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Volume 172

Characters of Finite Groups. Part 1

Ya. G. Berkovich
E. M. Zhmud′

American Mathematical Society

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Characters of Finite Groups. Part 1

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E. M. Zhmud'

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Я. Г. Беркович, Э. М. Жмудь

ТЕОРИЯ ХАРАКТЕРОВ. ЧАСТЬ I

Translated by P. Shumyatsky and V. Zobina
from the original Russian manuscript.
Translation edited by David Louvish
The translation, editing, and keyboarding of the material for this book was done in the framework of the joint project between the AMS and Tel-Aviv University, Israel.

1991 *Mathematics Subject Classification.* Primary 20C15.

ABSTRACT. The goal of this book is to place character theory and its applications to finite groups within the reach of people with a relatively modest special background, exceeding the standard algebra course only with respect to finite groups. Starting with basic notions and theorems in character theory, the authors present a vast variety of results on the properties of complex-valued characters and the applications of this theory to the theory of finite groups. Most of the results in the book are presented in the monograph form for the first time. Numerous exercises offer additional information on the topics discussed in the book and help the reader to understand the main concepts and results.

The book can be used by researchers and graduate students working in algebra, in particular in the theory of finite groups and their representations.

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To the memory of our dear friend
Professor Samuel D. Berman (1922–1987)

Contents

Preface

The representation theory of finite groups is now more than 100 years old. Its foundations were laid down by Frobenius, Burnside, Schur and, later, Brauer. It was Frobenius and Burnside who first realized the importance of representation theory for analyzing the structure of finite groups. Their classical papers amaze us even today with their depth and originality, and experts are still pondering the same fundamental problems. It suffices to note that Frobenius, in his very first paper on character theory, constructed the character table of the group $\mathrm{PSL}(2,p)$, a result regarded even now as highly nontrivial. The representation theory of finite groups is still developing vigorously, with the active participation of prominent mathematicians.

Characters constitute one of the main tools of the representation theory of finite groups over the complex field (we will consider only such representations).

The goal of this book is to place character theory and its applications to finite groups within the reach of people with a comparatively modest mathematical background, exceeding the usual algebra course only with respect to finite groups. In our opinion, it should not be very difficult for people with a good knowledge of the theory of finite groups to read this book (which is indeed intended primarily for such readers). But even people with a rather superficial knowledge of finite groups will be able to master the basics of representation theory if they read the first sections of Chapters 1–8.

The book consists of two parts. The present Part 1 contains Chapters 1–13, Part 2 (currently scheduled to be published in 1998) contains Chapters 14–31.

Although a very detailed table of contents is provided, we think it is appropriate to survey the contents of the book. In this survey we will describe the structure of the book, emphasize its main themes and point out connections between chapters.

Chapter 1. We introduce the main notions, prove such facts of primary importance as Schur's Lemma and Maschke's Theorem, and study the group $\mathrm{Lin}(G)$ of all irreducible representations of a finite abelian group over the complex field. §1.10 contains some important corollaries and applications of Maschke's Theorem.

Chapter 2. We prove the orthogonality relations and deduce their simplest corollaries. The we begin the study the relations between the character table $\mathrm{X}(G)$ of a group G and the properties of the group (this theme recurs through the whole book). For example, it is shown in [**Gar2**] that the character table of a solvable group enables one to determine its Frattini subgroup (i.e., to describe all G-classes that belong to it); but, as Garrison has shown, the character table does not generally determine the Frattini subgroup.

Chapter 3. Starting from the fact that the values of characters are algebraic integers, we deduce a series of deep theorems, the most important of which is Burnside's Theorem on the solvability of $\{p,q\}$-groups. Recently Kazarin [**Kaz1**] obtained a substantial improvement of the p^α-Lemma that implies Burnside's Theorem. Namely, he showed that any element of G, the index of whose centralizer is a prime power, belongs to a solvable radical of G (his proof uses modular theory). Note that in the 1970s there appeared a proof of Burnside's $\{p, q\}$-Theorem that does not use character theory (Goldschmidt, Bender, Matsuyama).

We present a short introduction to the theory of rational groups, i.e., groups all of whose characters are rational-valued. The chapter ends with an extensive survey of the characters of p-groups (this survey is continued in Chapter 31; many of the results that we present about the characters of p-groups are due to A. Mann).

Chapter 4 begins with a short introduction to multilinear algebra. We define some operations with representations and characters and prove the important Burnside–Brauer Theorem on powers of faithful characters, as well as an analogous result on powers of conjugacy classes (Garrison). These two results inspired a large amount of work on analogies between irreducible characters and conjugacy classes (Arad, Brenner, Mann, Blau, Chillag, Herzog, and others; see [**Arad2**]). We introduce the Frobenius-Schur indicator and present a formula for the number of involutions in a group. The irreducible characters of direct products and their kernels and quasikernels are then studied in detail. From these results we deduce an important corollary, due to Schur, stating that the degree of an irreducible character divides the index of the center (the proof we give is due to Tate). The chapter is concluded with Frobenius' proof of his fundamental theorem on the number of solutions of the equation $x^n = 1$ in a group (in Chapter 5 we present two more proofs of this theorem, one of them not using characters).

Chapters 5 and 7 are the central chapters of the book. Some of the results proved there are among those we refer to most often in the sequel.

Chapter 5. Our presentation of the theory of induced characters follows an approach outlined in the well-known paper of Brauer and Tate [**Brau10**], according to which the Reciprocity Law is postulated. Nevertheless, the chapter ends with another way to develop the theory of induced representations, due to Mackey. We prove the important Mackey Theorems on restrictions of induced characters [**Mac1**]; these theorems are then used to deduce irreducibility criteria for induced characters, due to Mackey and Shoda. As a nice application of the theory we present a proof of the Brauer–Suzuki–Wall Theorem on groups with elementary abelian centralizers of involutions [**Brau9**] — historically speaking, this was one of the first characterization results. We must mention that this theorem appeared in 1900 in a paper of Burnside, which was forgotten afterwards. Another proof of this theorem, using Bender's method, is presented in Chapter 15. Incidentally, the whole direction of characterization results in the theory of finite groups is rather poorly represented in the book. We believe that the theorems on intersections of kernels of certain characters are of some interest (see Chapter 14 for more results of this type). The chapter ends with a short survey of results about the number of elements of a given order and the number of subgroups of given structure in a group.

Chapter 6. We develop a theory of projective representations, study Schur multipliers and representation groups, and calculate multipliers of abelian groups (Schur). We prove that an abelian group possesses a faithful projective representation with at most two irreducible components (Zhmud′). From this result we deduce a description of the abelian groups that admit faithful irreducible projective representations (Frucht [**Fru**]). We also prove a realization theorem (a projective representation of a group G may be realized over the cyclotomic field generated by the $|G|$th root of unity; the proof uses Brauer's theorem on the realization of ordinary representations; see Chapter 8). We study p-groups with large multipliers. For example, we show that a group of order p^n has a multiplier of order $p^{n(n-1)/2}$ (i.e., of maximal possible order) if and only if it is an elementary abelian group; we also describe the p-groups satisfying the same condition with a multiplier of order $p^{n(n-1)/2-1}$ [**Ber13**]. We also prove the Gaschütz–Neubüser–Ti Yen estimate for the order of the multiplier of a p-group; this proof, due to the second author, does not use cohomology theory. The following result merits attention: if $|G/\mathrm{Z}(G)| = p^n$ and $|G'| = p^{n(n-1)/2}$, then $G/\mathrm{Z}(G)$ is an elementary abelian or nonabelian group of order p^3 and exponent p [**Ber13**] (the remark about the order is due to A. Mann).

Chapter 7 presents Clifford's classical work [**Cli**]. His result on the ramification index of an irreducible character over a normal subgroup will be especially important further on. This result implies Ito's Theorem 7.7, which states that the degree of an irreducible character divides the index of an abelian normal subgroup [**Ito1**]. Similarly we prove a more general (though less often used) theorem of Reynolds, which states that the degree of an irreducible character ϕ of a group G divides $|G : H|\phi(1)$, where $H \trianglelefteq G$ and $\phi \in \mathrm{Irr}(G)$. Gallagher's fundamental results (see [**Gal4**]) on the extension of invariant characters of a normal subgroup are presented. As a corollary, we obtain the inequality $\mathrm{k}(G) \le \mathrm{k}(H)\mathrm{k}(G/H)$, where $H \trianglelefteq G$ (see [**Gal6**]). Among other important results we want to emphasize Tate's p-nilpotency criterion [**Tat**]. The chapter ends with a classification of the nilpotent subgroups of class at most two and order at most $(p^{n/2} - 1)^2$ in $\mathrm{GL}(n, p)$ (our proof is a modification of arguments due to Glauberman).

Chapter 8. We prove the Brauer Induction Theorems. These theorems, together with their corollary, the Realization Theorem, are among the most important achievements of character theory. The rest of the chapter is devoted to various applications of these theorems. In particular, we present a complete exposition of Brauer's important paper [**Brau3**] on quotient groups of finite groups. The Induction Theorems undoubtedly have a potential far beyond this.

Chapter 9 gives a necessary and sufficient condition for a group to admit a faithful representation with at most k irreducible constituents [**Zhm1, Zhm3**]. We believe that special attention should be paid to the theorem stating that the number of kernels equals the number of antikernels (by "kernel" we mean the kernel of an irreducible character, and by "antikernel", the subgroup generated by a conjugacy class). For example, if distinct irreducible characters have distinct kernels (such a group is called a CM-group) and H is an antikernel, then the class generating H is uniquely determined. We study the structure of CM-groups and their generalizations. CM-groups are rational. All the results of §§1,3 were first proved by the second author [**Zhm9, Zhm21**], but our presentation in §3 differs considerably from the original one (see [**Ber27**]).

Chapter 10. The chapter revolves around Frobenius' famous theorem on transitive groups in which the stabilizer of any two points is trivial. We also consider other types of groups arising naturally in this connection. We prove several generalizations and converse theorems. Frobenius' Theorem treats an important particular case of the following problem also formulated by Frobenius: If a natural number n divides $|G|$ and the number of solutions of the equation $x^n = 1$ in G is n, is it true that the solutions constitute a subgroup? This problem was recently solved (in the affirmative) using the classification of finite simple groups [**Iiy**]. It is shown that the first column $\mathrm{X}_1(G)$ of the character table enables one to decide whether G is a Frobenius group. We give a brief introduction to the theory of exceptional characters and Suzuki's Theorem on the solvability of CA-groups of odd order (a group is called a CA-group if the centralizer of any element other than the identity is abelian). We note that $\mathrm{X}_1(G)$ determines the complex group algebra $\mathbb{C}G$ and vice versa. We consider several examples illustrating the influence of $\mathrm{X}_1(G)$ on the structure of G. For example, we prove Isaacs' Theorem which asserts that $\mathrm{X}_1(G)$ enables one to decide whether G is p-nilpotent. This approach is generalized in Chapter 11.

Chapter 11. We introduce the functions $\mathrm{T}(G)$ (= the sum of degrees of irreducible characters of G), $\mathrm{f}(G) = \mathrm{T}(G)/|G|$, and $\mathrm{mc}(G) = \mathrm{k}(G)/|G|$. Of course, the knowledge of $\mathrm{X}_1(G)$ permits one to calculate these functions (but the converse is not true). The Main Theorem classifies those groups G for which $\mathrm{f}(G) > 1/p$, where p is the smallest prime divisor of $|G|$. Note that $\mathrm{T}(G) \geq |\{x \in G \mid x^2 = 1\}|$ (by the Frobenius–Schur formula; see Chapter 4), and this makes it possible to use the Main Theorem to obtain a description of groups at least half of whose elements are involutions (see [**Wall**]; the proof presented is due to K. G. Nekrasov). It is further shown that if $H \trianglelefteq G$, $\phi \in \mathrm{Irr}(H)$, and $|\mathrm{Irr}(\phi^G)| \geq |G : H|/4$, then G/H is solvable (see [**Ber15**]).

Chapter 12. Blichfeldt was the first to study groups that possess a faithful irreducible character of relatively small degree. We analyze the structure of a p-solvable group of p-length 1 that has a faithful irreducible character of degree smaller than the exponent of its Sylow p-subgroup.

Chapter 13. We prove the Brauer–Suzuki Theorem on groups whose Sylow 2-group is a generalized quaternion group [**Brau8**], [**Gla3**].

Chapter 14 is devoted to one of the central themes of the book, the connection between the degrees and kernels of irreducible characters. It is shown that the quasikernel (and, therefore, the kernel) of an irreducible character of maximal degree is nilpotent. The same can be also said about the minimum (with respect to inclusion) quasikernels and kernels. We prove Thompson's Theorem [**Tho2**] on the p-nilpotency of a group such that all its nonlinear irreducible characters are of degree divisible by a fixed prime p, and present some related results. Incidentally, Chapter 25 contains a proof of the fact that the groups arising in Thompson's Theorem are even solvable (here we use the classification of simple groups; see Proposition 25.9 and Remark 1 after it). As a simple corollary of Thompson's Theorem one obtains another theorem, also due to Thompson, which asserts that G has an ordered Sylow tower if $\mathrm{cd}\, G$ is a chain with respect to divisibility. A similar situation, in which $\mathrm{cd}\, G - \{\chi(1)\}$ is a chain for a certain $\chi \in \mathrm{Irr}_1(G)$, and in addition it is assumed that $\chi(1)$ is prime to any element of the set $\mathrm{cd}\, G - \{\chi(1)\}$, is much more difficult; nevertheless, here too can achieve a good description of G. Note that Tate's Theorem on p-nilpotency plays a considerable role in the proof of the latter result. Thompson's Theorem is also an easy corollary of Tate's Theorem. Tate's Theorem is also used to prove Isaacs' Theorem on the solvability of a group G with $|\mathrm{cd}\, G| \leq 3$. A criterion for π-closure is formulated in terms of characters. Three appendices to the chapter are of nidependent interest.

Chapter 15. We present results of important papers by Brauer–Fowler [**Brau7**] and Bender [**Ben3**] on groups of even order and give some applications.

Chapter 16. We prove a theorem of Veitsblit, which gives a classification of groups with two infinitely distant involutions (the distance between two nonidentity elements of a group G is defined in Chapter 15).

Chapter 17. We prove Nagao's Theorem [**Nag1**] on the definability of a symmetric group S_n by its character table. Oyama [**Oya**] proved an analogous theorem for alternating groups. We are sure that analogous results may be obtained for any simple group, by using the classification of simple groups. Definability of a simple group by the first column of its character table is more difficult. The first column of the character table is not sufficient to determine whether the group is supersolvable (T. Hawkes). Many small groups are definable by the first column of the character table (this follows from the classification of all groups with class number at most 12 [**Ber3**]).

Chapter 18. We obtain a description of the irreducible linear groups G of degree p, where p is the least prime divisor of $|G|$; we also prove the important Jordan Theorem on linear groups (the proof presented here is due to Frobenius).

Chapter 19. The first half of the chapter is an introduction to the character theory of multiply transitive groups. In the last section we present the first steps of Young's approach to representations of symmetric groups.

Chapter 20. We construct the character table of $\mathrm{SL}(2, p^n)$. Construction of the character table is one of the most important parts of character theory. Long ago, Frobenius proposed a method to construct the character tables for symmetric, alternating, and some other groups (independently, Young developed the representation theory of symmetric groups).

Chapter 21. As far as we know, this is the most complete presentation of the theme "zeros of characters"(Karpilovsky's book [**Kar3**] contains a special chapter on this topic). Long ago, Burnside showed that any nonlinear irreducible character has a zero (i.e., an element of the group on which the character takes the value zero). Further results on zeros, obtained by Gallagher [**Gal5**], are complemented and strengthened in this chapter. An important role is played by Veitsblit's inequality, which gives an estimate for the number of zeros of an irreducible character [**Vei1**]. The chapter contains an extensive survey of the theory of the so-called S-characters, a notion due to the second author (as are all results of the survey).

Chapter 22 is an elementary introduction to the theory of the Schur index, constituting an important part of the theme "arithmetic properties of characters". This material was moved from Chapter 3 to Part II for reasons of continuity only.

Chapter 23. We study groups that satisfy the following condition.

If $\{1\} < N \leq G'$ *and* $N \trianglelefteq G$ *and* $\phi \in \mathrm{Irr}(N) - \{1_N\}$, *then the irreducible components of the character* ϕ^G *have pairwise distinct degrees.*

We classify the solvable groups that satisfy this condition; among these groups are the groups all of whose nonlinear irreducible characters are of pairwise distinct degrees (we also present here a classification of these groups due to Berkovich, Chillag, and Herzog [**Ber23**]).

Chapter 24. We study groups that possess only two nonlinear irreducible characters of the same degree (D_1-groups). Solvable D_1-groups are classified. It follows from the result of Kazarin and the first author [**Ber17**] that $\mathrm{PSL}(2,5)$ and $\mathrm{PSL}(2,7)$ are the only unsolvable D_1-groups. An important role is played here by the characterizations of Frobenius groups proved in Chapter 10.

Chapter 25. By Thompson's Theorem (Theorem 14.11(a)), any non-p-nilpotent group G possesses a nonlinear irreducible character of p'-degree. In this chapter we estimate the sum of degrees of nonlinear irreducible characters of p'-degrees and study the structure of groups for which the estimate is attained (cf. [**Bra1**]). In Proposition 25.9 we prove some properties (not mentioned earlier in literature) of groups in Thompson's Theorem (in particular, this proposition implies that such groups are solvable). The Appendix to this chapter treats a generalization of Frobenius kernels.

Chapter 26. Let $H < G$. We study pairs of groups for which the difference $\mathrm{T}(G) - \mathrm{T}(H)$ is small ($\mathrm{T}(G)$ is the sum of degrees of irreducible characters of G). The results of the chapter are taken from [**Ber25**].

Chapter 27. We study groups all of whose nonlinear irreducible characters take exactly three values, also proving some related results. All the results of this chapter were proved jointly by Chillag and the authors (see [**Ber24**]).

Chapter 28. We study groups G in which the number of involutions is at least $\frac{1}{4}|G|$. This result is deduced from a much more general result concerning the function $\mathrm{mc}(G)$.

Chapter 29. We classify the groups in which any two different kernels of nonlinear irreducible characters are nonincident.

Chapter 30 is a continuation of Chapter 27. We study the groups whose monolithic characters take at most three values (a character χ is called monolithic if it is irreducible and $G/\ker\chi$ is a monolith, i.e., contains only one minimal normal subgroup). In Proposition 30.18 we generalize some well-known results of character theory. Appendix B to Chapter 30, based on a paper of M. Roitman [**Roi**], contains elementary proofs of Zsigmondy's fundamental number-theoretic theorem and Feit's theorems on large Zsigmondy primes.

Chapter 31. We give a classification of groups G with $\mathrm{n}(G) \le 3$, where $\mathrm{n}(G)$ is the number of nonlinear irreducible characters of G. We study groups all of whose nonlinear irreducible characters are algebraically conjugate, and also groups G for which $\mathrm{Lin}(G)$ acts transitively on $\mathrm{Irr}_1(G)$.

It is evident that we concentrate mostly on applications, while purely theoretic questions occupy a relatively modest part of the book. A reader interested in a more detailed study of the theory is referred to the books by Isaacs, Feit, Dornhoff, Huppert, Gorenstein, Suzuki, Collins, Curtis–Reiner, and also to the books of Karpilovsky, which can be viewed as an encyclopedia of representation theory. Moreover, we regard our book as a complement to those mentioned above. It is especially useful to read it in parallel with one of them (especially those of Isaacs and Karpilovsky).

In both the contents and the style of the presentation we were greatly influenced by Isaacs' text, while the influence of other authors is comparatively small. Karpilovsky's multivolume treatise, the most complete textbook of character theory (at the time of writing the present book), was published after the present book had been written, and therefore could not have influenced us.

The material of Chapters 9, 11, 12, 14, 16, 17, 21, 23–31 is presented in monograph form for the first time. Other chapters also contain much new material.

Our presentation is fairly detailed. Since we have restricted ourselves to ordinary representations, the mathematical prerequisites are rather modest. We hope that readers acquainted with the basics of the theory of finite groups will not find the book difficult.

The exercises scattered through the whole book form an important part of the presentation. They are of varying degrees of difficulty — from purely technical ones, used in the main text, to really hard, often unsolved problems. In addition, we provide a long list of open problems at the end of Part II, written by the first author (many unsolved questions are formulated in the main text of the book as well). Most of the problems in the list were posed by the first author, but there are also some known problems. We also append a list of frequently met concepts and notations (containing definitions).

The authors of the results are mentioned wherever their names are known to us (unfortunately, the literature sometimes shows discrepancies on this point). The bibliography at the end of the book, though very incomplete, nevertheless contains many important works. Many articles are included in the Bibliography and, we recommend that you familiarize yourself with them. Some works in the list are devoted to modular theory, which is not presented in the text (but we hope to add a large chapter, "Modular Characters", in a future edition of this book).

We shall be grateful for critical remarks.

During the almost fourteen years of our work we have enjoyed the help and support of many of our colleagues. A. E. Zalesski read the text originally prepared for Rostov University Press (that edition never materialized, though typesetting began) and made some substantial remarks. Our contacts with A. I. Saksonov were very useful; he read some chapters and made useful supplements (his theorem, presented in Chapter 10, appeared in the process). While writing Chapter 11 we received great help from K. G. Nekrasov (the main theorem of that chapter was proved by him and the first author). He also prepared the text on which Chapter 17 is based. Over the last five years the first author has been actively collaborating with A. Mann, and the entire book includes many interesting results due to the him (for example, Chapter 26 is a presentation of a joint paper by Mann and the first author). Chapters 23 and 24 are generalizations of a joint paper by Berkovich, Chillag and Herzog. L. S. Kazarin and M. Roitman read the final text and made numerous useful remarks. We wish to express our gratitude to all of these colleagues. We are greatly indebted to the editors of *Zentralblatt für Mathematik*, who sent us about 100 papers and dissertations which were unavailable to us.

We remember our late friend Samuil Davidovich Berman (1922–1987) with special warmth. Our contacts with this outstanding mathematician and remarkable person had great influence on our mathematical growth. Discussions of some chapters of this book at his seminar were especially useful. We dedicate this book to his memory.

Since 1991 the work of the first author has been supported by the Ministry of Absorption and the Ministry of Science and Technology of Israel. During the last three years the work of the second author has been supported by a grant from the American Mathematical Society. We are grateful to all of them. The first author is also indebted to Professor J. Arazy, Head of the Afula Research Institute, for his constant interest and support of this work.

The Russian version of the book was accepted by the Rostov University Press in 1990 and the typesetting work began; however, for various extraneous reasons the book was not published. For the present edition, initiated by S. I. Gelfand, we have substantially improved the old version of the book. It is a great honor for us to have our book published by the American Mathematical Society.

Chapters 1–4 were translated from the Russian by P. Shumyatsky; the rest of Part 1 and Chapter 20 was translated by V. Zobina. Chapters 14–19 and 21–31 were translated by the first author. We very much appreciate numerous remarks and suggestions by N. Zobin. The English version was edited by D. Louvish.

Yakov Berkovich (Afula, Israel)
Emmanuel Zhmud′ (Kharkov, Ukraine)

List of Notation

Set Theory

$|M|$ is the cardinality of a set M (if G is a group, then $|G|$ is called the order of G).

$x \in M$ means that x is an element of M. $N \subseteq M$ means that N is a subset of M; if $N \neq M$ we write $N \subset M$.

$\emptyset$ is the empty set.

N is called a nontrivial subset of M, if $N \neq \emptyset$ and $N \subset M$. If $N \subset M$ we say that N is a proper subset of M.

$M \cap N$ is the intersection and $M \cup N$ is the union of sets M and N. If M, N are sets, then $N - M$ is the difference of N and M.

$\mathbb{C}$ is the set (field) of complex numbers.

$\mathbb{R}$ is the set (field) of real numbers.

$\mathbb{Q}$ is the set (field) of rational numbers.

$\mathbb{Z}$ is the set (ring) of integers: $\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$.

$\mathbb{N}$ is the set of natural numbers.

Number Theory and General Algebra

p is always a prime number.

m, n are always natural numbers.

(m, n) is the greatest common divisor of m and n.

$m \mid n$ should be read as: m divides n.

$\pi(m)$ is the set of all prime divisors of m.

π is a set of primes (it may be the empty set).

π' is the set of primes not contained in π.

m_π is the number satisfying the following conditions:

$$\pi(m_\pi) \subseteq \pi, \quad m_\pi \mid m, \quad \pi(m/m_\pi) \subset \pi'.$$

We write m_p, p' instead of $m_{\{p\}}$, $\{p\}'$, respectively.

m is a π-number, if $\pi(m) \subseteq \pi$ (or $m_\pi = m$).

$\mathrm{GF}(p^m)$ is the finite field containing p^m elements.

F^* is the multiplicative group of a field F.

F^n is the n-dimensional vector space over F.

F_n is the set of all $n \times n$ matrices over F.

If A is a square matrix, then $\det A$ and $\operatorname{tr} A$ are the determinant and the trace of A (that is, the sum of elements on its principal diagonal), respectively.

I_n is the $n \times n$ identity matrix.

$\bar{\alpha}$ is the number conjugate to $\alpha \in \mathbb{C}$.

$[x]$ is the integer part of $x \in \mathbb{R}$.

Groups

G is always a finite group.

$H \le G$ means that H is a subgroup of G.

$H < G$ means that $H \le G$ and $H \ne G$ (in this case H is called a proper subgroup of G). $\{1\}$ denotes the group of order 1. H is a nontrivial subgroup of G if $\{1\} < H < G$.

H is a maximal subgroup of G if $H < G$ and $H \le M < G$ imply that $H = M$.

$H \trianglelefteq G$ means that H is a normal subgroup of G; moreover, if $H \ne G$ we write $H \triangleleft G$ and say that H is a proper normal subgroup of G. $H \triangleleft G$ is called a nontrivial normal subgroup of G if $|H| > 1$.

H is a minimal normal subgroup of G if (a) $H \trianglelefteq G$; (b) $H > \{1\}$; (c) $N \triangleleft G$ and $N < H$ imply $N = \{1\}$. Thus, $\{1\}$ has no minimal normal subgroups.

G is simple if it is a minimal normal subgroup of G (in particular, $|G| > 1$).

H is a maximal normal subgroup of G if G/H is simple.

G is a monolith if $G = \{1\}$ or if G contains only one minimal normal subgroup.

The subgroup generated by all minimal normal subgroups of G is called the socle of G and is denoted by $\mathrm{Sc}(G)$. One can represent $\mathrm{Sc}(G)$ as the direct product of certain minimal normal subgroups of G. We put $\mathrm{Sc}(\{1\}) = \{1\}$. Obviously, $\mathrm{Sc}(G)$ is a characteristic subgroup of G.

$\mathrm{N}_G(M) = \{x \in G \mid x^{-1}Mx = M\}$ is the normalizer of a subset M in G.

$\mathrm{C}_G(x)$ is the centralizer of an element x in $G : \mathrm{C}_G(x) = \{z \in G \mid zx = xz\}$.

$\mathrm{C}_G(M) = \bigcap_{x \in M} \mathrm{C}_G(x)$ is the centralizer of a subset M in G.

Aut G is the group of all automorphisms of G (the automorphism group of G).

Inn G is the group of all inner automorphisms of G.

$\mathrm{Out}(G) = \mathrm{Aut}\, G / \mathrm{Inn}\, G$.

$[x, y] = x^{-1}y^{-1}xy$ is the commutator of elements x, y of G. If $M, N \subset G$ then $[M, N] = \langle [x, y] \mid x \in M, y \in N \rangle$. (However, in Chapter 11 $[M, N] = \{[x, y] \mid x \in M, y \in N\}$.)

If $M \subseteq G$, then $\langle M \rangle$ is the subgroup of G generated by M.

G' is the subgroup generated by all commutators $[x, y]$, $x, y \in G$ (i.e., $G' = [G, G]$), $G'' = (G')'$, $G''' = (G'')'$ and so on.

$\mathrm{Z}(G) = \bigcap_{x \in G} \mathrm{C}_G(x)$ is the center of G.

$\Phi(G)$ is the Frattini subgroup of G (the intersection of all maximal subgroups of G).

$\mathrm{F}(G)$ is the Fitting subgroup of G (the maximal normal nilpotent subgroup of G).

$\mathrm{S}(G)$ is the solvable radical of G (the maximal solvable subgroup of G).

$\exp G$ is the exponent of G (the least common multiple of the orders of the elements of G).

$o(x)$ is the order of an element x of G.

$\mathrm{k}(G)$ is the number of conjugacy classes of G (= G-classes), the class number of G.

If $M \subseteq G$, then $\mathrm{k}_G(M)$ is the number of G-classes containing elements of M.

$\pi(G) = \pi(|G|)$.

$\mathrm{O}_\pi(G)$ is the maximal normal π-subgroup of G, $\mathrm{O}(G) = \mathrm{O}_{2'}(G)$ (obviously, $\mathrm{O}_p(G) \in \mathrm{Syl}_p(\mathrm{F}(G))$).

$\mathrm{O}^\pi(G)$ is the subgroup generated by all π'-elements of G.

$\mathrm{C}(m)$ is the cyclic group of order m.

$A \times B$ is the direct product of groups A and B.

$A * B$ is a central product of groups A and B.

$G^0 = \{1\}$; G^m is the direct product of m copies of G.

$\mathrm{E}(p^m) = \mathrm{C}(p)^m$ is the elementary abelian group of order p^m.

A group G is said to be homocyclic if it is a direct product of isomorphic cyclic subgroups (obviously, elementary abelian p-groups are homocyclic).

$\mathrm{ES}(m,p)$ is an extraspecial group of order p^{1+2m} (a p-group G is said to be extraspecial if $G' = \Phi(G) = \mathrm{Z}(G)$ is of order p).

A special p-group is a nonabelian p-group G such that $G' = \Phi(G) = \mathrm{Z}(G)$ is elementary abelian.

(A, B) is a Frobenius group with kernel B and Frobenius complement A (A and B do not determine (A, B) up to isomorphism).

$\mathrm{D}(2m)$ is the dihedral group of order $2m$, $m > 2$.

$\mathrm{Q}(2^m)$ is the generalized quaternion group of order $2^m \geq 2^3$.

$\mathrm{SD}(2^m)$ is the semidihedral group of order $2^m \geq 2^4$.

$\mathrm{cl}\, G$ is the nilpotency class of a p-group G.

$\mathrm{CL}\ G$ is the set of all G-classes.

A p-group of maximal class is a nonabelian group G of order p^m with $\mathrm{cl}\, G = m - 1$.

If G is a p-group, then $\Omega_m(G) = \langle x \in G \mid x^{p^m} = 1 \rangle$.

$m \cdot G = \langle x^m \mid x \in G \rangle$.

$\mathrm{Syl}(G)$ is the set of all Sylow subgroups of G.

$\mathrm{Syl}_p(G)$ is the set of all Sylow p-subgroups of G.

H is a Hall subgroup of G if $(|H|, |G : H|) = 1$.

H is a π-Hall subgroup of G if $|H| = |G|_\pi$.

S_n is the symmetric group of degree n.

A_n is the alternating group of degree n.

$\mathrm{GL}(n, F)$ is the set of all nonsingular $n \times n$ matrices with entries in a field F, the general linear group over F.

$\mathrm{SL}(n, F) = \{A \in \mathrm{GL}(n, F) \mid \det A = 1 \in F\}$, the special linear group over F.

$\mathrm{PGL}(m, F) = \mathrm{GL}(n, F)/\mathrm{Z}(GL(n, F))$.

$\mathrm{PSL}(n, F) = \mathrm{SL}(n, F)/\mathrm{Z}(\mathrm{SL}(n, F))$.

$\mathrm{AGL}(n, F)$ is the natural extension of F^n by $\mathrm{GL}(n, F)$, the affine general linear group.

$\mathrm{Sz}(2^m)$ is the simple Suzuki group, $m > 1$ being odd.

For $H \leq G$, $H_G = \bigcap_{x \in G} x^{-1} H x$ is called the core of the subgroup H in G. Obviously, $H_G \trianglelefteq G$.

An element $x \in G$ is a π-element if $\pi(o(x)) \subseteq \pi$.

G is a π-group, if $\pi(G) \subseteq \pi$. Obviously, G is a π-group if and only if all its elements are π-elements.

$\mathrm{O}^\pi(G) = \langle x \in G \mid \pi(o(x)) \subseteq \pi' \}$.

$\mathrm{O}^{\pi,\sigma}(G) = \mathrm{O}^\sigma(\mathrm{O}^\pi(G))$.

A group G is an extension of $N \trianglelefteq G$ by a group H if $G/N \cong H$. A group G splits over N if $G = H \cdot N$ with $H \leq G$ and $H \cap N = \{1\}$ (in that case, G is a semidirect product of H and N with kernel N).

A group G is p-solvable if all indices of its composition series are equal to p or are p'-numbers. A group G is π-solvable if it is p-solvable for all $p \in \pi$. A group G is π-separable if all indices of its composition series are π- or π'-numbers.

If $M \subseteq G$, $x \in G$, then $M^x = x^{-1} M x = \{x^{-1} a x \mid a \in M\}$.

H is a TI-subgroup of G if $H \cap H^x = \{1\}$ for all $x \in G - \mathrm{N}_G(H)$. M is a TI-subset of G if $M \cap M^x \subseteq \{1\}$ for all $x \in G - \mathrm{N}_G(M)$

$H^\# = H - \{e_H\}$, where e_H is the identity element of the group H. If $M \subseteq G$, then $M^\# = M - \{e_G\}$.

A permutation σ of a set M is regular if $\sigma(x) \neq x$ for all $x \in M$. An automorphism α of G is regular (= fixed-point-free) if it induces a regular permutation on $G^\#$.

If $x, y \in G$, then the expression "$x \sim y$ in G" means that x, y are conjugate in G. Similarly, "$M \sim N$ in G" means that subsets M, N are conjugate in G.

An involution is an element of order 2 in a group.

An element $x \in G$ is real if $x \sim x^{-1}$ in G. An element x is rational if all generators of the subgroup $\langle x \rangle$ are conjugate in G. An involution is a real and rational element.

A section of a group G is an epimorphic image of some subgroup of G.

A group G is p-closed if $|\,\mathrm{Syl}_p(G)| = 1$ (i.e., $\mathrm{O}_p(G) \in \mathrm{Syl}_p(G)$).

A group G is p-nilpotent if it has a normal p-complement, i.e., a normal subgroup H of order $|G|_{p'}$.

An $\mathrm{S}(p^a, q^b, q^c)$-group is a q-closed minimal nonnilpotent group G of order $p^a q^{b+c}$ with $|\mathrm{Z}(G)| = p^{a-1} q^c$ (see Chapter 11).

If $F = \mathrm{GF}(p^n)$, then we write $\mathrm{GL}(m, p^n) \dots$ instead of $\mathrm{GL}(m, F) \dots$.

If $M \subseteq G$, then M^G or $\langle\langle M \rangle\rangle$ is the normal closure of M in G.

Characters and Representations

$\mathrm{F}[G]$ is the set of all functions from G to $\mathbb{C}$.

$\mathrm{CF}[G]$ is the set of all central (=class) functions from G to $\mathbb{C}$.

$\mathrm{Char}(G)$ is the set of all complex characters of G. It is convenient to consider the zero function $0_{G \to \mathbb{C}}$ as an element of the set $\mathrm{Char}(G)$.

$\mathrm{Irr}(G)$ is the set of all irreducible characters of G.

A character of degree 1 is said to be linear.

$\mathrm{Lin}(G)$ is the set of all linear characters of G (obviously, $\mathrm{Lin}(G) \subseteq \mathrm{Irr}(G)$).

$\mathrm{Irr}_1(G) = \mathrm{Irr}(G) - \mathrm{Lin}(G)$ is the set of all nonlinear irreducible characters of G. $\mathrm{n}(G) = |Irr_1(G)|$ is the number of nonlinear irreducible characters of G.

A class function θ is said to be a generalized character of G, if $\theta = \chi_1 - \chi_2$, where $\chi_1, \chi_2 \in \mathrm{Char}(G)$.

$\mathrm{Ch}(G)$ is the set of all generalized characters of G.

If $\theta, \lambda \in \mathrm{F}[G]$, $x \in G$, then $(\theta\lambda)(x) = \theta(x)\lambda(x)$.

FG is the group algebra of G over the field F.

$\chi(1)$ is the degree of a character χ of G; $\deg T$ is the degree of a representation T of G.

If $\chi \in \mathrm{Char}(G)$, $\phi \in \mathrm{Char}(H)$, $H \leq G$, then χ_H is the restriction of χ to H, and ϕ^G is the induced character ($\phi^G \in \mathrm{Char}(G)$).

If $\vartheta, \psi \in \mathrm{CF}[G]$, then

$$\langle \vartheta, \psi \rangle = |G|^{-1} \sum_{x \in G} \vartheta(x)\overline{\psi(x)}$$

is the scalar (or inner) product of ϑ and ψ.

If $H \lhd G$, $\phi \in \mathrm{Irr}(H)$, then $\mathrm{I}_G(\phi) = \{x \in G \mid \phi^x = \phi\}$ is the inertia group of ϕ in G (where $\phi^x(h) = \phi(xhx^{-1})$ for $h \in H$).

If $H \le G$ and $\phi \in \mathrm{CF}[H]$, then $\dot{\phi}$ is the function in $\mathrm{CF}[G]$ that coincides with ϕ on H and vanishes on $G - H$.

1_G is the principal character of G ($1_G(x) = 1$ for all $x \in G$).

ρ_G is the regular character of G.

$\mathrm{Irr}(\chi)$ is the set of all irreducible constituents of a character χ of G, $\mathrm{Irr}_1(\chi) = \mathrm{Irr}(\chi) \cap \mathrm{Irr}_1(G)$. (The expression $\psi \in \mathrm{Irr}(\chi)$ means that the character ψ is a constituent of χ.)

$\mathrm{X}(G)$ is the character table of G, and $\mathrm{X}_1(G)$ is its first column (consisting of the degrees of irreducible characters, counting multiplicities).

$\mathrm{M}(G)$ is the Schur multiplier of G.

If M is a set, the Kronecker symbol $\delta : M \times M \to \{0, 1\}$ is defined as follows:

$$\delta_{a,b} = \begin{cases} 1 & \text{if } a = b; \\ 0 & \text{if } a \neq b. \end{cases}$$

$\mathrm{cd}\ G = \{\chi(1) \mid \chi \in \mathrm{Irr}(G)\}$.

$\mathrm{cd}_1\ G = \{\chi(1) \mid \chi \in \mathrm{Irr}_1(G)\} = \mathrm{cd}\ G - \{1\}$.

$\mathrm{b}(G) = \max\{n \mid n \in \mathrm{cd}\ G\}$.

$\ker T$ is the kernel of a representation T.

$\ker \chi$ is the kernel of a character χ.

$\mathrm{Z}(\chi) = \{x \in G \mid |\chi(x)| = \chi(1)\}$ is the quasikernel of $\chi \in \mathrm{Char}(G)$.

$\mathrm{T}(\chi) = \{x \in G \mid \chi(x) = 0\}$ is the set of zeroes of $\chi \in \mathrm{Ch}(G)$.

$\mathrm{U}(\chi) = \{x \in G \mid |\chi(x)| = 1\}$ is the set of χ-unitary elements of G (where $\chi \in \mathrm{Ch}(G)$).

Let $N \trianglelefteq G$. Then $\mathrm{Irr}_N(G) = \{\chi \in \mathrm{Irr}(G) \mid N \le \ker \chi\}$. We often identify the sets $\mathrm{Irr}_N(G)$ and $\mathrm{Irr}(G/N)$. Next, $\mathrm{Irr}(G, N) = \mathrm{Irr}(G) - \mathrm{Irr}(G/N)$.

$\mathrm{Lin}_N(G) = \mathrm{Lin}(G) \cap \mathrm{Irr}_N(G)$.

$\mathrm{Irr}_\phi(G) = \{\chi \in \mathrm{Irr}(G) \mid \langle \chi_N, \phi \rangle > 0\}$, where $N \trianglelefteq G$, $\phi \in \mathrm{Irr}(N)$.

Let $H \le G$, $\phi \in \mathrm{Irr}(H)$, $\chi \in \mathrm{Irr}(G)$. Then χ is an extension of ϕ to G if $\chi_H = \phi$.

$\nu_2(\chi)$ is the Frobenius-Schur indicator of $\chi \in \mathrm{Irr}(G)$ (see Chapter 4).

$\mathrm{mc}(G) = \mathrm{k}(G)/|G|$ is the measure of commutativity of G.

$\mathrm{T}(G) = \sum_{\chi \in \mathrm{Irr}(G)} \chi(1)$, and $\mathrm{f}(G) = \mathrm{T}(G)/|G|$.

Let T be a representation, affording the character χ of G. Then the function $\det \chi : G \to \mathbb{C}^*$ is defined by $(\det \chi)(x) = \det T(x)$, $x \in G$. Obviously $\det \chi \in \mathrm{Lin}(G)$.

If $\chi \in \mathrm{CF}(G)$, then $\bar{\chi} : G \to \mathbb{C}$ is defined by $\bar{\chi}(x) = \overline{\chi(x)}$, $x \in G$.

If $X \subseteq \mathrm{Irr}(G)$, then $X^\# = X - \{1_G\}$. In particular, $\mathrm{Irr}^\#(G)$ is the set of all nonprincipal characters of G.

$\mathrm{Irr}_1(G, p') = \{\chi \in \mathrm{Irr}_1(G) \mid p \nmid \chi(1)\}$.

$\mathrm{T}_1(G, p') = \sum_{\chi \in \mathrm{Irr}_1(G,p')} \chi(1)$.

If $P \in \mathrm{Syl}_p(G)$, then $\mathrm{T}_1(G, P, p') = \sum_{\chi \in \mathrm{Irr}_1(G,p')\, P \not\leq / \ker \chi} \chi(1)$.

$\mathrm{Kern}\ G = \{\ker \chi \mid \chi \in \mathrm{Irr}_1(G)\}$.

$\mathrm{v}(x) = |\{\chi(x) \mid x \in G\}|$.

A character χ of G is monolithic if $\chi \in \mathrm{Irr}(G)$ and $G/\ker \chi$ is a monolith. $\mathrm{Irr}_m(G)$ is the set of all monolithic characters of G, $\mathrm{Irr}_{1,m}(G) = \mathrm{Irr}_m(G) \cap \mathrm{Irr}_1(G)$.

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Author Index

In this index, "**2**.1" means "Chapter 2, §1", and just "**2**" means the whole of Chapter 2.

Subject Index

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