

Translations of  
**MATHEMATICAL  
MONOGRAPHS**

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Volume 194

**Records:  
Mathematical Theory**

Valery B. Nevzorov



**American Mathematical Society**

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Records:  
Mathematical Theory

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**American Mathematical Society**  
Providence, Rhode Island

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## РЕКОРДЫ: МАТЕМАТИЧЕСКАЯ ТЕОРИЯ

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ABSTRACT. The book presents the theory of records which has been the subject of an intense research activity since 1952. It may be viewed as a research monograph with an extensive bibliography for probabilists and statisticians, actuarial mathematicians, meteorologists, hydrologists, reliability engineers, sports and market analysts. At the same time the book includes a wide variety of exercises and is written in a form which allows it to be used as a textbook for graduate students.

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To my parents

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## Preface

In 2002 the mathematical theory of records will mark its 50th anniversary. The first paper on this subject by Chandler (1952) attracted the attention of many researchers and inspired many new publications. It happens sometimes that, by novelty of its subject or brilliant results, a mathematical paper arouses and maintains interest of numerous researchers to a “fashionable” problem for several years or even decades, but as the “vein of gold” is exhausted this interest drops off sharply. This was not the case for records. The number of publications on this subject has been increasing exponentially, doubling about every 10 years (around 80 in 1977, about 160 in 1987, and more than 300 by the beginning of 1998). In the Introduction, we try to explain this phenomenon. Here we only point out that numerous models of records provide a convenient object for application of various mathematical methods; on the other hand, there is a lot of numerical data on records in sports (national, Olympic, world records), hydrology (for instance, the floods in St. Petersburg have been recorded for 300 years), meteorology (the reader of course has heard on radio or TV something like “the air temperature today in our city was the lowest for the last 95 years”), etc., which motivate the mathematicians to build models compatible with the available record observations and to try to predict the future record values.

It is difficult to separate the theory of records from the theory of order statistics. Records are especially closely related to extremal order statistics. A systematic exposition of the theory of order statistics and extremes can be found in books by H. A. David “Order Statistics” (1970, 1981) and J. Galambos “The Asymptotic Theory of Extreme Order Statistics” (1978, 1987). Regarding records, a comparatively detailed review of results (without proofs) and related bibliography can be found only in the form of articles (Nevzorov (1987), Nagaraja (1988), Nevzorov and Balakrishnan (1998)). As mentioned above, the number of publications has practically doubled for the last 10 years. Hence it becomes necessary to relate the classical results for records with the latest advances. This book can be viewed as an “Introduction into the Theory of Records.” We tried to present the material in the form combining the features of a textbook and a survey of literature. Thus, after learning the basic methods utilized in the theory of records, the reader will find in Appendix 1 bibliographical notes which, together with the list of references comprising about 300 papers on records and related topics, will allow him to acquire a deeper knowledge of the subject. Moreover, this book contains about 50 exercises which will allow the reader to assess the degree of his mastering the material. Hints and solutions are collected in Appendix 2.

The book is written on an intermediate level which presumes the knowledge of only standard courses of probability theory and mathematical statistics. In the first part of the book we present briefly the necessary material on order statistics

which is used in the theory of records. The reader interested in a more detailed knowledge of the theory of order statistics and its applications is referred to the book by David (1970) mentioned above and references therein.

The book can be used for preparing courses on order statistics and records, as well as for studying these areas of probability theory and mathematical statistics on one's own. A large part of the material was employed by the author in lecture courses on order statistics and records at the Faculty of Mathematics and Mechanics at the St. Petersburg State University and the Department of Statistics at Ohio State University.

I hope that the methods for analysis of records, numerous record models, and the various applications of records treated in the book will attract the attention not only of mathematicians, but also of engineers (especially those engaged in reliability of constructions), actuaries, sport statisticians, specialists in hydrology, meteorology, gerontology, and many others dealing with analysis of extremal values of various random variables and processes.

Writing the book gives me an opportunity to set out systematically the results of my research on order statistics and records. It is my pleasure to express gratitude to V. V. Petrov from whom I always received attention and support beginning with my first steps in science. I am indebted to my former students A. V. Stepanov and S. V. Malov, now scientists themselves, who have been the most scrupulous readers of my works, and with whom constant communication stimulated my research in this area. Of great importance for my scientific activity was an exchange of ideas with colleagues. I am thankful to M. Ahsanullah, N. Balakrishnan, P. Deheuvels, V. A. Egorov, G. Haiman, and H. N. Nagaraja with whom I obtained a number of joint results included in this book.

Comments useful in improving the text were made by S. M. Ananievskii, D. M. Chibisov, O. V. Rusakov, and V. V. Slavova. Advice from A. M. Trevgoda and assistance from my wife Lyudmila and my son Igor helped to speed up the process of typesetting the manuscript.

V. B. Nevzorov

St. Petersburg, April 2000

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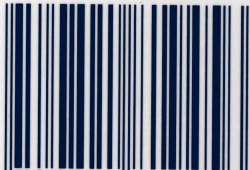
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