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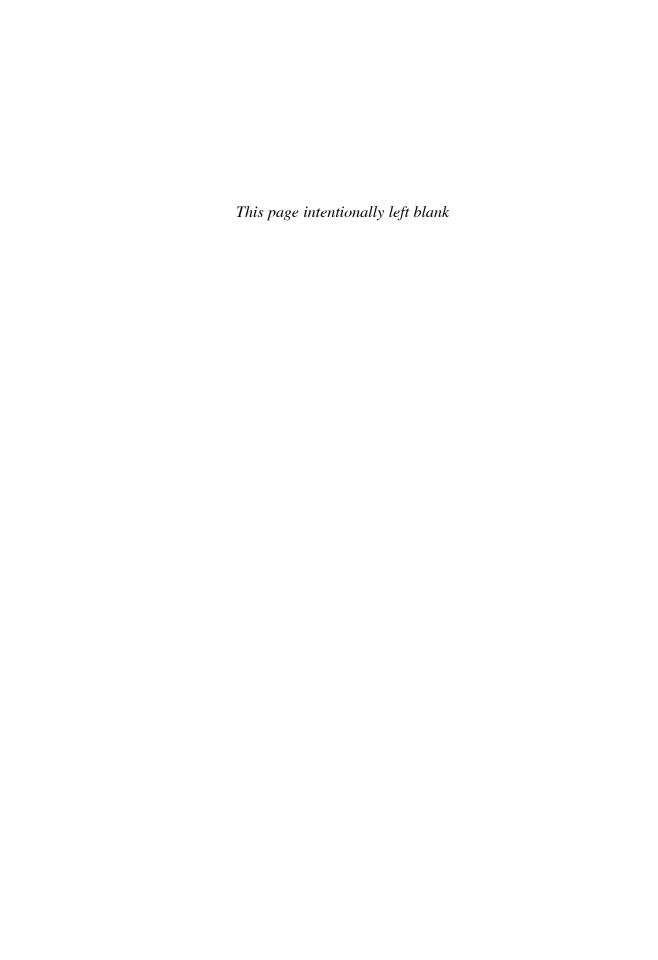
Ordinary Differential Equations with Constant Coefficient

S. K. Godunov



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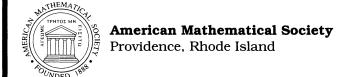


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Ordinary Differential Equations with Constant Coefficient

S. K. Godunov



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С. К. ГОДУНОВ

ОБЫКНОВЕННЫЕ ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ С ПОСТОЯННЫМИ КОЭФФИЦИЕНТАМИ

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ABSTRACT. This book presents the theory of ordinary differential equations with constant coefficients. The exposition is based on the matrix calculus. Boundary-value problems, Green matrices, the Lopatinskii condition, and the Lyapunov stability are considered. Some qualitative aspects connected with the use of computers for the analysis and solution of boundary-value problems are discussed. Control problems reduced to systems of ordinary equations with constant coefficients are also discussed. The book can be used by researchers and students working in the theory of ordinary differential equations, in particular in its computational aspects.

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Preface

This book presents the theory of ordinary differential equations with constant coefficients and follows lectures given by the author at Novosibirsk State University. The first version of the book was based on the ideas in the article [14] which develops the Gelfand–Shilov theorem [6] on the polynomial representation of a matrix exponential. The course was constantly improved by V. I. Kostin, V. M. Gordienko, G. V. Demidenko, S. I. Fadeev, E. V. Zolotareva, and G. A. Chumakov. Remarks by Yu. F. Borisov were also very useful.

Chapters 1 and 2 can be regarded as an extension of the books by the author [8, 9] published by Novosibirsk State University in 1982–1983. It turned out that the exposition of material suggested there is convenient for practical study of ordinary differential equations using computers. Therefore Chapter 3 is devoted to algorithmic and qualitative questions and contains computational procedures for solving boundary-value problems. In particular, the orthogonal sweep method is described in detail. Based on this method, some algorithms that allow us to clarify the dichotomy of the spectrum of the matrix of coefficients are also explained here. S. I. Fadeev provided substantial help in preparing this chapter.

Chapter 4 deals with stationary optimal control systems that are described by systems of differential equations with constant coefficients. In this chapter, the notions of controllability, observability, and stabilizability are analyzed. Some results of the analysis were published in [3, 10, 11], where some questions on the matrix Lur'e–Riccati equations were studied.

This book is a translation of the manuscript "Ordinary differential equations", submitted for publication to the Novosibirsk State University. The manuscript consists of two volumes. The publication of both volumes was approved by Novosibirsk State University, and they were scheduled to be published in 1989. However, Novosibirsk State University was able to publish only the first volume (Chapters 1–3), and only in 1994. The second volume remains unpublished.

This book is a translation of the published first volume and a chapter (Chapter 4 in this book) from the unpublished second volume in Russian. In the English version of the book some misprints and inaccuracies are corrected. Some style changes are also made in order to facilitate the reader's understanding.

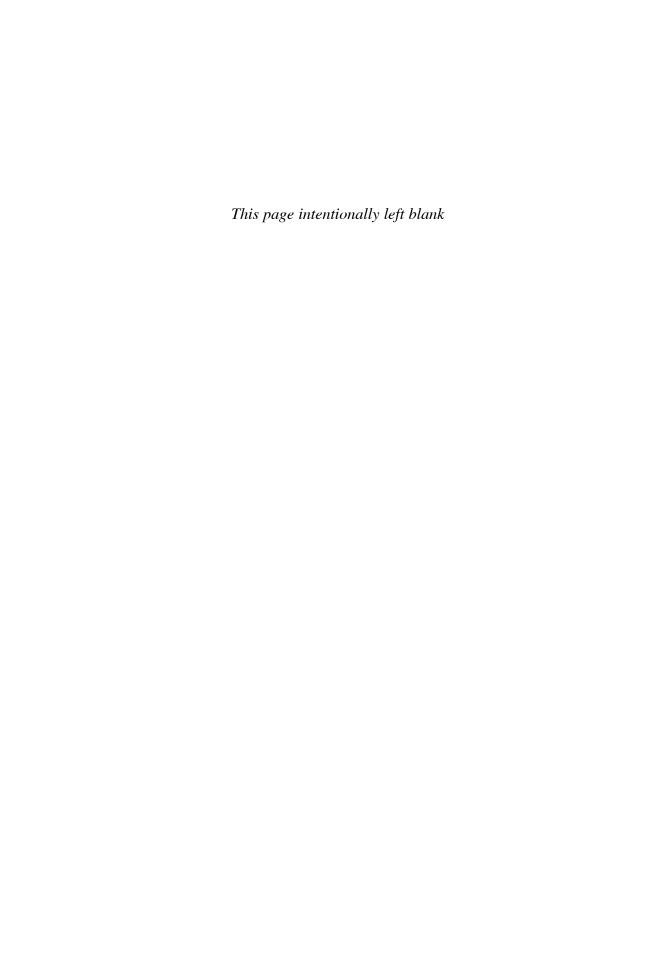
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References

- A. Ya. Bulgakov, An effectively calculable parameter for the stability property of a system of linear differential equations with constant coefficients, Sibirsk. Mat. Zh. 21 (1980), no. 3, 32-41; English transl., Siberian Math. J. 21 (1980), 339-346.
- 2. _____, The basis of guaranteed accuracy in the problem of separation of invariant subspaces for non-self-adjoint matrices, Trudy Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR 15 (1989), 12–92; English transl., Siberian Advances in Math. 1 (1991), no. 1,2, 64–108, 1–56.
- 3. _____, An algorithm for solving the Lur'e-Riccati matrix equation with guaranteed accuracy, Trudy Inst. Mat. Sibirsk. Otdel. Rus. Akad. Nauk **24** (1994), 63–104; English transl., Siberian Advances in Math. **5** (1995), no. 4, 1–49.
- F. R. Gantmacher, The theory of matrices, vol. I, II, Chelsea Publishing Company, New York, 1959.
- I. M. Gelfand and S. V. Fomin, Calculus of variations, Fizmatgiz, Moscow, 1961; English transl., Prentice-Hall, Englewood Cliffs, NJ, 1963.
- I. M. Gelfand and G. E. Shilov, Some questions of the theory of differential equations, Fizmatgiz, Moscow, 1958; English transl., Academic Press, New York and London, 1967.
- S. K. Godunov, Problem of the dichotomy of the spectrum of a matrix, Sibirsk. Mat. Zh. 27 (1986), no. 5, 24-37; English transl., Siberian Math. J. 27 (1986), no. 5, 649-660.
- 8. _____, Quadratic Lyapunov functions, Novosibirsk State University, Novosibirsk, 1982.
- 9. ______, Matrix exponential, the Green matrix, and the Lopatinskii conditions, Novosibirsk State University, Novosibirsk, 1983.
- An estimate for Green's matrix of a Hamiltonian system in the optimal control problem, Sibirsk. Mat. Zh. 34 (1993), no. 4, 70–80; English transl., Siberian Math. J. 34 (1993), no. 4, 653–662.
- 11. _____, Norms of solutions to the Lur'e-Riccati matrix equation as a criterion for quality of stabilizability and detectability, Trudy Inst. Mat. Sibirsk. Otdel. Rus. Akad. Nauk 22 (1992), 3–22; English transl., Siberian Advances in Math. 2 (1992), no. 3, 135–158.
- 12. _____, Current aspects of linear algebra, "Nauchnaya kniga", Novosibirsk, 1997.
- S. K. Godunov, A. G. Antonov, O. P. Kirilyuk, and V. I. Kostin, Guaranteed accuracy for solutions of system of linear equations in Euclidean spaces, "Nauka", Novosibirsk, 1988; English transl., Kluwer, Dordrecht, 1993.
- S. K. Godunov and V. M. Gordienko, The Green matrix of the boundary-value problem for ordinary differential equations, Uspekhi Mat Nauk 39 (1984), no. 1, 39–76; English transl. in Russian Math. Surveys 39 (1984).
- A. N. Kolmogorov and S. V. Fomin, Introductory real analysis, "Nauka", Moscow, 1981;
 English transl. of the 2nd edition, Dover, New York, 1975.
- 16. A. G. Kurosh, Higher algebra, "Nauka", Moscow, 1971. (Russian)
- S. V. Kuznetsov, Solution of boundary-value problems for ordinary differential equations, Trudy Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR 6 (1985), 85-110.
- L. A. Lyusternik and V. I. Sobolev, A short course in functional analysis, "Vyssh. Shkola", Moscow, 1982; English transl. of the 1st edition, Halsted Press, New York, 1974.
- A. N. Malyshev, Guaranteed accuracy in spectral problems of linear algebra, Trudy Inst. Mat. Sibirsk. Otdel. Rus. Akad. Nauk 17 (1991), 19–103; English transl., Siberian Advances in Math. 2 (1992), no. 1, 144–197.
- 20. _____, Introduction in linear algebra, "Nauka", Novosibirsk, 1991. (Russian)
- G. I. Petrovskii, Lectures on the theory of ordinary differential equations, "Nauka", Moscow, 1964; English transl., Prentice-Hall, Englewood Cliffs, NJ, 1966.
- 22. J. H. Wilkinson, The algebraic eigenvalue problem, Clarendon Press, Oxford, 1965.



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