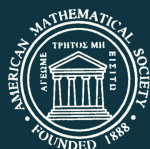


Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 156

**Qualitative Topics
in Integer Linear
Programming**

V. N. Shevchenko




American Mathematical Society

Selected Titles in This Series

- 156 **V. N. Shevchenko**, Qualitative topics in integer linear programming, 1996
- 155 **Yu. Safarov and D. Vassiliev**, The asymptotic distribution of eigenvalues of partial differential operators, 1996
- 154 **V. V. Prasolov and A. B. Sossinsky**, Knots, links, braids and 3-manifolds. An introduction to the new invariants in low-dimensional topology, 1996
- 153 **S. Kh. Aranson, G. R. Belitsky, and E. V. Zhuzhoma**, Introduction to the qualitative theory of dynamical systems on surfaces, 1996
- 152 **R. S. Ismagilov**, Representations of infinite-dimensional groups, 1996
- 151 **S. Yu. Slavyanov**, Asymptotic solutions of the one-dimensional Schrödinger equation, 1996
- 150 **B. Ya. Levin**, Lectures on entire functions, 1996
- 149 **Takashi Sakai**, Riemannian geometry, 1996
- 148 **Vladimir I. Piterbarg**, Asymptotic methods in the theory of Gaussian processes and fields, 1996
- 147 **S. G. Gindikin and L. R. Volevich**, Mixed problem for partial differential equations with quasihomogeneous principal part, 1996
- 146 **L. Ya. Adrianova**, Introduction to linear systems of differential equations, 1995
- 145 **A. N. Andrianov and V. G. Zhuravlev**, Modular forms and Hecke operators, 1995
- 144 **O. V. Troshkin**, Nontraditional methods in mathematical hydrodynamics, 1995
- 143 **V. A. Malyshev and R. A. Minlos**, Linear infinite-particle operators, 1995
- 142 **N. V. Krylov**, Introduction to the theory of diffusion processes, 1995
- 141 **A. A. Davydov**, Qualitative theory of control systems, 1994
- 140 **Aizik I. Volpert, Vitaly A. Volpert, and Vladimir A. Volpert**, Traveling wave solutions of parabolic systems, 1994
- 139 **I. V. Skrypnik**, Methods for analysis of nonlinear elliptic boundary value problems, 1994
- 138 **Yu. P. Razmyslov**, Identities of algebras and their representations, 1994
- 137 **F. I. Karpelevich and A. Ya. Kreinin**, Heavy traffic limits for multiphase queues, 1994
- 136 **Masayoshi Miyanishi**, Algebraic geometry, 1994
- 135 **Masaru Takeuchi**, Modern spherical functions, 1994
- 134 **V. V. Prasolov**, Problems and theorems in linear algebra, 1994
- 133 **P. I. Naumkin and I. A. Shishmarev**, Nonlinear nonlocal equations in the theory of waves, 1994
- 132 **Hajime Urakawa**, Calculus of variations and harmonic maps, 1993
- 131 **V. V. Sharko**, Functions on manifolds: Algebraic and topological aspects, 1993
- 130 **V. V. Vershinin**, Cobordisms and spectral sequences, 1993
- 129 **Mitsuo Morimoto**, An introduction to Sato's hyperfunctions, 1993
- 128 **V. P. Orevkov**, Complexity of proofs and their transformations in axiomatic theories, 1993
- 127 **F. L. Zak**, Tangents and secants of algebraic varieties, 1993
- 126 **M. L. Agranovskii**, Invariant function spaces on homogeneous manifolds of Lie groups and applications, 1993
- 125 **Masayoshi Nagata**, Theory of commutative fields, 1993
- 124 **Masahisa Adachi**, Embeddings and immersions, 1993
- 123 **M. A. Akivis and B. A. Rosenfeld**, Élie Cartan (1869–1951), 1993
- 122 **Zhang Guan-Hou**, Theory of entire and meromorphic functions: Deficient and asymptotic values and singular directions, 1993
- 121 **I. B. Fesenko and S. V. Vostokov**, Local fields and their extensions: A constructive approach, 1993

(See the AMS catalog for earlier titles)

This page intentionally left blank



Qualitative Topics in Integer Linear Programming

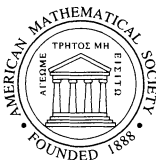
This page intentionally left blank

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 156

**Qualitative Topics
in Integer Linear
Programming**

V. N. Shevchenko



American Mathematical Society
Providence, Rhode Island

В. Н. ШЕВЧЕНКО

КАЧЕСТВЕННЫЕ ВОПРОСЫ
ЦЕЛОЧИСЛЕННОГО ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ

Translated by H. H. McFaden from an original Russian manuscript

EDITORIAL COMMITTEE

AMS Subcommittee

Robert D. MacPherson

Grigorii A. Margulis

James D. Stasheff (Chair)

ASL Subcommittee Steffen Lempp (Chair)

IMS Subcommittee Mark I. Freidlin (Chair)

1991 *Mathematics Subject Classification*. Primary 90-02, 90C10;

Secondary 90C05, 90C08, 90C11, 68Q25, 90C90, 90C60.

ABSTRACT. Integer solutions are considered for systems of linear inequalities, equations, and congruences, along with the construction and theoretical analysis of integer programming algorithms. The complexity of algorithms is analyzed in dependence on two parameters: the dimension, and the maximal modulus of the coefficients describing the conditions of the problem. The analysis is based on a thorough treatment of the qualitative and quantitative aspects of integer programming, in particular, on bounds obtained by the author for the number of extreme points. This makes progress possible in many cases when the traditional approach, in which complexity is regarded as a function only of the length of the input, leads to a negative result. The book is intended for mathematicians and other specialists in the theory and applications of discrete optimization, as well as for students and post-graduates specializing in mathematical cybernetics and information science.

Library of Congress Cataloging-in-Publication Data

Shevchenko, V. N. (Valery N.)

[Kachestvennye voprosy tselochislennogo linejnogo programmirovaniâ. English]

Qualitative topics in integer linear programming / V. N. Shevchenko.

p. cm. — (Translations of mathematical monographs ; v. 156)

Includes bibliographical references.

ISBN 0-8218-0535-5 (alk. paper)

1. Linear programming. I. Title. II. Series.

T57.74.S54213 1996

519.7'7—dc20

96-32702

CIP

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Assistant to the Publisher, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@ams.org.

© 1997 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

10 9 8 7 6 5 4 3 2 1 02 01 00 99 98 97

Contents

Notation	ix
Foreword	xi
Chapter 1. Intersection of a Convex Polyhedral Cone with the Integer Lattice	1
§1.1. Finite generation of the intersection of a convex polyhedral cone with a sublattice of the integer lattice	1
§1.2. Decomposition of a cone into unimodular cones	5
§1.3. Algebraic characterization of a semigroup generated by a finite set of vectors	10
§1.4. On conditions for consistency of a system of linear equations in nonnegative integers	12
§1.5. Quantitative aspect of Carathéodory's theorem	17
Chapter 2. A Discrete Analogue of the Farkas Theorem, and the Problem of Aggregation of a System of Linear Integer Equations	25
§2.1. A discrete analogue of the Farkas theorem	25
§2.2. Aggregation of systems of linear equations	27
§2.3. On the growth of the coefficients of an aggregating equation	29
§2.4. Other approaches to aggregation	33
Chapter 3. Intersection of a Convex Polyhedral Set with the Integer Lattice	35
§3.1. The convex hull of the set of integer solutions of a system of linear equations and inequalities	35
§3.2. Determination of upper bounds on the number of extreme points	38
§3.3. Cramer systems of linear inequalities, and a linear equation on an Abelian group	42
§3.4. The polyhedron of the knapsack problem	47
§3.5. On the attainability of upper bounds for the number of extreme points	49
§3.6. Determination of upper bounds independent of the right-hand sides for the number of vertices	53
§3.7. Bounds on the number of vertices in the mixed integer case	58
Chapter 4. Cut Methods in Integer Programming	61
§4.1. Systems of congruences and regular cuts	61
§4.2. Classification of cut methods	66
§4.3. Algorithms for solving the elementary ILPP	70
§4.4. The mean value of the minors of certain matrices	75

Chapter 5. Complexity Questions in Integer Linear Programming	79
§5.1. NP-completeness of the knapsack problem and problems close to it	79
§5.2. Pseudopolynomial algorithms and quasipolynomial algorithms	81
§5.3. Determination of the successive minima of an integer lattice and of a lattice vector closest to a given one	83
§5.4. Construction and use of a c -reduced basis	88
Appendix 1. Solution of Systems of Linear Equations and Congruences in Integers	97
Appendix 2. Examples of Applied Problems Related to the Topic of the Book	101
Appendix 3. Investigation of Minor and Permanent Characteristics of Certain Boolean Matrices	109
Appendix 4. Threshold Functions of Many-Valued Logic and Their Deciphering	125
Bibliography	133

Notation

\mathbf{R}	the field of real numbers,
\mathbf{Q}	the field of rational numbers,
\mathbf{Z}	the ring of integers,
P^n	the set of n -dimensional arithmetic vectors with components in P ,
$P^{m \times n}$	the set of matrices with m rows and n columns with elements in P ,
A_+	the set of nonnegative elements of a set A ,
$ A $	the cardinality of a set A ,
$\text{Co } A$	the convex hull of a subset A of \mathbf{R}^n ,
$\dim A$	the dimension of a set $A \subset \mathbf{R}^n$, that is, the maximal number of linearly independent vectors in A ,
$\lceil \alpha \rceil$	the maximal integer not exceeding the number α ,
$\lfloor \alpha \rfloor$	the minimal integer not less than α ,
$\{\alpha\} = \alpha - \lceil \alpha \rceil$	the fractional part of α ,
$\lceil \alpha \rceil$	the integer closest to α : $\lceil \alpha \rceil = \lceil \alpha \rceil$ if $\alpha - \lceil \alpha \rceil \leq 1/2$, and $\lceil \alpha \rceil = \lfloor \alpha \rfloor$ otherwise,
$\text{res}_a b$	the residual after division of $b \in \mathbf{Z}$ by $a \in \mathbf{Z}$, $a \neq 0$,
$\log \alpha$	the logarithm of α to the base 2,
$x \equiv y \pmod{d}$	means for nonzero $d \in \mathbf{Z}$ and $x, y \in \mathbf{Z}^n$ that each component of the vector $x - y$ is divisible by d ,
$\ x\ $	the Euclidean norm of a vector x ,
A^\top	the transpose of a matrix A ,
$\text{rk } A$	the rank of a matrix A ,
e_j	the vector in \mathbf{Z}^n with j th coordinate 1 and the other coordinates 0 ($j = 1, \dots, n$),
$\binom{m}{n}$	the number $\frac{m!}{n!(m-n)!}$ of combinations of m things, taken n at a time.

This page intentionally left blank

Foreword

The main object of investigation in this book is the set M of integer solutions of a system of linear inequalities, equations, or congruences, and its convex hull $\text{Co } M$. Such mathematical objects arise in integer programming, which is the sufficiently universal language of discrete optimization, an area with a large sphere of applications in mathematical cybernetics, economics, and other disciplines.

We pose the following problems: 1) determine whether M is nonempty; 2) give a description of $\text{Co } M$, that is, find the set of its extreme points and (or) a list of its faces; 3) solve the integer linear programming problem (ILPP), that is, find in M a point x at which a specified linear function attains a maximum; 4) estimate the complexity of algorithms giving answers in the preceding cases.

It is presently common to estimate the complexity as a function of the length l of the input, the function being interpreted as the number of binary digits required to store all the input parameters of the problem. In our case these are the coefficients a_{ij} on the left- and right-hand sides of the system describing the set M . We assume them to be integers unless otherwise stated, and then $l = \sum [2 + \log |a_{ij}|]$, where the sum is over all coefficients. It is common to regard an algorithm as efficient if its complexity is bounded above by a polynomial in l (then one says that it is a polynomial algorithm, and that the problem solved by it is polynomially solvable).

No polynomial algorithms are known at present for even the first of the problems posed. Moreover, if we accept the well-known hypothesis that $P \neq NP$, where P and NP are the classes of problems polynomially solvable on a determinate and indeterminate Turing machine, respectively (precise definitions and an exposition of these questions can be found, for example, in [50]), then there are no such algorithms.

The problem P_1 of determining whether a system of linear equations in non-negative integers is consistent is a universal or NP-complete problem, that is, any problem P_0 in the class NP can be polynomially reduced to it, and hence the polynomial solvability of the problem P_1 would mean the existence of a polynomial algorithm also for any problem P_0 .

The approach regarding the complexity as a function only of the single parameter l is apparently too general. In integer linear programming it is expedient to consider two parameters, the dimension n and $\alpha = \max |a_{ij}|$, which it is natural to take as guides for distinguishing efficiently (polynomially) solvable subclasses of problems. It is known that polynomial algorithms for the problem P_1 exist for the two subclasses described by the respective conditions: a) n increases without bound, while α and the number m of equations are arbitrary but fixed; b) n is fixed, while α increases without bound. The first of these is easy to get by the dynamic

programming method, and the second was obtained by Lenstra; it is the latter that will receive most of our attention.

The foregoing gives the reader some idea of what this book covers. To make this idea more concrete we go through the contents by chapters.

In Chapter 1 we consider the intersection of a convex polyhedral cone with the integer lattice \mathbf{Z}^n and prove that it is finitely generated. This provides a theoretical basis for the subsequent constructions. Here we pose the question of conditions on the right-hand side of a system of linear equations that are necessary and sufficient for the system to be consistent in the nonnegative integers \mathbf{Z}_+ .

In Chapter 2 we solve the problem posed in Chapter 1 by presenting a discrete analogue of the Farkas theorem and solving the closely related problem of aggregation of a system of linear equations $Ax = b$. The latter problem consists in finding a linear equation whose set of nonnegative integer solutions coincides with the set $\mathfrak{M}(A, b)$ of nonnegative integer solutions of the original system. We derive necessary and sufficient conditions for the existence of such an equation, and we prove that its coefficients must grow exponentially with respect to m .

The central part of the book is Chapter 3, in which we consider the intersection M of a polyhedron with the integer lattice \mathbf{Z}^n or with a sublattice of it. For various ways of specifying the polyhedron we prove that M and its convex hull $\text{Co } M$ are finitely generated, and we present bounds for the number of vectors in generating sets and their components. On the basis of the properties obtained for the set N of extreme points of $\text{Co } M$ we then give a method for obtaining upper bounds for their number $|N|$. In particular, it is shown that $|N|$ is bounded above by a polynomial in $\log \alpha$ for any fixed dimension n , that is, $|N|$ is essentially less than $|M|$, and this difference can be increasingly significant, the larger α is. For some important special cases, for example, for the knapsack problem, it is shown that our upper bounds cannot be improved with respect to order, and examples are given in which $|N|$ grows exponentially with n .

In Chapter 4 we use the results of Chapter 1 on decomposition of cones to analyze the existing cut methods in integer programming from a unified point of view, and we present new ones. We reveal the role of the asymptotic or elementary ILPP (in which the constraints are given by a square system of linear inequalities $Az \leq b$, where $\Delta = |\det A| \neq 0$), which is equivalent to the group minimization problem (GMP; introduced by Gomory) consisting in the minimization of a linear function on the set of nonnegative integer solutions of the equation $\sum_{j=1}^n g_j x_j = g_0$, where the g_j are elements of an additive Abelian group of order Δ . The equivalence of these two problems allows methods for solving each of them to be carried over to the other, and gives two natural parameters n and Δ as a function of which the complexity of various algorithms can be estimated. In this connection we investigate the mean values of the minors of various classes of matrices.

Complexity questions of linear programming are examined in Chapter 5. In particular, it is shown that the asymptotic ILPP and the knapsack problem (more precisely, the problem P_1 for $m = 1$) are NP-complete. An algorithm that is polynomial for a fixed dimension is constructed for finding all the extreme points and faces of $\text{Co } \mathfrak{M}(A, b)$. Analogous results are presented also for the case when the polyhedron is described by a system of inequalities if their number is bounded by a polynomial in n . It is shown that if the growth of n is bounded above by a quantity proportional to $(\log \log \alpha)^{1/2}$, then the polynomial property of all the estimates

is preserved. It is also shown that all the estimates can be made independent of the right-hand sides. These results enable us to distinguish new subclasses of polynomially solvable ILPP's.

The appendices contain some facts about solving systems of linear equations and congruences in integers (references to such assertions in Appendix 1 are given by the letter A with a corresponding number), a combinatorial-statistical analysis of determinants and permanents of certain (0-1)-matrices, some functions of many-valued logic connected with the ILPP (in particular, a solution of the problem of deciphering threshold functions), models of certain practical problems reducing to the ILPP, and methods for solving them.

Since 1970 the author has been head of the Department of Mathematical Logic and Higher Algebra at Nizhni Novgorod State University and has worked on integer programming problems. In this monograph broad use is made of results obtained by the author and his students and colleagues. The author cannot, of course, claim completeness for his presentation of the topics touched upon here, and the bibliography reflects his tastes and, to a greater extent, his opportunities. In particular, the list of publications contains a series of auxiliary textbooks for graduate courses given yearly by the author for students in the Department of Computational Mathematics and Cybernetics.

The author is grateful to his department colleagues S. I. Veselov, A. P. Il'ichev, and A. Yu. Chirkov, who took part in the writing of some sections of the book, as well as to V. A. Talanov for useful discussions, and to M. Yu. Preobrazhenskaya and I. V. Suslova for the technical preparation of the manuscript for printing at the publishing house "Nauka" in 1991. Unfortunately, well-known changes in our country hindered publication of the book, which has become possible only thanks to the support of the Russian Foundation for Fundamental Research, to which the author expresses his gratitude.

The insertions in the text reflecting recently obtained results necessitated an extra list of references, beginning with [333].

This page intentionally left blank

This page intentionally left blank

Bibliography

1. N. N. Aizenberg and Yu. L. Ivas'kiv, *Many-valued threshold logic*, "Naukova Dumka", Kiev, 1977. (Russian)
2. V. B. Alekseev, *On deciphering of certain classes of monotone many-valued functions*, Zh. Vychisl. Mat. i Mat. Fiz. **16** (1976), 189–198; English transl. in USSR Comput. Math. and Math. Phys. **16** (1976).
3. T. T. Arkhipova and I. V. Sergienko, *On conditions for the coincidence of local and global extrema in optimization problems*, Kibernetika (Kiev) **1975**, no. 1, 13–15; English transl. in Cybernetics **11** (1975).
4. L. G. Babat, *On integer linear aggregation*, Èkonom. i Mat. Metody **13** (1977), 599–601. (Russian)
5. E. Balas, *An additive algorithm for solving linear programs with zero-one variables*, Oper. Res. **13** (1965), 517–546.
6. Robert G. Busacker and Thomas L. Saaty, *Finite graphs and networks: An introduction with applications*, McGraw-Hill, New York, 1965.
7. E. G. Belousov, *Introduction to convex analysis and integer programming*, Izdat. Moskov. Gos. Univ., Moscow, 1977. (Russian)
8. Claude Berge, *Théorie des graphes et ses applications*, Collection Univ. Math., II, Dunod, Paris, 1958; English transl., Wiley, New York, 1962.
9. V. N. Bol'shakov, *On the spectrum of a permanent on Λ_n^k* , Materials All-Union Sem. Discrete Mathematics and its Applications, Izdat. Moskov. Gos. Univ., Moscow, 1986, pp. 65–73. (Russian)
10. P. L. Buzyskiĭ and A. A. Votyakov, *A direct algorithm for integer programming*, Investigations in Discrete Optimization, "Nauka", Moscow, 1976, pp. 68–88. (Russian)
11. A. A. Bukhshtab, *Number theory*, "Prosveshchenie", Moscow, 1966. (Russian)
12. B. L. van der Waerden, *Algebra*, Vols. I (7th ed.), II (5th ed.), Ungar, New York, 1970.
13. I. V. Vengerova and Yu. Yu. Finkel'shteĭn, *On the efficiency of the branch and bound method*, Èkonom. i Mat. Metody **11** (1975), 186–193. (Russian)
14. B. S. Verkhovskii, *On a multi-index transportation problem with axial sums*, Dokl. Akad. Nauk SSSR **156** (1964), 282–285; English transl. in Soviet Math. Dokl. **5** (1964).
15. A. M. Vershik and P. V. Sporyshev, *A bound for the mean number of steps of the simplex method, and problems of asymptotic integral geometry*, Dokl. Akad. Nauk SSSR **271** (1983), 1044–1048; English transl. in Soviet Math. Dokl. **28** (1983).
16. S. I. Veselov, *On the number of extreme points of the master polyhedron in the group minimization problem*, Kibernetika (Kiev) **1981**, no. 6, 137; English transl. in Cybernetics **17** (1981).
17. ———, *A lower bound for the mean number of irreducible and extreme points in two discrete programming problems*, Manuscript No. 619–84, deposited at VINITI, Moscow, 1984. (Russian)
18. ———, *A new way of reducing the general integer programming problem to the knapsack problem*, Manuscript No. 620–84, deposited at VINITI, Moscow, 1984. (Russian)
19. ———, *Construction of the set of feasible solutions in certain discrete programming problems*, Author's summary of Candidate's dissertation, Minsk, 1985. (Russian)
20. ———, *Aggregation of linear integer equations*, Kibernetika (Kiev) **1985**, no. 4, 58–60; English transl. in Cybernetics **21** (1985).
21. ———, *Determination of the convex hull of the integer points of a polyhedron in the plane*, Manuscript No. 8624-B88, deposited at VINITI, Moscow, 1988. (Russian)

22. S. I. Veselov, I. E. Parubochii, and V. N. Shevchenko, *A program for finding the skeleton of the cone of nonnegative solutions of a system of linear inequalities*, Systems and Applied Programs, Part 2, Izdat. Gor'kov. Univ., Gorki, 1984, pp. 83–92. (Russian)
23. S. I. Veselov and A. Yu. Chirkov, *On the problem of integer programming with a bimodular matrix*, Combinatorial-Algebraic and Probabilistic Methods and their Applications, Izdat. Gor'kov. Univ., Gorki, 1990, pp. 107–110. (Russian)
24. S. I. Veselov and V. N. Shevchenko, *On the exponential growth of the coefficients of an aggregating equation*, Fourth All-Union Conf. on Problems of Theoretical Cybernetics, Abstracts of Reports, Novosibirsk, 1977, pp. 51–52. (Russian)
25. ———, *On the exponential growth of the coefficients of an aggregating equation*, Kibernetika (Kiev) **1978**, no. 4, 78–79; English transl. in Cybernetics **14** (1978).
26. ———, *On the number of extreme points of a quadratic system of linear inequalities*, Manuscript No. 450–79, deposited at VINITI, Moscow, 1978. (Russian)
27. ———, *Some consequences of the Minkowski theorem on convex bodies, and aggregation of linear Diophantine equations*, Fifth All-Union Conf. on Problems of Theoretical Cybernetics, Abstracts of Reports, Novosibirsk, 1980, pp. 47–49. (Russian)
28. ———, *Bounds for the maximal distance between the points of certain integer lattices*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1980, pp. 26–33. (Russian)
29. ———, *On faces and extreme points of discrete programming problems*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1981, pp. 39–49. (Russian)
30. ———, *On the complexity of the determination of an integer point in a polyhedron*, Third All-Union School in Discrete Optimization and Computers, Abstracts of Reports, Izdat. Tsentral. Èkonom.-Mat. Inst. Akad. Nauk SSSR, Moscow, 1987, p. 19. (Russian)
31. I. M. Vinogradov, *Foundations of the theory of numbers*, 8th ed., “Nauka”, Moscow, 1972; English transl. of 6th ed., Pergamon Press, New York, 1955.
32. V. V. Voevodin and Yu. A. Kuznetsov, *Matrices and computations*, “Nauka”, Moscow, 1984. (Russian)
33. V. L. Volkovich and A. F. Voloshin, *On a general scheme for sequential analysis and elimination of variants*, Kibernetika (Kiev) **1978**, no. 5, 98–105; English transl. in Cybernetics **14** (1978).
34. A. A. Votyakov, *On a class of effectively solvable problems of integer linear programming*, First Conf. on Optimal Planning and Regulation of the National Economy, Abstracts of Reports, Moscow, 1971, pp. 86–92. (Russian)
35. ———, *On problems invariant with respect to z -roundoff*, Èkonom. i Mat. Metody **7** (1971), 259–264. (Russian)
36. ———, *Equivalent transformations of a linear integer problem*, Investigations in Discrete Mathematics, “Nauka”, Moscow, 1973, pp. 9–26. (Russian)
37. A. A. Votyakov and M. A. Frumkin, *An algorithm for finding the general integer solution of a system of linear equations*, Investigations in Discrete Optimization, “Nauka”, Moscow, 1976, pp. 128–140. (Russian)
38. F. R. Gantmakher, *The theory of matrices*, 2nd ed., “Nauka”, Moscow, 1966; English transl. of 1st ed., Vols. 1, 2, Chelsea, New York, 1959.
39. Ya. E. Garyaev, V. N. Shevchenko, and A. Yu. Chirkov, *On Lenstra's algorithm for reducing the basis of a lattice*, Manuscript No. 7165–85, deposited at VINITI, Moscow, 1985. (Russian)
40. G. V. Gens and E. V. Levner, *Discrete optimization problems and efficient approximation algorithms*, Izv. Akad. Nauk SSSR Tekhn. Kibernet. **1979**, no. 6, 84–92; English transl. in Engrg. Cybernetics **17** (1979).
41. ———, *Efficient approximation algorithms for combinatorial problems*, Preprint No. 52, Tsentral. Èkonom.-Mat. Inst. Akad. Nauk SSSR, Moscow, 1981. (Russian)
42. È. Kh. Gimadi, N. I. Glebov, and V. A. Perepelitsa, *Algorithms with bounds for discrete optimization problems*, Problemy Kibernet. vyp. 31 (1975), 35–42. (Russian)
43. S. Ginsburg, *The mathematical theory of context-free languages*, McGraw-Hill, New York, 1966.
44. N. I. Glebov, *On a class of problems of convex integer programming*, Upravlyaemye Sistemy vyp. 11 (1973), 38–42. (Russian)

45. ———, *On applicability of the method of coordinatewise descent to some problems of convex integer programming*, *Upravlyaemye Sistemy vvp.* 17 (1978), 52–59. (Russian)
46. E. G. Gol'shtein and D. B. Yudin, *New directions in linear programming*, "Sov. Radio", Moscow, 1966. (Russian)
47. V. P. Grishukhin, *An estimate of the complexity of the Balas algorithm*, *Mathematical Methods in the Solution of Economics Problems*, No. 3, "Nauka", Moscow, 1972, pp. 93–105. (Russian)
48. ———, *On the mean number of iterations of the Balas algorithm*, *Investigations in Discrete Mathematics*, "Nauka", Moscow, 1973, pp. 58–68. (Russian)
49. ———, *Efficiency of the branch and bound method in problems with Boolean variables*, *Investigations in Discrete Optimization*, "Nauka", Moscow, 1976, pp. 203–230. (Russian)
50. Michael R. Garey and David S. Johnson, *Computers and intractability*, Freeman, San Francisco, CA, 1979.
51. V. A. Danilov et al., *Mathematical analysis (functions, limits, quotients)*, Fizmatgiz, Moscow, 1961. (Russian)
52. George B. Dantzig, *Linear programming and extensions*, Princeton Univ. Press, Princeton, NJ, 1963.
53. M. Dertouzos, *Threshold logic: A synthesis approach*, M.I.T. Press, Cambridge, MA, 1965.
54. S. V. Yablonskii and O. B. Lupanov (eds.), *Discrete mathematics and mathematical questions of cybernetics*, Vol. 1, "Nauka", Moscow, 1974; German transl., Birkhäuser, Basel, 1980.
55. V. A. Emelichev, *Discrete optimization. Sequential solution schemes*, *Kibernetika (Kiev)* **1971**, no. 6, 97–110; **1972**, no. 2, 109–121; English transl. in *Cybernetics* **7** (1971); **8** (1972).
56. V. A. Emelichev and M. M. Kovalev, *Polyhedral aspects of discrete optimization*, *Kibernetika (Kiev)* **1982**, no. 6, 54–62; English transl. in *Cybernetics* **18** (1982).
57. V. A. Emelichev, M. M. Kovalev, and M. K. Kravtsov, *Polytopes, graphs, optimization*, "Nauka", Moscow, 1981; English transl., Cambridge Univ. Press, Cambridge, 1984.
58. V. A. Emelichev and V. I. Komlik, *A method for constructing a sequence of plans for solving discrete optimization problems*, "Nauka", Moscow, 1981. (Russian)
59. V. A. Emelichev and V. G. Ovchinnikov, *On the theory of optimization on anticircuits having the Steinitz substitution property*, *Kibernetika (Kiev)* **1985**, no. 2, 55–58; English transl. in *Cybernetics* **21** (1985).
60. V. A. Emelichev, D. A. Suprunenko, and V. S. Tanaev, *On the work of Belorussian mathematicians in discrete optimization*, *Izv. Akad. Nauk SSSR Tekh. Kibernet.* **1982**, no. 6, 25–45; English transl. in *Engrg. Cybernetics* **20** (1982).
61. I. I. Eremin and N. N. Astaf'ev, *Introduction to the theory of linear and convex programming*, "Nauka", Moscow, 1976. (Russian)
62. Yu. I. Zhuravlev, *Set-theoretic methods in the algebra of logic*, *Problemy Kibernet.* 8 (1962), 5–44; English transl. in *Problems of Cybernetics* 8 (1962).
63. ———, *An estimate of the complexity of algorithms for constructing minimal disjunctive normal forms for functions of the algebra of logic*, *Diskret. Anal. vvp.* 3 (1964), 44–77. (Russian)
64. ———, *Local algorithms for computing information*, *Kibernetika (Kiev)* **1965**, no. 1, 12–19; **1966**, no. 2, 1–11; English transl. in *Cybernetics* **1** (1965); **2** (1966).
65. Yu. I. Zhuravlev and Yu. Yu. Finkel'shtein, *Local algorithms for integer linear programming problems*, *Problemy Kibernet.* vvp. 14 (1965), 289–295. (Russian)
66. ———, *The sphere of applications of discrete programming methods*, *The Use of Operations Research in Economics*, "Èkonomika", Moscow, 1977, pp. 26–69. (Russian)
67. Yu. A. Zuev, *Asymptotics of the logarithm of the number of threshold functions of the algebra of logic*, *Dokl. Akad. Nauk SSSR* **306** (1989), 528–530; English transl. in *Soviet Math. Dokl.* **39** (1989).
68. ———, *Combinatorial-probabilistic and geometric methods in threshold logic*, *Diskret. Mat.* **3** (1991), no. 2, 47–57; English transl. in *Discrete Math. Appl.* **3** (1991).
69. Yu. A. Zuev and V. N. Trishin, *On the connection between linear inequalities and monotone Boolean functions*, *Zh. Vychisl. Mat. i Mat. Fiz.* **24** (1984), 780–781; English transl. in *USSR Comput. Math. and Math. Phys.* **24** (1984).
70. N. N. Ivanov, *On a method of aggregation of the integer linear programming problem*, *Vesti Akad. Navuk BSSR Ser. Fiz.-Mat. Navuk* **1975**, no. 2, 96–98. (Russian)

71. ———, *Integer points of convex polyhedral sets*, Author's summary of Candidate's dissertation, Minsk, 1976. (Russian)
72. N. N. Ivanov and V. N. Shevchenko, *Structure of finitely generated semilattices*, Dokl. Akad. Nauk BSSR **19** (1975), 773–774. (Russian)
73. ———, *The inevitability of exponential growth upon aggregation*, Fifth Republican Conf. of Belorussian Mathematicians, Abstracts of Reports, Grodno, 1980, p. 56. (Russian)
74. A. P. Il'ichev, G. N. Oblezov, and V. N. Shevchenko, *The choice of a rational technological process for manufacturing components*, Software for Automation of a System of Plan Calculations, NIIÈ i ÈMMP pri Gosplane BSSR, Minsk, 1981, pp. 140–141. (Russian)
75. A. P. Il'ichev, M. A. Roshchina, and V. N. Shevchenko, *On the mean values of the minors of certain classes of Boolean matrices*, Seventh All-Union Conf. on Problems of Theoretical Cybernetics, Abstracts of Reports, Part 1, Irkutsk, 1985, p. 82. (Russian)
76. ———, *On the mean values of the minors of certain matrices*, Manuscript No. 2485-B86, deposited at VINITI, Moscow, 1986. (Russian)
77. A. P. Il'ichev and V. N. Shevchenko, *On the extreme points of polyhedra in multi-index transportation problems*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1981, pp. 66–72. (Russian)
78. R. M. Karp, *Reducibility of combinatorial problems*, Complexity of Computer Computations, Plenum Press, New York, 1972, pp. 85–103.
79. J. W. S. Cassels, *An introduction to the geometry of numbers*, Springer-Verlag, Berlin, 1959.
80. ———, *Rational quadratic forms*, Academic Press, New York, 1978.
81. N. N. Katerinokhina, *Searching for a maximal upper zero for a certain class of monotone functions of k -valued logic*, Zh. Vychisl. Mat. i Mat. Fiz. **21** (1981), 470–481; English transl. in USSR Comput. Math. and Math. Phys. **21** (1981).
82. V. F. Kiselev, S. I. Veselov, A. P. Il'ichev, and V. N. Shevchenko, *On a certain problem of optimal repair maintenance of radioelectronic equipment*, Tekhnika Sredstv Svyazi. Ser. Tekhnika Radiosvyazi vyp.9 (1983), 34–37. (Russian)
83. V. F. Kiselev and V. N. Shevchenko, *On an optimal assortment of radio facilities at a mobile communications center*, Tekhnika Sredstv Svyazi. Ser. Tekhnika Radiosvyazi vyp. 9 (1983), 38–41. (Russian)
84. A. N. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. II, Amer. Math. Soc., Providence, RI, 1967.
85. Donald E. Knuth, *The art of computer programming*, Vol. 2, Addison-Wesley, Reading, MA, 1969.
86. P. M. Knyazev, *The method of aggregation of systems of integer equations*, Manuscript No. 2305–80, deposited at VINITI, Moscow, 1980. (Russian)
87. M. M. Kovalev, *Discrete optimization (integer programming)*, Izdat. Belorus. Univ., Minsk, 1977. (Russian)
88. ———, *The method of partial orders*, Dokl. Akad. Nauk BSSR **24** (1980), 113–116. (Russian)
89. ———, *Matroids in discrete optimization*, Izdat. Belorus. Univ., Minsk, 1987. (Russian)
90. M. M. Kovalev and V. M. Kotov, *Analysis of the gradient method for solving the traveling salesman problem*, Zh. Vychisl. Mat. i Mat. Fiz. **21** (1981), 1035–1038; English transl. in USSR Comput. Math. and Math. Phys. **21** (1981).
91. M. M. Kovalev and A. V. Moshchanskiĭ, *Optimal search for extrema of convex functions on lattices*, Diskret. Mat. **2** (1990), no. 1, 130–141; English transl. in Discrete Math. Appl. **2** (1990).
92. ———, *Searching for a maximal upper zero of a threshold function of many-valued logic*, Zh. Vychisl. Mat. i Mat. Fiz. **30** (1990), 620–622; English transl. in USSR Comput. Math. and Math. Phys. **30** (1990).
93. L. N. Kozerskaya, T. T. Lebedeva, and I. V. Sergienko, *Stability questions. Parametric and post-optimal analysis of discrete optimization problems*, Kibernetika (Kiev) **1983**, no. 4, 83–92; English transl. in Cybernetics **19** (1983).
94. M. K. Kozlov, S. P. Tarasov, and L. G. Khachiyan, *Polynomial solvability of convex quadratic programming*, Dokl. Akad. Nauk SSSR **248** (1979), 1049–1051; English transl. in Soviet Math. Dokl. **20** (1979).
95. A. A. Kolokolov, *On the number of cutting planes in the first Gomory algorithm*, Problems of Analysis of Discrete Information, Part 1, Novosibirsk, 1975, pp. 84–96. (Russian)

96. ———, *Some algorithms and bounds in integer programming*, Author's summary of Candidate's dissertation, Novosibirsk, 1978. (Russian)
97. ———, *Regular cuts in solving integer optimization problems*, Upravlyaemye Sistemy vyp. 21 (1981), 18–25. (Russian)
98. ———, *A lower bound for the number of iterations for a certain class of cut algorithms*, Upravlyaemye Sistemy vyp. 23 (1983), 64–69. (Russian)
99. A. A. Korbut, U. Kh. Malkov, I. Kh. Sigal, and Yu. Yu. Finkel'shtein, *On the contemporary state and prospects of development of computational methods and programs for solving integer linear programming problems*, Optimal Decisions in Economic Systems, Izdat. Gor'kov. Univ., Gorki, 1982, pp. 3–30. (Russian)
100. A. A. Korbut, I. Kh. Sigal, and Yu. Yu. Finkel'shtein, *On the effectiveness of combinatorial methods in discrete programming*, The Contemporary State of the Theory of Operations Research, "Nauka", Moscow, 1979, pp. 237–264. (Russian)
101. A. A. Korbut and Yu. Yu. Finkel'shtein, *Discrete programming*, "Nauka", Moscow, 1969; German transl., Akademie Verlag, Berlin, 1971.
102. V. K. Korobkov, *Counterexamples for efficiency bounds of the Balas algorithm*, Kibernet. Sb. **6** (1969), 253–258. (Russian)
103. ———, *On certain integer linear programming problems*, Problemy Kibernet. vyp. 14 (1965), 279–299. (Russian)
104. Stephen A. Cook, *The complexity of theorem-proving procedures*, Third ACM Sympos. Theory of Computing, Assoc. Comput. Mach., New York, 1971, pp. 5–15.
105. A. G. Kurosh, *A course in higher algebra*, 10th ed., "Nauka", Moscow, 1971; English transl., "Mir", Moscow, 1975.
106. S. S. Lebedev, *Integer programming and Lagrange multipliers*, Èkonom. i Mat. Metody **10** (1974), 592–610. (Russian)
107. S. S. Lebedev and O. K. Sheĭman, *Duality in integer programming*, Èkonom. i Mat. Metody **17** (1981), 593–608. (Russian)
108. L. A. Levin, *Universal enumeration problems*, Problemy Peredachi Informatsii **9** (1973), no. 3, 115–117; English transl. in Problems Inform. Transmission **9** (1973).
109. V. K. Leont'ev, *Discrete extremal problems*, Itogi Nauki i Tekhniki: Teor. Veroyatnost., Mat. Statist., Teoret. Kibernet., vol. 16, VINITI, Moscow, 1979, pp. 39–101; English transl. in J. Soviet Math. **15** (1981), no. 2.
110. E. S. Lyapin, *Semigroups*, Fizmatgiz, Moscow, 1960; English transl., Amer. Math. Soc., Providence, RI, 1963; rev. ed., 1972.
111. G. I. Malashonok, *Solution of a system of linear equations in an integer ring*, Zh. Vychisl. Mat. i Mat. Fiz. **23** (1983), 1497–1500; English transl. in USSR Comput. Math. and Math. Phys. **23** (1983).
112. V. S. Mikhalevich, *Sequential optimization algorithms and their applications*, Kibernetika (Kiev) **1965**, no. 1, 45–55; no. 2, 85–89; English transl. in Cybernetics **1** (1965).
113. V. S. Mikhalevich and V. P. Volkovich, *Computational methods in the investigation and design of complex systems*, "Nauka", Moscow, 1982. (Russian)
114. V. S. Mikhalevich and A. I. Kuksa, *Methods of sequential optimization in discrete network problems of optimal allocation of resources*, "Nauka", Moscow, 1983. (Russian)
115. V. S. Mikhalevich, I. V. Sergienko, and N. Z. Shor, *Investigations of methods for solving optimization problems, and their applications*, Kibernetika (Kiev) **1981**, no. 4, 89–113; English transl. in Cybernetics **17** (1981).
116. A. P. Mishina and I. V. Proskuryakov, *Higher algebra. Linear algebra, polynomials, general algebra*, 2nd ed., "Nauka", Moscow, 1965; English transl. of 1st ed., Pergamon Press, Oxford, 1965.
117. N. M. Morzhakov and V. N. Shevchenko, *On infinite cyclic processes in integer linear programming*, Manuscript No. 1363–81, deposited at VINITI, Moscow, 1981. (Russian)
118. A. S. Nemirovskii and D. B. Yudin, *Complexity of problems and effectiveness of optimization methods*, "Nauka", Moscow, 1979. (Russian)
119. È. I. Nechiporuk, *On synthesis of circuits from threshold elements*, Problemy Kibernet. vyp. 11 (1964), 49–62. (Russian)
120. R. G. Nigmatullin, *On approximation algorithms with bounded absolute error for discrete optimal problems*, Kibernetika (Kiev) **1978**, no. 1, 95–101; English transl. in Cybernetics **14** (1978).

121. Hukukane Nikaidô, *Convex structures and economic theory*, Academic Press, New York, 1968.
122. Christos H. Papadimitrou and Kenneth Steinglitz, *Combinatorial optimization: Algorithms and complexity*, Prentice-Hall, Englewood Cliffs, NJ, 1982.
123. I. E. Parubochiĭ, V. N. Shevchenko, and S. I. Veselov, *On an algorithm for constructing the skeleton of a convex cone*, Third Conf. Young Scholars Sci. Res. Inst. of Appl. Math. and Cybernetics and the Dept. of Computational Math. and Cybernetics of Gorki Univ., Part 1, Izdat. Gor'kov. Univ., Gorki, 1983, pp. 193–198; Manuscript No. 5835–84, deposited at VINITI, Moscow, 1984. (Russian)
124. V. A. Perepelitsa, *An asymptotic approach to the solution of certain extremal problems on graphs*, Problemy Kibernet. vyp. 26 (1973), 291–314. (Russian)
125. G. L. Petrova, *Conditions for being finitely generated for a class of semigroups*, VestiĀkad. Navuk BSSR Ser. Fiz.-Mat. Navuk **1976**, no. 2, 101–103. (Russian)
126. A. V. Potemkina, *On solving the integer convex programming problem*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1980, pp. 139–160. (Russian)
127. ———, *On finite determination of the convex hull of the intersection of a convex set and an integer lattice*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1985, pp. 119–132. (Russian)
128. A. V. Potemkina and V. N. Shevchenko, *Construction of regular cuts in convex integer programming*, Ėkonom. i Mat. Metody **17** (1981), 390–394. (Russian)
129. I. B. Rogachev and O. A. Zdravomyslova, *On the number of incompatible problems of transportation type connected with optimization of radio communication systems*, Tekhnika Sredstv Svyazi. Ser. Tekhnika Radiosvyazi **1978**, no. 9 (25), 12–15. (Russian)
130. R. Tyrrell Rockafellar, *Convex analysis*, Princeton Univ. Press, Princeton, NJ, 1970.
131. L. N. Rybakov On a certain class of commutative semigroups, Mat. Sb. (N.S.) **5** (47) (1939), 521–536. (Russian)
132. Thomas L. Saaty, *Mathematical methods of operations research*, McGraw-Hill, New York, 1959.
133. V. N. Sachkov, *Introduction to combinatorial methods of discrete mathematics*, “Nauka”, Moscow, 1982; English transl., *Combinatorial methods in discrete mathematics*, Cambridge Univ. Press, Cambridge, 1996.
134. I. V. Sergienko, *Use of the method of vector descent for solving integer programming problems*, Upravlyayushchie Sistemy i Mashiny **1975**, no. 3, 92–98. (Russian)
135. ———, *On some directions in the development of methods of discrete optimization and their software*, Kibernetika (Kiev) **1982**, no. 6, 45–53; English transl. in *Cybernetics* **18** (1982).
136. ———, *Mathematical models and methods for the solution of discrete optimization problems*, “Naukova Dumka”, Kiev, 1985. (Russian)
137. I. V. Sergienko and M. F. Kaspshitskaya, *Models and methods for solving combinatorial optimization problems on a computer*, “Naukova Dumka”, Kiev, 1981. (Russian)
138. I. V. Sergienko, T. T. Lebedeva, and V. A. Roshchin, *Approximation methods for solving discrete optimization problems*, “Naukova Dumka”, Kiev, 1980. (Russian)
139. I. V. Sergienko, V. P. Soltan, and T. T. Lebedeva, *On the question of conditions for coincidence of local and global minima in discrete optimization problems*, Kibernetika (Kiev) **1984**, no. 5, 58–65; English transl. in *Cybernetics* **20** (1984).
140. A. V. Serzhantov, *An optimal algorithm for deciphering certain classes of monotone functions*, Zh. Vychisl. Mat. i Mat. Fiz. **23** (1983), 206–212; English transl. in *USSR Comput. Math. and Math. Phys.* **23** (1983).
141. A. N. Smirnov and V. N. Shevchenko, *Martin's algorithm and regular cuts*, Zh. Vychisl. Mat. i Mat. Fiz. **20** (1980), 505–509; English transl. in *USSR Comput. Math. and Math. Phys.* **20** (1980).
142. N. A. Sokolov, *Searching for a maximal upper zero for a certain class of monotone functions of finite-valued logic*, Zh. Vychisl. Mat. i Mat. Fiz. **21** (1981), 1552–1565; English transl. in *USSR Comput. Math. and Math. Phys.* **21** (1981).
143. ———, *Optimal deciphering of monotone Boolean functions*, Zh. Vychisl. Mat. i Mat. Fiz. **27** (1987), 1878–1887; English transl. in *USSR Comput. Math. and Math. Phys.* **27** (1987).
144. V. P. Soltan, *Introduction to the axiomatic theory of convexity*, “Shtiintsa”, Kishinev, 1984. (Russian)

145. V. P. Soltan and P. S. Soltan, *Convex functions*, Dokl. Akad. Nauk SSSR **249** (1979), 555–558; English transl. in Soviet Math. Dokl. **20** (1979).
146. P. S. Soltan, *Extremal problems on convex sets*, “Shtiintsa”, Kishinev, 1976. (Russian)
147. P. S. Soltan, D. K. Zambitskiĭ, and K. F. Prisakaru, *Extremal problems on graphs, and algorithms for solving them*, “Shtiintsa”, Kishinev, 1973. (Russian)
148. D. A. Suprunenko, *On the traveling salesman problem*, Kibernetika (Kiev) **1975**, no. 5, 121–124; English transl. in Cybernetics **11** (1975).
149. Alexander Schrijver, *Theory of linear and integer programming*, Wiley-Interscience, New York, 1986.
150. V. A. Talanov and V. N. Shevchenko, *A problem on a dynamic transportation network*, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **15** (1972), 1113–1114; English transl. in Radiophys. and Quantum Electronics **15** (1972).
151. ———, *Systems of equations of transportation type with an application to combinatorics*, Izdat. Gor’kov. Univ., Gorki, 1978. (Russian)
152. ———, *On a generalization of the assignment problem*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor’kov. Univ., Gorki, 1979, pp. 101–103. (Russian)
153. V. E. Tarakanov, *Combinatorial problems on binary matrices*, Combinatorial Analysis vyp. 5, Izdat. Moskov. Gos. Univ., Moscow, 1980, pp. 4–15. (Russian)
154. V. A. Trubin, *On a method for solving integer linear programming problems of a special form*, Dokl. Akad. Nauk SSSR **189** (1969), 952–954; English transl. in Soviet Math. Dokl. **10** (1969).
155. A. P. Uzdemir, *A sequential decomposition scheme in optimization problems*, Avtomat. i Telemekhan. **1980**, no. 11, 94–105; English transl. in Automat. Remote Control **1980**.
156. V. B. Ferster, *A decreasing algorithm for the cutting plane method for solving integer programming problems*, Investigations in Discrete Optimization, “Nauka”, Moscow, 1976, pp. 50–53. (Russian)
157. Yu. Yu. Finkel’shtein, *A theoretical bound for the maximal number of iterations for Gomory’s all-integer algorithm*, Problemy Kibernet. vyp. 26 (1973), 315–326. (Russian)
158. ———, *Approximation and applied discrete programming problems*, “Nauka”, Moscow, 1976. (Russian)
159. G. M. Fikhtengol’ts, *A course in differential and integral calculus*, Vol. II, 7th ed., “Nauka”, Moscow, 1969; German transl. of 5th ed., VEB Deutscher Verlag Wiss., Berlin, 1966.
160. L. R. Ford, Jr. and D. R. Fulkerson, *Flows in networks*, Princeton Univ. Press, Princeton, NJ, 1962.
161. A. A. Fridman, *On some contemporary directions in discrete optimization*, Èkonom. i Mat. Metody **13** (1977), 1115–1131. (Russian)
162. ———, *Discrete problems and the branch and bound method*, Èkonom. i Mat. Metody **10** (1974), 611–620. (Russian)
163. M. A. Frumkin, *Power algorithms in the theory of systems of linear Diophantine equations*, Uspekhi Mat. Nauk **30** (1975), no. 4 (184), 263–264. (Russian)
164. ———, *Use of modular arithmetic and construction of algorithms for solving systems of linear equations*, Dokl. Akad. Nauk SSSR **229** (1976), 1067–1070; English transl. in Soviet Math. Dokl. **17** (1976).
165. ———, *Power algorithms in integer programming*, Author’s summary of Candidate’s dissertation, Moscow, 1977. (Russian)
166. Frank Harary, *Graph theory*, Addison-Wesley, Reading, MA, 1969.
167. V. R. Khachaturov, *An approximation-combinatorial method and some of its applications*, Zh. Vychisl. Mat. i Mat. Fiz. **14** (1974), 1464–1487; English transl. in USSR Comput. Math. and Math. Phys. **14** (1974).
168. L. G. Khachiyan, *A polynomial algorithm in linear programming*, Dokl. Akad. Nauk SSSR **244** (1979), 1093–1096; English transl. in Soviet Math. Dokl. **20** (1979).
169. ———, *Convexity and algorithmic complexity of the solution of the polynomial programming problem*, Izv. Akad. Nauk SSSR Tekhn. Kibernet. **1982**, no. 6, 26–52; English transl. in Engrg. Cybernetics **20** (1982).
170. ———, *Polynomial solvability of the problem of convex Diophantine programming with a fixed number of variables*, Izv. Akad. Nauk SSSR Tekhn. Kibernet. **1983**, no. 1, 177–181; English transl. in Engrg. Cybernetics **21** (1983).
171. G. Hadley, *Nonlinear and dynamic programming*, Addison-Wesley, Reading, MA, 1964.

172. T. C. Hu, *Integer programming and network flows*, Addison-Wesley, Reading, MA, 1969.
173. Yu. Yu. Chervak, *On a cut method for discrete problems*, Ukrain. Mat. Zh. **23** (1971), 839–843; English transl. in Ukrainian Math. J. **23** (1971).
174. ———, *Discrete programming, geometric methods for constructing cuts*, Èkonom. i Mat. Metody **10** (1974), 957–963. (Russian)
175. ———, *The returning algorithm of the cut method and the branch and bound method*, Èkonom. i Mat. Metody **14** (1978), 1002–1005. (Russian)
176. ———, *Methods of lexicographic optimization and their applications*, Author's summary of Doctoral dissertation, Kiev, 1981. (Russian)
177. V. P. Cherenin, *Solution of some combinatorial problems of optimal planning by the method of sequential calculations*, Materials, Conf. on Experience and Prospective Applications of Mathematical Methods and Computers in Planning, Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1962, pp. 41–54. (Russian)
178. V. P. Cherenin and V. R. Khachaturov, *Solution of a class of problems on allocation of production by the method of sequential calculations*, Èkonom. i Mat. Metody **1** (1965), no. 2, 161–167. (Russian)
179. S. N. Chernikov, *Linear inequalities*, "Nauka", Moscow, 1968; German transl., VEB Deutscher Verlag Wiss., Berlin, 1971.
180. A. Yu. Chirkov and V. N. Shevchenko, *On quasipolynomial algorithms for certain integer programming problems*, Republican Sem. on Discrete Optimization (Uzhgorod, May 28–30, 1985), Abstracts of Reports, Inst. Kibernet. Akad. Nauk UkrSSR, Kiev, 1985, pp. 124–125. (Russian)
181. ———, *On finding successive minima of an integer lattice and finding a lattice vector closest to a given vector*, Kibernetika (Kiev) **1987**, no. 4, 46–49; English transl. in Cybernetics **23** (1987).
182. V. N. Shevchenko, *The problem of optimal scheduling with a constraint on the number of workers*, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **8** (1965), 635–637; English transl. in Radiophys. and Quantum Electronics **8** (1965).
183. ———, *The problem of a uniform distribution of lost time (several shifts)*, Èkonom. i Mat. Metody **3** (1967), 619–623. (Russian)
184. ———, *The problem of optimal task scheduling with n machine tools*, Problemy Kibernet. vyp. 18 (1967), 129–146; English transl. in Systems Theory Res. **18** (1968).
185. ———, *On the intersection of a convex polyhedral cone with the integer lattice*, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **13** (1970), 1264–1266; English transl. in Radiophys. and Quantum Electronics **13** (1970).
186. ———, *On a dual description of the cone generated over the integers by a finite set of vectors*, Mat. Zametki **14** (1973), 523–526; English transl. in Math. Notes **14** (1973).
187. ———, *Construction of regular cuts in integer linear programming*, Fifth Winter School-Sympos. on Mathematical Programming and Related Topics, part 2, Moscow, 1973, pp. 266–272. (Russian)
188. ———, *On a modification of the third Gomory algorithm*, Third All-Union Conf. on Problems of Theoretical Cybernetics, Abstracts of Reports, Novosibirsk, 1974, pp. 97–99. (Russian)
189. ———, *On solution of an elementary integer linear programming problem*, Upravlyaemye Sistemy vyp. 14 (1975), 69–73. (Russian)
190. ———, *A discrete analogue of a theorem of Farkas, and the problem of aggregation of a system of linear equations*, Kibernetika (Kiev) **1976**, no. 2, 99–101; English transl. in Cybernetics **12** (1976).
191. ———, *Linear and integer linear programming*, Part 1, Izdat. Gor'kov. Univ., Gorki, 1976. (Russian)
192. ———, *Linear programming and the theory of linear inequalities*, Izdat. Gor'kov. Univ., Gorki, 1977. (Russian)
193. ———, *On the structure of the convex hull of the set of integer solutions of a system of linear inequalities*, Fourth All-Union Conf. on Problems of Theoretical Cybernetics, Abstracts of Reports, Novosibirsk, 1977, pp. 81–82. (Russian)
194. ———, *Semigroups connected with integer programming problems*, Second All-Union Sympos. Theory of Semigroups, Abstracts of Reports, Sverdlovsk, 1978, p. 101. (Russian)

195. ———, *A bound on the number of extreme points in certain integer programming problems*, Third All-Union Conf. on Operations Research, Abstracts of Reports, Izdat. Gor'kov. Univ., Gorki, 1978, pp. 248–249. (Russian)
196. ———, *Convex polyhedral cones, systems of congruences, and regular cuts in integer programming*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1979, pp. 109–119. (Russian)
197. ———, *On the number of extreme points in integer programming*, Kibernetika (Kiev) **1981**, no. 2, 133–134; English transl. in Cybernetics **17** (1981).
198. ———, *On the property of periodicity in the knapsack problem*, Analysis and Modeling of Economic Processes, Izdat. Gor'kov. Univ., Gorki, 1981, pp. 36–38. (Russian)
199. ———, *The exchange problem, the Frobenius problem, and the group minimization problem*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1982, pp. 166–179. (Russian)
200. ———, *Sets of integer solutions of a square system of linear inequalities*, auxiliary textbook, Izdat. Gor'kov. Univ., Gorki, 1983. (Russian)
201. ———, *An algebraic approach in integer programming*, Kibernetika (Kiev) **1984**, no. 4, 36–41; English transl. in Cybernetics **20** (1984).
202. ———, *On some functions of many-valued logic connected with integer programming*, Methods of Discrete Analysis in the Theory of Graphs and Circuits vyp. 42, Novosibirsk, 1985, pp. 99–102. (Russian)
203. ———, *Deciphering of a threshold function of many-valued logic*, Combinatorial-Algebraic Methods in Applied Mathematics, Izdat. Gor'kov. Univ., Gorki, 1987, pp. 155–163. (Russian)
204. ———, *Pseudo- and quasipolynomial algorithms and integer programming*, Third All-Union School in Discrete Optimization and Computers, Abstracts of Reports, Izdat. Tsentral. Ekonom.-Mat Inst. Akad. Nauk SSSR, Moscow, 1987, pp. 71–72. (Russian)
205. ———, *Enumeration of $(0, 1)$ -determinants and permanents with two 1's in each column*, Eighth All-Union Conf. on Problems of Theoretical Cybernetics, Abstracts of Reports, Izdat. Gor'kov. Univ., Gorki, 1988, pp. 162–163. (Russian)
206. ———, *Upper bounds for the number of extreme points in integer programming*, Mathematical Problems in Cybernetics, no. 4, "Nauka", Moscow, 1992, pp. 65–72. (Russian)
207. V. N. Shevchenko and S. I. Veselov, *Deciphering of functions of many-valued logic, Theory and Program Realization of Discrete Optimization Methods*, Izdat. Inst. Kibernet. Akad. Nauk UkrSSR, Kiev, 1989, pp. 30–34. (Russian)
208. V. N. Shevchenko and N. N. Ivanov, *On representation of a semigroup by a semigroup generated by a finite set of vectors*, Vestsi Akad. Navuk BSSR Ser. Fiz.-Mat. Navuk **1976**, no. 2, 98–100. (Russian)
209. V. N. Shevchenko and A. P. Il'ichev, *On permanents and determinants of square submatrices of incidence matrices of homogeneous hypergraphs*, Permanents: Theory and Applications, Krasnoyarsk. Politekhn. Inst., Krasnoyarsk, 1990, pp. 126–135. (Russian)
210. ———, *On minors and permanents of certain $(0, 1)$ -matrices*, Diskret. Mat. **3** (1991), no. 2, 96–102; English transl. in Discrete Math. Appl. **3** (1991).
211. V. N. Shevchenko and A. A. Pavlyuchonok, *Computation of the permanent of the incidence matrix of a complete graph*, Fourth Scientific Conf. on Mathematical Methods of Programming and Software, Abstracts of Reports, Izdat. Inst. Mat. Mekh. Ural. Otdel. Akad. Nauk SSSR, Sverdlovsk, 1989, pp. 218–219. (Russian)
212. V. N. Shevchenko and O. L. Remizova, *On the structure of regular cuts in integer linear programming*, Uchen. Zap. Gor'kov. Univ. Teor. Kolebaniĭ, Prikl. Mat. i Kibernet. vyp. 166 (1973), 199–206. (Russian)
213. A. I. Shevchuk, *Programming realization of a modification of the Gomory algorithm*, State registr. No. P005843, Algorithms and Programs. Information Bulletin, no. 7, Izdat. VNTITs, Moscow, 1982, p. 53. (Russian)
214. A. I. Shevchuk and V. N. Shevchenko, *The Euclidean algorithm and extreme points of the set of integer solutions of a system of linear inequalities*, Scientific-Applied Conf. on Research of Scholars in the Service of the Five-Year Plan, Part 1 (Physical and Mathematical Sciences), Abstracts of Reports, Grodno. Univ., Grodno, 1980, pp. 108–110. (Russian)
215. G. E. Shilov, *Mathematical analysis. Finite-dimensional linear spaces*, "Nauka", Moscow, 1969; English transl., Prentice-Hall, Englewood Cliffs, NJ, 1971.

216. V. A. Shlyk, *On the faces of master Gomory polyhedra*, *Vestsī Akad. Navuk BSSR Ser. Fiz.-Mat. Navuk* **1978**, 5–11. (Russian)
217. N. Z. Shor, *The cut method with stretching of the space for solving convex programming problems*, *Kibernetika* (Kiev) **1977**, no. 1, 94–95; English transl. in *Cybernetics* **13** (1977).
218. ———, *Minimization methods for nondifferentiable functions*, “Naukova Dumka”, Kiev, 1979; English transl., Springer-Verlag, Berlin, 1985.
219. *Encyclopedia of elementary mathematics*, Vol. 5: *Geometry*, “Nauka”, Moscow, 1966; German transl., VEB Deutscher Verlag Wiss., Berlin, 1971.
220. D. B. Yudin and E. G. Gol’shtein, *Linear programming: Theory and finite methods*, Fizmatgiz, Moscow, 1963; English transl., Israel Program Sci. Transl., Jerusalem, 1965.
221. D. B. Yudin, A. P. Goryashko, and A. S. Nemirovskii, *Mathematical methods for optimization of automatic control system devices and algorithms*, “Radio i Svyaz”, Moscow, 1982. (Russian)
222. S. V. Yablonskiĭ, *On algorithmic difficulties in the design of minimal contact circuits*, *Problemy Kibernet.* vyp. 2 (1959), 75–121; English transl. in *Problems of Cybernetics* **2** (1959).
223. D. A. Babaev and F. Glower, *Aggregation of nonnegative integer-valued equations*, *Discrete Appl. Math.* **1** (1984), 125–130.
224. L. Babai, *On Lovász’ lattice reduction and the nearest lattice point problem*, *Combinatorica* **6** (1986), 1–13.
225. M. Balinski, *Integer programming: Methods, uses, computation*, *Management Sci.* **12** (1965), 253–313.
226. D. E. Bell, *A theorem concerning the integer lattice*, *Stud. Appl. Math.* **56** (1976/77), 187–188.
227. P. van Emde Boas, *Another NP-complete partition problem and the complexity of computing short vectors in a lattice*, *Reprot 81–04*, Dept. Math. Univ. Amsterdam, Amsterdam, 1981.
228. J. Borosh and A. S. Frankel, *Exact solutions of linear equations with rational coefficients by congruence techniques*, *Math. Comp.* **20** (1966), 107–112.
229. J. Borosh and L. B. Treibig, *Bounds on positive integer solutions of linear Diophantine equations*, *Proc. Amer. Math. Soc.* **55** (1976), 299–304.
230. J. H. Bradley, *Transformation of integer programs to knapsack problems*, *Discrete Math.* **1** (1971), 29–45.
231. A. Brauer, *On a problem of partitions*, *Amer. J. Math.* **64** (1942), 299–314.
232. A. Brauer and J. E. Shockley, *On a problem of Frobenius*, *J. Reine Angew. Math.* **211** (1962), 399–408.
233. C. Carathéodory, *Über den Variabilitätsbereich der Koeffizienten von Potenzreihen, die gegebene Werte nicht annehmen*, *Math. Ann.* **64** (1907), 95–115.
234. V. Chvátal, *Hard knapsack problems*, *Oper. Res.* **28** (1980), 1402–1411.
235. ———, *Edmonds polytopes and a hierarchy of combinatorial problems*, *Discrete Math.* **4** (1973), 305–337.
236. ———, *Edmonds polytopes and weakly Hamiltonian graphs*, *Math. Programming* **5** (1973), 29–40.
237. V. Chvátal and P. L. Hammer, *Aggregation of inequalities in integer programming*, *Ann. Discrete Math.* **1** (1977), 145–162.
238. W. Cook, J. Fonlupt, and A. Schrijver, *An integer analogue of Carathéodory’s theorems*, *J. Combin. Theory Ser. B* **40** (1986), 63–70.
239. W. Cook, A. M. N. Gerards, A. Schrijver, and E. Tardos, *Sensitivity theorems in integer linear programming*, *Math. Programming* **34** (1986), 251–264.
240. G. B. Dantzig, *Discrete-variable extremum problems*, *Oper. Res.* **5** (1957), 266–277.
241. ———, *Note on solving linear programs in integers*, *Naval Res. Logist. Quart.* **6** (1959), 75–76.
242. L. E. Dickson, *Finiteness of the odd perfect and primitive abundant numbers with n distinct prime factors*, *Amer. J. Math.* **35** (1913), 413–422.
243. D. Dobkin and R. J. Lipton, *A lower bound of $(1/2)n^2$ on linear search programs for knapsack problems*, *J. Comput. System Sci.* **16** (1978), 413–417.
244. J. Edmonds, *Maximum matching and a polyhedron with 0, 1-vertices*, *J. Res. Nat. Bur. Standards (B)* **69** (1965), 125–130.
245. ———, *Systems of distinct representatives and linear algebra*, *J. Res. Nat. Bur. Standards (B)* **71** (1967), 241–245.

246. J. Farkas, *Über die Theorie der einfachen Ungleichungen*, J. Reine Angew. Math. **124** (1902), 1–24.
247. S. D. Feit, *A fast algorithm for the two-variable integer programming problems*, J. Assoc. Comput. Mach. **31** (1984), 99–113.
248. M. R. Garey and D. S. Johnson, *Strong NP-completeness results: motivation, examples, and implications*, J. Assoc. Comput. Mach. **25** (1978), 499–508.
249. R. S. Garfinkel and G. L. Nemhauser, *Integer programming*, Wiley, New York, 1972.
250. A. M. Geoffrion and R. E. Marsten, *Integer programming algorithms: A framework and state of the art survey*, Management Sci. **18** (1972), 465–491.
251. P. C. Gilmore and R. E. Gomory, *The theory of computation of the knapsack functions*, Oper. Res. **14** (1968), 1045–1074.
252. F. Glover, *A new foundation for a simplified primal integer programming algorithm*, J. Oper. Res. Soc. Amer. **16** (1968), 727–740.
253. ———, *Integer programming over a finite additive group*, SIAM J. Control **7** (1969), 213–231.
254. R. E. Gomory, *Outline of an algorithm for integer solutions to linear programs*, Bull. Amer. Math. Soc. **64** (1958), 275–278.
255. ———, *An all-integer integer programming algorithm*, Industrial Scheduling, Prentice–Hall, Englewood Cliffs, NJ, 1963, pp. 193–206.
256. ———, *On the relation between integer and noninteger solutions to linear programs*, Proc. Nat. Acad. Sci. USA **53** (1965), 260–265.
257. ———, *Integer faces of a polyhedron*, Proc. Nat. Acad. Sci. USA **57** (1967), 16–18.
258. ———, *Some polyhedra related to combinatorial problems*, Linear Algebra Appl. **2** (1969), 451–558.
259. R. E. Gomory and W. J. Baumol, *Integer programming and pricing*, Econometrica **28** (1960), 521–550.
260. R. E. Gomory and A. J. Hoffman, *On the convergence of an integer programming process*, Naval Res. Logist. Quart. **10** (1963), 121–124.
261. G. A. Gorry, W. D. Northup, and J. F. Shapiro, *Computational experience with a group theoretical integer programming algorithm*, Math. Programming **4** (1973), 171–192.
262. G. A. Gorry and J. F. Shapiro, *An adaptive group-theoretic algorithm for integer programming*, Management Sci. **17** (1971), 285–306.
263. J. H. Grace and A. Young, *The algebra of invariants*, Chelsea, New York, 1964.
264. H. Greenberg, *An algorithm for a linear Diophantine equation and a problem of Frobenius*, Numer. Math. **34** (1980), 349–352.
265. ———, *Solution to a linear Diophantine equation for nonnegative integers*, J. Algorithms **9** (1988), 343–353.
266. R. T. Gregory and E. V. Krishnamurty, *Methods and applications of error-free computation*, Springer–Verlag, Berlin, 1984.
267. M. Grötschel, L. Lovász, and A. Schrijver, *The ellipsoid method and its consequences in combinatorial optimization*, Combinatorica **1** (1981), 169–197.
268. ———, *Geometric algorithms and combinatorial optimization*, Springer–Verlag, Berlin, 1988.
269. P. L. Hammer, T. Ibaraki, and U. M. Peled, *Threshold number and threshold completions*, Ann. Discrete Math. **5** (1981), 125–145.
270. A. C. Hayes and D. C. Larman, *The vertices of the knapsack polytope*, Discrete Appl. Math. **6** (1983), 135–138.
271. D. S. Hirschberg and C. K. Wong, *A polynomial-time algorithm for the knapsack problem with two variables*, J. Assoc. Comput. Mach. **23** (1976), 147–154.
272. M. Hujter and B. Vizvari, *The exact solution to the Frobenius problem with three variables*, J. Ramanujan Math. Soc. **12** (1987), 117–143.
273. O. H. Ibarra and C. E. Kim, *Fast approximation algorithms for the knapsack and sum of subset problems*, J. Assoc. Comput. Mach. **22** (1975), 463–468.
274. R. Jeroslow, *Cutting plane theory: algebraic method*, Discrete Math. **23** (1978), 121–150.
275. ———, *Some basis theorems for integer monoids*, Math. Oper. Res. **3** (1978), 145–154.
276. ———, *Trivial integer programs unsolvable by branch and bound*, Math. Programming **6** (1974), 105–109.
277. ———, *Representations of unbounded optimization problems as integer programs*, J. Optim. Theory Appl. **30** (1980), 339–351.

278. E. L. Johnson, *On the group problem and a subadditive approach to integer programming*, Ann. Discrete Math. **5** (1979), 97–112.
279. ———, *Cyclic groups, cutting planes, shortest paths*, Mathematical Programming, Academic Press, New York, 1973, pp. 185–211.
280. S. M. Johnson, *A linear Diophantine problem*, Canad. J. Math. **12** (1960), 390–398.
281. R. Kannan, *A polynomial algorithm for the two-variable integer programming problem*, J. Assoc. Comput. Mach. **27** (1980), 118–122.
282. ———, *Polynomial-time aggregation of integer programming problems*, J. Assoc. Comput. Mach. **30** (1983), 133–145.
283. ———, *Minkowski's convex body theorem and integer programming*, Math. Oper. Res. **12** (1987), 415–440.
284. ———, *Solution of the Frobenius problem and its generalization*, Report, Carnegie Mellon Univ., Pittsburgh, PA, 1991.
285. R. Kannan and L. Lovász, *Covering minima and lattice point free convex bodies*, Ann. of Math. (2) **128** (1988), 577–602.
286. R. Kannan and C. L. Monma, *On computational complexity of integer programming problems*, Lecture Notes in Econom. and Math. Systems **157** (1978), 161–162.
287. K. Kendall and St. Zions, *Solving integer programming problems by aggregation constraints*, Oper. Res. **25** (1977), 346–351.
288. V. Klee and G. J. Minty, *How good is the simplex algorithm?*, Inequalities, Academic Press, New York, 1972, pp. 159–175.
289. M. J. Knight, *A generalization of a result of Sylvester's*, J. Number Theory **12** (1980), 364–366.
290. H. Krawczyk and A. Paz, *The Diophantine problem of Frobenius: A close bound*, Discrete Appl. Math. **23** (1989), 289–291.
291. H. W. Lenstra, *Integer programming with a fixed number of variables*, Report 81–03, Dept. Math. Univ. Amsterdam, Amsterdam, 1981.
292. ———, *Integer programming with a fixed number of variables*, Math. Oper. Res. **8** (1983), 538–548.
293. A. K. Lenstra, H. W. Lenstra, and L. Lovász, *Factoring polynomials with rational coefficients*, Math. Ann. **261** (1982), 513–534.
294. C. Links and D. S. Rubin, *Finite primal cutting plane algorithms for integer programs*, SIAM J. Control Optim. **17** (1979), 47–55.
295. L. Lovász, *An algorithmic theory of numbers, graphs, and convexity*, Report 85368–OR, Univ. Bonn, Bonn, 1985.
296. G. T. Martin, *An accelerated Euclidean algorithm for integer linear programming*, Recent Advances in Mathematical Programming, McGraw–Hill, New York, 1963, pp. 311–317.
297. J. B. Mathews, *On the partition of numbers*, Proc. London Math. Soc. **28** (1897).
298. S. I. Mathis, *A counterexample to the rudimentary primal integer programming algorithm*, Oper. Res. **19** (1971), 1518–1522.
299. R. R. Meyer, *On the existence of optimal solutions to integer and mixed-integer programming problems*, Math. Programming **7** (1974), 223–235.
300. R. R. Meyer and M. L. Wage, *On the polyhedrality of the convex hull of the feasible set of an integer program*, SIAM J. Control Optim. **16** (1978), 682–687.
301. P. B. Milanov, *A polynomial-time algorithm for a large subclass of the integer knapsack problem*, C. R. Acad. Bulgare Sci. **37** (1984), 309–312.
302. P. B. Milanov and V. E. Brimkov, *On the knapsack problem*, Dokl. Bolgarsk. Akad. Nauk **40** (1987), no. 3, 25–27.
303. P. B. Milanov, *On the threshold independence systems*, Report 87487-OR, Univ. Bonn, Bonn, 1987.
304. J. Moravek, *On the complexity of discrete programming problems*, Apl. Mat. **14** (1969), 442–474.
305. J. R. Oystein, *On a linear Diophantine problem of Frobenius*, J. Reine Angew. Math. **301** (1978), 171–178.
306. M. W. Padberg, *Equivalent knapsack-type formulation of bounded integer linear programs: An alternative approach*, Naval Res. Logist. Quart. **19** (1973), 699–707.
307. C. H. Papadimitriou, *On the complexity of integer programming*, J. Assoc. Comput. Mach. **28** (1981), 765–768.

308. J. Pehler, *Über die Verschärfung von Schnitten in der Methode von Gomory bei der reinganzahligen linearen Optimierung*, Math. Operationsforsch. und Statist. **1** (1970), 207–216.
309. ———, *Gomory-Schnitte und diophantische Gleichungen*, Math. Operationsforsch. und Statist. **5** (1974), 259–268.
310. G. Plateau and M. T. Guerch, *Aggregation of equalities in integer programming*, Lecture Notes in Control and Inform. Sci. **59** (1984), 183–192.
311. O. J. Rodseth, *On a linear Diophantine problem of Frobenius*, J. Reine Angew. Math. **301** (1978), 171–178.
312. D. S. Rubin, *On the unlimited number of faces in integer hulls of linear programs with a single constraint*, Oper. Res. **18** (1970), 940–945.
313. S. Sahni, *General techniques for combinatorial approximation*, Oper. Res. **25** (1977), 920–936.
314. A. Schrijver, *Theory of linear and integer programming*, Wiley, New York, 1986.
315. J. Seelander, *Über eine Methode zur Kopplung dualer Schnittebenenverfahren in der ganzzahligen linearen Optimierung*, Math. Operationsforsch. und Statist. Ser. Optim. **8** (1977), 307–316.
316. ———, *Einige Bemerkungen zur Bestimmung von Stabilitätsbereichen in der reinganzahligen linearen Optimierung*, Math. Operationsforsch. und Statist. Ser. Optim. **11** (1980), 261–271.
317. E. S. Selmer, *On the linear Diophantine problem of Frobenius*, J. Reine Angew. Math. **293/294** (1977), 1–17.
318. E. S. Selmer and O. Beyer, *On the linear Diophantine problem of Frobenius in three variables*, J. Reine Angew. Math. **301** (1978), 161–170.
319. J. F. Shapiro, *Turnpike theorems for integer programming problems*, Oper. Res. **18** (1970), 432–440.
320. S. Smale, *On the mean speed of the simplex method of linear programming*, Math. Programming **27** (1983), 241–262.
321. R. P. Stanley, *Linear homogeneous Diophantine equations and magic labelling of graphs*, Duke Math. J. **40** (1973), 607–632.
322. J. Sylvester, *Mathematical questions with their solutions*, The Educational Times **41** (1884), 21.
323. E. Tardos, *A strongly polynomial algorithm to solve combinatorial linear programs*, Report 84360–OR, Inst. Economic and Oper. Res., Bonn, 1984.
324. ———, *A strongly polynomial minimum cost circulation algorithm*, Combinatorica **5** (1985), 247–255.
325. D. J. Vaaler, *A geometric inequality with applications to linear forms*, Pacific J. Math. **83** (1979), 543–553.
326. ———, *On linear forms and Diophantine approximation*, Pacific J. Math. **90** (1980), 475–482.
327. D. J. Vaaler and A. J. van der Poorten, *Bounds for solutions of systems of linear equations*, Bull. Austral. Math. Soc. **25** (1982), 125–132.
328. B. L. van der Waerden, *Die Reductionstheorie der positiven quadratischen Formen*, Acta Math. **96** (1956), 265–309.
329. M. Vlach, *Conditions for the existence of solutions of the three-dimensional planar transportation problem*, Discrete Appl. Math. **13** (1986), 61–78.
330. S. Yajima and T. Ibaraki, *A lower bound of the number of threshold functions*, IEEE Trans. on Electronic Computers **EC14** (1965), 926–929.
331. R. D. Young, *A primal (all-integer) integer programming algorithm*, J. Res. Nat. Bur. Standards: Ser. B. Math. and Math. Phys. **69** (1965), 213–250.
332. ———, *A simplified primal (all-integer) integer programming algorithm*, Oper. Res. **16** (1968), 213–250.
333. E. Artin, *Geometric algebra*, Interscience, New York, 1957.
334. Alfred V. Aho, John E. Hopcroft, and Jeffrey D. Ullman, *The design and analysis of computational algorithms*, Addison-Wesley, Reading, MA, 1975.
335. S. I. Veselev, *Proof of a conjecture on Diophantine linear equations*, Manuscript No. 667–B93, deposited at VINITI, Moscow, 1993. (Russian)

336. N. Yu. Zolotykh, *An algorithm for deciphering a threshold function of k -valued logic on the plane with $O(\log k)$ calls on an oracle*, First Internat. Conf. Mathematical Algorithms, Izdat. Nizhegorodsk. Univ., Nizhni Novgorod, 1995, pp. 21–26. (Russian)
337. A. P. Il'ichev, G. P. Kogan, V. N. Shevchenko, and A. A. Pavlyuchonok, *Polynomial computational algorithms and the asymptotics of permanents of certain matrices*, First Internat. Conf. Mathematical Algorithms, Izdat. Nizhegorodsk. Univ., Nizhni Novgorod, 1995, pp. 27–32. (Russian)
338. Henryk Minc, *Permanents*, Addison–Wesley, Reading, MA, 1978.
339. K. A. Rybnikov, *Combinatorial analysis. Problems and exercises*, “Nauka”, Moscow, 1982. (Russian)
340. A. Yu. Chirkov, *Carathéodory's theorem and coverings of a polyhedron by simplexes*, preprint, Nizhegorodsk. Univ., Nizhni Novgorod, 1993; Manuscript No. 668–B93, deposited at VINITI, Moscow, 1993. (Russian)
341. ———, *A lower bound on the number of vertices of the convex hull of the intersection of a polyhedron with an integer lattice*, preprint, Nizhegorodsk. Univ., Nizhni Novgorod, 1994; Manuscript No. 1381–B94, deposited at VINITI, Moscow, 1994. (Russian)
342. A. Yu. Chirkov and V. N. Shevchenko, *On the number of vertices of the convex hull of the intersection of a polyhedron with an integer lattice*, preprint, Nizhegorodsk. Univ., Nizhni Novgorod, 1993; Manuscript No. 2165–B93, deposited at VINITI, Moscow, 1993. (Russian)
343. A. Yu. Chirkov and A. A. Pavlyuchonok, *An upper bound for the number of vertices of an integer polyhedron*, First Internat. Conf. Mathematical Algorithms, Izdat. Nizhegorodsk. Univ., Nizhni Novgorod, 1995, p. 135. (Russian)
344. V. N. Shevchenko and N. Yu. Zolotykh, *A quasipolynomial algorithm for deciphering a threshold function of k -valued logic*, Information Bulletin of the Association of Mathematical Programming, No. 5, Ural. Otdel. Ross. Akad. Nauk, Ekaterinburg, 1995. (Russian)
345. I. Barany, R. Howe, and L. Lovász, *On integer points in polyhedra: a lower bound*, *Combinatorica* **12** (1992), 135–142.
346. W. J. Bultmann and W. Maas, *Fast identification of geometric objects with membership queries*, Fourth Annual Workshop Computational Learning Theory, San Mateo, CA, 1991, pp. 337–353.
347. W. Cook, M. Hartmann, R. Kannan, and C. McDiarmid, *On integer points in polyhedra*, *Combinatorica* **12** (1992), 27–37.
348. W. Maas and G. Turan, *How fast can a threshold gate learn?*, Report 321, IIG Report Series, Graz Univ. of Technology, Graz, 1991.
349. V. N. Shevchenko, *The polynomial algorithms in the integer programming*, First Internat. Conf. Mathematical Algorithms, Izdat. Nizhegorodsk. Univ., Nizhni Novgorod, 1995, pp. 136–137.

ISBN 0-8218-0535-5



9 780821 805350