back in 1949, when A. N. Kolmogorov obtained sufficient conditions on continuity of a stochastic process. Having done that, A. N. Kolmogorov formulated a problem of obtaining necessary and sufficient conditions for the trajectories of a Gaussian process to be continuous with probability one *in terms of its covariance or spectrum*. A large group of mathematicians had been attacking this problem ever since the first work on this subject by Yu. K. Belyaev, in which he studied analytic processes appeared in 1959. This group included V. N. Sudakov, R. Dudley, X. Fernique, and others. The problem was finally solved and powerful methods of studying Gaussian distributions were developed as a by-product. These methods contributed not only to the problem itself, but rather to the theory in whole.

Another example of such interaction, the analysis of asymptotic properties of functionals of supremum type, number-of-crossings type, and several other types, is presented in this book. The first "Gaussian" result obtained after the above-mentioned paper by V. A. Volkonskij and Yu. A. Rozanov was a Poisson limit theorem for the number of high excursions without the condition of strong mixing. Yu. K. Belyaev and H. Cramer were the first to prove it. Follow-up results in this and related directions were obtained by M. P. Leadbetter, V. P. Nosko, J. Pickands, G. Lingren, S. Berman, and several others including the author of this book. A fairly complete literature review on this subject can be found in articles **[72]** and **[63]**.

Even though this subject has constantly been in the area of interests of many mathematicians and researchers who use the theory of stochastic processes in applications, a significant number of natural and principal questions is still unanswered. What is the broadest class of Gaussian processes and fields for which the exact asymptotic behavior of $\mathbf{P}(\sup X(t) > u)$ as $u \to \infty$ can be found? What are correction terms for this asymptotic in u, and also for the asymptotic behavior in the case of processes approximating Gaussian in a array scheme? What is the rate of convergence in the limit theorem for the maximum of a trajectory of a Gaussian process or field? How correction terms should be chosen in a array scheme of convergence to a Gaussian process? (The last two questions, in particular, are important for a non-parametric construction of confidence domains for distribution densities and for surfaces of regression.) Is it possible to find necessary and sufficient conditions in a Poisson limit theorem for high excursions of Gaussian processes? What correction terms do we have there? How can we associate conditions of mixing with a natural for Gaussian process conditions on the rate of decreasing of its covariance function?

New methods of asymptotical analysis of Gaussian processes and fields are systematically developed in this book. These methods allow us to obtain almost complete solutions to several basic problems (including those listed above) which concern the analysis of the limiting behavior of supremum type and number-of-crossings type functionals of Gaussian process and field trajectories.

Even though this book is targeted at mathematicians, one can find a large number of asymptotic formulas inside, useful in mathematical statistics applications, reliability theory, theory of rough surfaces, other areas of applications of stochastic processes.

All results presented in the book have been thoroughly discussed at the seminar "Selected problems of the theory of Gaussian processes and fields", which has been running in Moscow State University for many years. It is my pleasure to thank all participants of this seminar. Some of the participants are also co-authors of results presented here. Special thanks go to the seminar's head, my teacher Yurii Konstantinovich Belyaev. It is to a large extent his responsibility that this book has has been written.

Let us comment on some notations.[1] Letters L and C, with sub-, super-scripts or without, usually denote the constants exact form which is of no importance to us. Different constants may look alike for that matter. The object of analysis is a random element $X(t) = X(t, \omega)$ defined on $T \times \Omega$, where $(\Omega, \mathcal{U}, \mathbf{P})$ is our primary probabilistic space, (T, ρ) is a parametric metric space, usually a subset of $\mathbb{R}^n$ or $\mathbb{Z}^n$. Conditions imposed on the distribution of $X(t)$ always guarantee the existence of a version with almost surely (a.s.) continuous trajectories, and this version is always considered.

[1] See also the Preface to the English edition.

Preface to the English Edition

I was pleased to find out that the American Mathematical Society had decided to publish an English variant of my book. But it also reminded me that serious revision of the book was in order. Two reasons for that deserve to be mentioned. First, the general theory of Gaussian processes, as well as some of the aspects of the book itself, has undertaken significant changes from the moment of appearance of the original variant of the book, and I could not keep these changes out of the book. Second, the book was written as a "large article" with proofs comprehensible only by a small number of specialists. This style was deemed unsuitable by the publisher of the translation, as well as by me at the time we started talking about this project. Out of this discussion a new "one-dimensional" chapter, the Introduction, has emerged. This chapter was designed to be accessible by a wide audience, including upper-division students in mathematics. And, hopefully, it is. So, there are different strategies in attacking this, not at all easy, book. One reader can go through the Introduction alone, and then refer to the results in the main body with some understanding of where they basically come from. Another reader may be dissatisfied with incomplete proofs in the Introduction and read the whole book. The reader will be rewarded by a deeper understanding of the theory and will find a lot of interesting ideas not presented in the Introduction. Besides, if a multi-dimensional case were just a technical generalization of the one-dimensional one, it would not be worth publishing anyway.

Furthermore, the method of double sum was in its infancy when the book first appeared in Russian, even though potential advantages of this method were pretty much understood. By now, a lot of new results have been gathered, and the understanding of the method has significantly improved. It all forced me to rewrite the corresponding chapter completely, even though I tried to keep factual material (the results) as unchanged as possible. The remaining chapters were touched to a smaller degree. The results were just left unchanged, because even today they are unbeaten, and their proofs have been available only in Russian. In particular, it is true for the asymptotic integral-geometric expansion of the probability of high excursion of a stationary Gaussian field, for necessary and sufficient conditions in a Poisson limit theorem for large deviations of arbitrary Gaussian functions in discrete time, for the Rice method in a central limit theorem for sums of stochastic processes, and so on.

Other changes include additional titles in the bibliography; however, we keep intact the idea of having only the sources to which proofs refer, or with original proofs. Rare exceptions were made for titles with extensive bibliographies.

It is my pleasure to thank the American Mathematical Society and, in particular, S. I. Gelfand for his extensive help and remarks on the general plan of the translation. My thanks also go to the translator, V. V. Piterbarg, whose job was not made any easier by me making extensive revisions of the book, even though the electronic means of communication between Russia and America worked surprisingly well. I am grateful to V. Fatalov for his help in preparing the second chapter. I welcome your remarks and suggestions, in particular on the chapter "The Method of Double Sum", which is growing into a separate book.

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