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Yu. N. Lin'kov

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Asymptotic Statistical Methods for Stochastic Processes

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Providence, Rhode Island

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АСИМПТОТИЧЕСКИЕ МЕТОДЫ СТАТИСТИКИ
СЛУЧАЙНЫХ ПРОЦЕССОВ

НАУКОВА ДУМКА, КИЕВ, 1993

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ABSTRACT. For increasing number of observations, asymptotic methods of statistics of semimartingale type processes are considered. Local densities of probability measures generated by semimartingales are introduced, and their asymptotic properties are investigated for various types of asymptotic distinguishability of the corresponding families of statistical hypotheses. Certain problems of simple hypothesis testing and estimation of an unknown parameter from observation of semimartingales, as well as some asymptotic information-theoretic problems of statistics, are solved.

This book can be used by researchers and graduate students working in statistics of stochastic processes and its applications.

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Preface

The asymptotic properties of the likelihood ratio play an important part in solving problems in statistics for various schemes of observations. Wald [**195, 196**] and Le Cam [**168, 169**] were the first to commence developing asymptotic methods in mathematical statistics based on asymptotic properties of the likelihood ratio. At first, sequences of independent random variables were considered using the central limit theorem for the logarithm of the likelihood ratio. These investigations gave rise to the notion of the local asymptotic normality (LAN) of a family of probability measures generated by observed random variables [**169**]. Later on Hájek and Šidák [**14**], Chibisov [**125**], Roussas [**110**], Ibragimov and Has'minskii [**40**], Dzaparidze [**29**], and others developed a rather general asymptotic theory of parameter estimation and hypothesis testing based on asymptotic properties of the likelihood ratio for sequences of mutually dependent random variables.

The extension of statistical methods to time-continuous stochastic processes attracted the attention of many scientists. Among the first works in this field was the work by Ulf Grenander [**24**], which marked the beginning of active research on developing statistical methods for Gaussian and stationary processes [**2, 32, 37, 40, 41**]. Many asymptotic methods for estimating Gaussian diffusion-type processes based on the LAN property can be found in the book by Ibragimov and Has'minskii [**40**]. The extension of the asymptotic methods of mathematical statistics based on the central limit theorem and the LAN property to non-Gaussian and nonstationary stochastic processes gave rise to new ideas in the theory of stochastic processes.

In recent years convenient formulas for densities of probability measures generated by stochastic processes were obtained and limit theorems for various stochastic processes were proved. Eventually, the asymptotic methods became an important tool in studying diffusion-type [**57, 62, 68, 103**] and counting [**57, 71, 78, 183, 186**] processes, processes with independent increments [**73, 129, 163, 199**], Markov processes [**77, 132, 162**], and semimartingales [**83, 84, 119, 166**].

In this book we describe the asymptotic methods for parameter estimation and hypothesis testing based on asymptotic properties of the likelihood ratio in the case where an observed stochastic process is a semimartingale. Semimartingales form a rather wide class of stochastic processes, which include diffusion-type and counting processes, processes with independent increments, Markov processes, and others. In this book we consider only right-quasicontinuous semimartingales. This limitation allowed us to simplify the presentation of the asymptotic method and to make it accessible to engineers.

As was observed by Chibisov (see the corresponding remark in [**110**]) and Ibragimov and Has'minskii [**40**], the asymptotic method developed by Wald and Le Cam is rather general by its nature. It can be applied to any model of observations for which the likelihood ratio possesses the properties required by this method. Therefore, further development of the method of Wald and Le Cam and its application to particular models of observations reduce to finding specific restrictions that must be imposed on the likelihood ratio. In many cases these restrictions appear to be common to a variety of schemes of observations. Following [**40**], this fact was substantially used in this book. When discussing any statistical problem, we first deduce general results for observations of arbitrary nature and then modify them to the case of observations of martingales. Thereafter these results are applied to diffusion-type and counting processes, which are, in this book, the main models for demonstrating the efficiency and checking the correctness of our theory. The choice of these two particular schemes of observations is justified by their importance in solving various statistical problems [**12, 47, 50, 101, 120, 190**].

Chapter 1 contains general basic notions and results for stochastic processes, which are used throughout the rest of the book. The facts concerning the notion of a martingale and its generalizations are given. Certain classes of stochastic processes are introduced. Random measures; stochastic integrals with respect to local martingales, random measures, and semimartingales; and other notions are defined. The Itô formula for semimartingales and the Lenglart inequality are presented. Statistical experiments generated by observations of semimartingales are introduced, and formulas for the likelihood ratio are given. Limit theorems for semimartingales and, in particular, central limit theorems for local martingales are formulated.

Chapters 2 and 3 are devoted to the problem of distinguishing between two simple statistical hypotheses. In Chapter 2 a general scheme of statistical experiments is considered. Certain types of asymptotic distinguishability between families of hypotheses are introduced. These types are characterized in terms of the likelihood ratio, Hellinger integral of order ε, Kakutani–Hellinger distance, and the distance in variation between hypothetical measures, etc. The problem of complete asymptotic distinguishability is discussed. In the case of complete asymptotic distinguishability the behavior of error probabilities in the Neyman–Pearson test is investigated for various kinds of behavior of the likelihood ratio; namely under the following conditions: the law of large numbers is fulfilled, the theorem on large deviations of the logarithm of the likelihood ratio holds, properly centered and normalized logarithm of the likelihood ratio weakly converges. In the case of continuous families of hypotheses the behavior of error probabilities of the Neyman–Pearson test is investigated, provided the likelihood ratio weakly converges to some law under the null hypothesis. Asymptotic expansions of the likelihood ratio are considered for contiguous and noncontiguous families of hypothesis. At the end of Chapter 2 two reductions of the problem of hypothesis testing are considered. These reductions make it possible to take into account the behavior of the sets of singularity of hypothetical measures.

In Chapter 3 the results of Chapter 2 are used in statistical experiments generated by observations of semimartingales. All restrictions are formulated in predictable terms: either in terms of triples of predictable characteristics of semimartingales or in terms of Hellinger processes of order ε. Both the nonparametric case where there are two triples of characteristics and the parametric case where there is a family of predictable characteristics of a semimartingale that depend on an unknown parameter are considered. The results obtained are applied to studying various particular cases of diffusion-type and counting processes.

In Chapters 4 and 5 some problems of asymptotic estimation of unknown parameters are considered. In Chapter 4 the general limit theorems on asymptotic properties of maximum likelihood and Bayes estimates obtained by Ibragimov and Has'minskii [**40**] for observations of an arbitrary nature are applied to observations of semimartingales. The results obtained are used for studying diffusion-type and counting processes. In Chapter 5 an unknown parameter is assumed to be random, and under this condition certain information-theoretic problems of estimation of parameters are considered. The asymptotic behavior of the Shannon information contained in an observation of the unknown parameter is studied, and various methods of obtaining lower bounds for Shannon information contained in a statistical estimate for the unknown parameter are described. Based on these results, many informational inequalities for risk functions of statistical estimates analogous to the well-known Cramér–Rao and Hájek inequalities are derived. The results obtained are applied to general schemes of observations of semimartingales.

Clearly, it is impossible to consider all the problems of statistics of stochastic processes in one book. For example, we do not mention the problem of complex hypothesis testing, which is close to the results of Chapter 4. Some interesting results obtained in this area can be found in the works by Roussas [**110**], Ingster [**42, 44**], Burnashev [**11**], and others. In this connection the somewhat obsolete review [**6**] should also be mentioned.

The problems of statistics for the semimartingales that are not left-quasicontinuous have not been considered at all. The effect of the sets of singularity of hypothetical measures has not been discussed in detail: there is only one section devoted to the reduction of the problem of hypothesis testing. The important problem of applying the results of Chapter 5 to the study of asymptotic sufficiency has been omitted altogether.

The bibliography at the end of the book does not pretend to be complete. However the author tried to mention all the works that have played a significant part in the development of asymptotic methods in statistics of stochastic processes. For the English edition, the list of references was enlarged and the Bibliographical Notes revised.

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Basic Notation

$A = (A^{ij})$ .... matrix with elements A^{ij}
$A' = (A^{ji})$ .... the transpose of a matrix A
$|A|$ .... the norm of a matrix A, $|A| = (\operatorname{Tr} AA')^{1/2}$
$a \wedge b$ .... the minimum of two numbers $a, b \in \mathbf{R}$, $a \wedge 0 = -a^-$
$a \vee b$ .... the maximum of two numbers $a, b \in \mathbf{R}$, $a \vee 0 = a^+$
$\mathcal{B}(A)$ .... the Borel σ-algebra of subsets of A, $\mathcal{B}(\mathbf{R}^k) = \mathcal{B}^k$, $\mathcal{B}(\mathbf{R}_0^k) = \mathcal{B}_0^k$, $\mathcal{B}_0^1 = \mathcal{B}_0$, $\mathcal{B}^1 = \mathcal{B}$, $\mathcal{B}(\mathbf{R}_+) = \mathcal{B}_+$
$\mathbf{D}$, $\widetilde{\mathbf{D}}$, $\mathbf{D}_y$ .... variances with respect to measures P, $\widetilde{\mathsf{P}}$, P_y
$\det A$ .... determinant of a matrix A
E, $\widetilde{\mathsf{E}}$, E_y .... expectation with respect to measures P, $\widetilde{\mathsf{P}}$, P_y
$f = (f_x)_{x \in X}$ .... a function defined on X
I_k .... the unit matrix of order k
$\mathcal{L}(Y|\mathsf{P})$ .... the distribution law of a vector Y with respect to a measure P
$\mathbf{N}$ .... the set of positive integers
$\mathcal{N}(a, B)$ .... the normal (Gaussian) law with vector of means a and covariance matrix B
$\mathsf{P} \ll \mathbf{Q}$ .... absolute continuity of a measure P with respect to a measure $\mathbf{Q}$
$\mathsf{P} \sim \mathbf{Q}$ .... equivalence (mutual absolute continuity) of measures P and $\mathbf{Q}$
$\mathsf{P} \perp \mathbf{Q}$ .... singularity of measures P and $\mathbf{Q}$
$P^t\text{-}\lim_{t\to\infty} Y_t = c$ .... means that $\lim_{t\to\infty} P^t\{|Y_t - c| > \varepsilon\} = 0$ for all $\varepsilon > 0$
$\mathbf{R}^k$ .... k-dimensional Euclidean space with a fixed orthonormal basis with points $x = (x^1, x^2, \ldots, x^k)'$, $\mathbf{R}^1 = \mathbf{R}$, $\mathbf{R}^k \setminus \{0\} = \mathbf{R}_0^k$, $\mathbf{R}_0^1 = \mathbf{R}_0$, $\mathbf{R}_+ = [0, \infty)$, $\overline{\mathbf{R}}_+ = [0, \infty]$
$\operatorname{Tr} A$ .... the trace of a matrix A
$|x|_p$ .... the norm of a vector $x \in \mathbf{R}^k$ for $p \in (0, \infty)$, $|x|_p = \left(\sum_{i=1}^k |x^i|^p\right)^{1/p}$, $|x|_2 = |x|$
(x, y) .... scalar product of vectors $x, y \in \mathbf{R}^k$, $x'y = (x, y)$
δ^{ij} .... the Kronecker delta
Δf_x .... a jump of a function at a point $x \in \mathbf{R}$, $\Delta f_x = f_x - f_{x-}$
$(\Omega, \mathcal{F}, \mathsf{P})$ .... a probability space, where Ω is a set of points ω; $\mathcal{F}$, a σ-algebra of subsets of Ω; and P, a probability measure on $\mathcal{F}$
χ_A .... the indicator function, $\chi_A = \chi(A) = (\chi_x(A))_{x \in X}$
$\chi_x(A)$ .... the indicator of a set A, $\chi_x(A) = \chi(A; x)$
$\square$ .... the symbol of the end of a proof (Q.E.D.)

Classes of stochastic processes

$\mathcal{M}$ .... class of all uniformly integrable martingales
$\overline{\mathcal{M}}$ .... class of all martingales
$\mathcal{M}^c$ .... class of all uniformly integrable continuous martingales
$\overline{\mathcal{M}}^c$ .... class of all continuous martingales
$\mathcal{M}^d$ .... class of all uniformly integrable purely discontinuous martingales
$\overline{\mathcal{M}}^d$ .... class of all purely discontinuous martingales
$\mathcal{M}^2 = \{\xi \in \mathcal{M}: \sup_{t\in\mathbf{R}_+} \mathsf{E}\,|\xi_t|^2 < \infty\}$ .. class of all uniformly square-integrable martingales
$\overline{\mathcal{M}}^2 = \{\xi \in \overline{\mathcal{M}}: \mathsf{E}\,|\xi_t|^2 < \infty\ \forall t \in \mathbf{R}_+\}$ .. class of all square-integrable martingales
$\mathcal{V}^+$ .... class of all adapted right-continuous nondecreasing processes A with $A_0 = 0$ and $A_t < \infty$ (P-a.s.) for all $t \in \mathbf{R}_+$
$\mathcal{V} = \{\xi - \eta: \xi \in \mathcal{V}^+, \eta \in \mathcal{V}^+\}$ ... class of all processes with a locally bounded variation
$\mathcal{V}^c$ .... class of all processes of $\mathcal{V}$ with (P-a.s.) continuous trajectories
$\mathcal{A}^+ = \{\xi \in \mathcal{V}^+: \mathsf{E}\,\xi_\infty < \infty\}$ ... class of all integrable nondecreasing processes
$\mathcal{A} = \{\xi - \eta: \xi \in \mathcal{A}^+, \eta \in \mathcal{A}^+\}$ ... class of all processes with integrable variation
$\mathcal{K}(\mathbf{F}, \mathsf{P})$ .... class of all processes with respect to a filtration $\mathbf{F}$ and measure P
$\mathcal{K}_{\text{loc}}$ .... the local class
$\mathcal{M}_{\text{loc}}$ .... class of all local martingales
$\mathcal{M}^2_{\text{loc}}$ .... class of all square-integrable local martingales
$\mathcal{O}$ .... class of all optional processes
$\mathcal{P}$ .... class of all predictable processes
$\widetilde{\mathcal{O}}$ .... class of all $\mathcal{O} \times \mathcal{B}$-measurable functions
$\widetilde{\mathcal{P}}$ .... class of all $\mathcal{P} \times \mathcal{B}$-measurable functions
$\mathcal{S}$ .... class of all semimartingales
$\mathcal{S}_p$ .... class of all special semimartingales

Classes of integrable functions

$\mathcal{L}^2_{\text{loc}}(\xi) = \{f: f \in \mathcal{P}, f^2 \circ \langle\xi\rangle \in \mathcal{A}_{\text{loc}}\}$
$\mathcal{L}^1_{\text{loc}}(\xi) = \{f: f \in \mathcal{P}, (f^2 \circ [\xi, \xi])^{1/2} \in \mathcal{A}^+_{\text{loc}}\}$
$\mathcal{L}_{\text{loc}}(\xi) = \{f: f \in \mathcal{L}^1_{\text{loc}}(M), f \circ A \in \mathcal{V}\}$
$\mathcal{G}^i_{\text{loc}}(\nu) = \{f: f \in \widetilde{\mathcal{P}}, |f|^i * \nu \in \mathcal{A}^+_{\text{loc}}\}$, $i = 1, 2$
$\mathcal{G}_{\text{loc}}(\nu) = \{f: f \in \widetilde{\mathcal{P}}, |f|^2(1 + |f|)^{-1} * \nu \in \mathcal{A}^+_{\text{loc}}\}$

Classes of loss functions

$\mathbf{W}$ .... class of all loss functions
$\mathbf{W}'$ .... a subclass of loss functions in $\mathbf{W}$

$\mathbf{W}_p$ .... class of all loss functions in $\mathbf{W}$ that have a polynomial majorant

$\mathbf{W}'_p$ .... class of all functions in $\mathbf{W}'$ that have a polynomial majorant

$\widetilde{\mathbf{W}}'_p$ .... class of all functions $l(u)$ in $\mathbf{W}'_p$ for which $\inf_{|u|\geq A} l(u) \geq \sup_{|u|\leq A^\gamma} l(u)$, $\gamma > 0$, for all A

Some other symbols

$\mathfrak{M}(\mathbf{F})$ .... class of all finite stopping times with respect to $\mathbf{F}$

$\overline{\mathfrak{M}(\mathbf{F})}$ .... class of all stopping time with respect to $\mathbf{F}$

$\Phi(K)$ .... class of all normalizing matrices

$H \circ A$ .... the Lebesgue–Stieltjes integral

$f\cdot\xi$ .... the stochastic integral over a process ξ

$f * \mu$ .... the integral with respect to a random measure μ

$f * (\mu - \nu)$ .... the stochastic integral with respect to a local martingale measure $\mu - \nu$

$\delta_t^{+,\alpha}$ .... the Neumann–Pearson test of level α

$\hat{\theta}_t$ .... the maximum likelihood estimate of a parameter θ

$\tilde{\theta}_t$ .... the Bayes estimate of a parameter θ

$I(\xi^t, \theta)$ .... the amount of Shannon information contained in an observation ξ^t of an unknown parameter θ

$I(\bar{\theta}^t, \theta)$ .... the amount of Shannon information contained in an estimate $\bar{\theta}^t$ of parameter θ

$H_t(\varepsilon) = H(\varepsilon; \widetilde{P}^t, P^t)$ ..... the Hellinger integral of order ε for measures $\widetilde{P}^t$ and P^t

$I(P^t|\widetilde{P}^t)$ .... the entropy of a measure P^t with respect to a measure $\widetilde{P}^t$, the Kullback–Leibler information between measures P^t and $\widetilde{P}^t$

$(H^t) \vartriangle (\widetilde{H}^t)$ .... completely asymptotically distinguishable families of hypotheses

$(H^t) \overline{\vartriangle} (\widetilde{H}^t)$ .... families of hypotheses that are not completely asymptotically distinguishable

$(H^t) \cong (\widetilde{H}^t)$ .... completely asymptotically indistinguishable families of hypotheses

$(\widetilde{H}^t) \lhd (H^t)$ .... a family of hypotheses $(\widetilde{H}^t)$ is contiguous to a family of hypotheses (H^t)

$(\widetilde{H}^t) \overline{\lhd} (H^t)$ .... a family of hypotheses $(\widetilde{H}^t)$ is noncontiguous to a family of hypotheses (H^t)

$(H^t) \lhd\rhd (\widetilde{H}^t)$ .... mutually contiguous families of hypotheses

$(H^t) \overline{\lhd}\overline{\rhd} (\widetilde{H}^t)$ .... mutually noncontiguous families of hypotheses

$(H^t) \overline{\lhd\rhd} (\widetilde{H}^t)$ .... families of hypotheses are not mutually contiguous

$(P^t) \vartriangle (\widetilde{P}^t)$ .... completely asymptotically separable families of measures

$(P^t) \overline{\vartriangle} (\widetilde{P}^t)$ .... families of measures are not completely asymptotically separable

$(P^t) \cong (\widetilde{P}^t)$ .... completely asymptotically inseparable families of measures

$(\widetilde{P}^t) \triangleleft (P^t)$ .... a family of measures $(\widetilde{P}^t)$ is contiguous to a family of measures (P^t)

$(\widetilde{P}^t) \overline{\triangleleft} (P^t)$ .... a family of measures $(\widetilde{P}^t)$ is noncontiguous to a family of measures (P^t)

$(P^t) \triangleleft\triangleright (\widetilde{P}^t)$ .... mutually contiguous families of measures

$(P^t) \overline{\triangleleft}\,\overline{\triangleright} (\widetilde{P}^t)$ .... mutually noncontiguous families of measures

$(P^t) \overline{\triangleleft\triangleright} (\widetilde{P}^t)$ .... families of measures are not mutually contiguous

Bibliographical Notes

Chapter 1

1.1. In this section, we discuss notions and results of the general theory of stochastic processes and stochastic integration needed for the further presentation. More details on this subject are given in [**30**], [**138**], [**139**], [**153**], [**156**], [**96**], [**98**], [**128**], [**20**], [**21**], [**100**], [**207**]; also see [**1**], [**46**].

1.2. We follow Ibragimov and Khas'minskii [**40**] in describing general statistical experiments. In [**4**] and [**115**], statistical experiments are called *statistical structures*. Lemma 1.2.1 plays the crucial role in absolute continuity of probability measures. This lemma was proved by Girsanov [**17**] for Wiener processes; many other authors have later obtained this result for different types of stochastic processes; see [**25**], [**46**], [**96**], [**98**], [**113**], [**139**], [**153**], and [**156**]. Theorem 1.2.1 without the additional condition VIII) was proved in [**46**], [**153**], [**98**], and [**156**]. The local density was studied in many special cases; it was obtained in [**113**] for Markov processes, in [**96**] for diffusion type processes, in [**25**] for Markov type processes, and in [**45**] for counting processes. Similar results related to the exponential representation of distributions of stochastic processes are obtained in [**166**], [**231**]. Statistical experiments generated by stochastic processes and their properties are described in [**40**], [**146**], [**158**], and [**171**].

1.3. Theorems 1.3.1 and 1.3.2 were proved in [**84**]. A proof of Theorem 1.3.3 was given in [**83**]. Theorems 1.3.4–1.3.6 are new; for close results see [**76**], [**80**], and [**81**]. More details about semimartingales are given in [**156**], [**154**], [**27**]. Limit theorems for special cases of semimartingales can be found in [**51**], [**56**], [**71**], [**73**], [**77**], [**116**], [**119**], and [**149**].

Chapter 2

2.1. In this section we discuss some notions and results needed for the further presentation. More details are given in [**9**] and [**58**].

2.2. In this section we follow [**85**] and [**89**]. The complete group of types of the asymptotic distinguishability between families of hypotheses was introduced by the author [**85**]. The early paper [**70**] should also be mentioned, where types $\mathbf{a}_0$, $\mathbf{a}_1$, and $\mathbf{e}$ of the asymptotic distinguishability were introduced for similar diffusion processes. These types also form a complete group. Various definitions of the asymptotic distinguishability were considered by other authors: Krafft [**65**] and Ingster [**43**] gave a definition for families of hypotheses, and Liptser, Pukel'sheim, and Shiryaev [**95**], Eagleson [**141**], Eagleson and Mémin [**142**], and Le Cam and Traxler [**170**] introduced the same notion for the complete asymptotic discrimination of families of measures. Mutual contiguity for families of measures was introduced by Le Cam [**169**] (note that this notion is called simply *continuity* in that paper). Le Cam [**169**] and Roussas [**110**] gave various characterizations for the mutual contiguity. The contiguity of a family of measures to another family was defined by Hájek and Šidák [**14**]. Several characterizations for this kind of contiguity were given by Liptser, Pukel'sheim, and Shiryaev [**95**], Eagleson and Mémin [**142**], Hall and Loynes [**148**], and F. Liese [**236**]. These questions were also discussed in [**146**], [**171**], [**156**], [**174**] and [**127**].

2.3. Here we follow [**88**]. The results of this section can be found in [**87**] for the case of equivalent measures P^t and $\widetilde{P}^t$. Kullback and Leibler [**200**] introduced the relative entropy $I(P^t|\widetilde{P}^t)$ under the name *information for distinguishability*. Sanov [**201**] also used this notion in problems on large deviations for polynomial schemes (see also [**202**]–[**205**]). Theorem 2.3.1 is a generalization of a classical Stein lemma (see [**91**], [**198**], and [**219**] for other generalizations). The original proof of Stein's lemma is based on a result on large deviations. Later, Kullback [**53**] suggested another method using the law of large numbers (see also [**124**]). Theorems 2.3.2 and 2.3.3 belong to Krafft and Plachky [**164**] for the case of sequences of independent identically distributed random variables. A generalization of their results to the minimax risk in the case of composite hypotheses was given in [**206**]. Another proof of Theorem 2.3.2 was obtained by Kolomiets [**49**]. Theorems 2.3.3 and Corollary 2.3.4 are related to results on large deviations for Λ_t as $t \to \infty$. If the condition $\lim_{t\to\infty} \chi_t^{-1} \ln \alpha_t = -a$ is assumed for $a > 0$ instead of $\alpha 1'$ and condition Λ^* is satisfied, then, using Lemma 2.3.4, one can show that $\lim_{t\to\infty} \chi_t^{-1} \ln \beta(\delta_t^{+,\alpha_t}) = -b(a)$, where $b(a)$ is a certain function [**208**]. The first results on large deviations were obtained by Khintchine [**209**], Smirnoff [**210**], Cramér [**211**], and Chernoff [**212**], who studdied the case of sums of independent identically distributed random variables. For further developments see [**201**], [**205**], [**213**], [**189**], [**191**], [**143**], [**144**], [**214**]. The function $b(a)$ was studied in [**215**] and [**216**] for observations of independent identically distributed random variables and in [**217**] and [**218**] for general binary statistical experiments.

2.4. In this section we present results of [**88**] and [**92**] in a somewhat different form. Results similar to Theorems 2.4.1 and 2.4.2 were obtained by Basawa and Scott [**131**], Hornik [**150**], and Janssen [**157**]. The idea of the proof of Theorems 2.4.3 and 2.4.4 is similar to that of Theorems 2.4.1 and 2.4.2. A close result was proved in [**197**]. Relation (2.4.25) was obtained in [**197**] for independent identically distributed random variables.

2.5. Here we follow [**92**] and [**93**]. Similar results with the Gaussian law in condition $\Lambda 6$ can be found in [**88**]. Close results were obtained by Roussas [**110**] and Hall and Loynes [**148**]. The main Lemma 2.3.1 was proved for the Gaussian law $\mathcal{S} = \mathcal{N}(a, \sigma^2)$ in [**110**] with $a = -\sigma^2/2$ and in [**88**] with $a < -\sigma^2/2$. Lemma 2.5.1 was proved in [**93**] for a general law (see also [**92**]). Theorem 2.5.4 is new. General results on relative compactness and tightness can be found in [**7**], [**127**], [**110**], and [**156**].

2.6. In this section, we present results of [**88**] and [**92**] in a somewhat modified form (see also [**89**], [**90**], [**176**], [**177**]). The asymptotic decomposition of the likelihood ratio in condition $\Lambda 8^*$ was obtained in [**70**] for unbounded vectors u_t. Condition $\Lambda 8^*$ with bounded vectors u_t for measures P^t and $\widetilde{P}^t$ belonging to a parametric family is known as the *local asymptotic normality condition* and was presented in various books, mainly devoted to the theory of parametric estimation; see [**169**], [**110**], [**40**], [**29**], [**57**], [**146**], [**9**], [**184**]. Condition $\Lambda 8'$ with bounded vectors u_t for measures P^t and $\widetilde{P}^t$ belonging to a parametric family is closely related to the notion of *local asymptotic mixed normality.*

2.7. The reductions of hypotheses testing considered in this section were studied in [**88**]. In presenting them, we follow [**92**] and [**177**]. More detail and special cases as well as further results can be found in [**88**], [**92**], and [**177**].

Chapter 3

3.1. In this section, we follow [**156**] and [**49**]. For properties and applications of Hellinger processes, see [**160**], [**174**], [**178**], [**179**], [**181**]. Representation (3.1.9) for a Hellinger process of order ε was obtained in [**86**] using the multiplicative decomposition of the process $Y(\varepsilon)$. If $\varepsilon_-^t = \inf\{\varepsilon\colon H_t(\varepsilon; \widetilde{\mathsf{P}}, \mathsf{P}) < \infty\}$ and $\varepsilon_+^t = \sup\{\varepsilon\colon H_t(\varepsilon; \widetilde{\mathsf{P}}, \mathsf{P}) < \infty\}$, then $\varepsilon_-^t \uparrow \varepsilon_- \leq 0$ and $\varepsilon_+^t \downarrow \varepsilon_+ \geq 1$ as $t \to \infty$; see [**223**], [**224**]. Therefore the Hellinger process $h(\varepsilon) = h(\varepsilon; \widetilde{\mathsf{P}}, \mathsf{P})$ is well defined by Theorem 3.1.1. This allows one to get assertions of this section for all $\varepsilon \in (\varepsilon_-, \varepsilon_+)$. More details are given in [**223**]–[**225**].

3.2. The law of large numbers was proved in [**84**]. Theorem 3.2.2 is obtained by Kolomiets [**49**]. Theorem 3.2.3 is based on Theorems 1–3 in [**86**]. The idea to split the space Ω by the Hellinger process was applied for the first time to diffusion processes in [**67**]; the same method was later used in [**81**] to estimate parameters of counting processes. Theorem 3.2.4 was proved in [**92**], [**176**]. Further references can be found in [**92**] and [**176**].

3.3. This section is based on [**83**] and [**84**]. Asymptotic relation (3.3.5) was obtained in [**75**] for diffusion type processes and in [**78**] for counting processes. This relation was discussed in [**77**] for Markov processes and in [**226**] for semimartingales with determinate triplets of predictable characteristics.

3.4. General limit theorems for diffusion processes were obtained in [**66**], [**70**], and [**71**]. Types $\mathbf{a}_0$, $\mathbf{a}_1$, and $\mathbf{e}$ of the asymptotic distinguishability between families of hypotheses were introduced in [**70**] in the case of similar hypotheses. Some results for the scheme "a signal in white noise" can be found in [**82**]. Example 3.4.2 is taken from [**92**]. Results for null recurrent processes were discussed in [**87**]. Similar asymptotic results for the minimax risk belong to Gushchin [**28**]. Sequential tests for distinguishing diffusion processes were given in [**126**] and [**96**]. Properties of homogeneous diffusion processes were described in more detail in [**19**]–[**20**], [**114**], [**122**]. Corresponding results for (and applications of) Hellinger integrals and processes of order ε can be found in [**28**], [**219**], [**237**], and [**240**].

3.5. Here we present some general limit theorems of [**71**] and [**78**]. There is an extensive literature on the problem of distinguishing counting processes with determinate compensators; see, for example, [**47**], [**134**], [**72**], and [**194**]. Results discussed in this section were obtained in [**94**] for the case of determinate compensators and renewal processes. The idea of the proof of Theorem 3.5.9 was introduced in [**86**]. Generalizations of Theorems 3.5.7, 3.5.8, 3.5.10, and 3.5.11 were obtained in [**228**] for renewal processes with discontinuous compensators. Theorems 3.5.1–3.5.4 were proved in [**227**] for counting processes with discontinuous compensators; see also the survey papers [**225**] and [**230**].

Chapter 4

4.1. This section contains results on properties of statistical estimates $\hat{\theta}_t$ and $\tilde{\theta}_t$ due to Ibragimov and Khas'minskii [**40**].

4.2. Theorems 4.2.1–4.2.6 are taken from [**83**], [**84**]; see also [**175**], [**176**]. Properties of the likelihood ratio for semimartingales with determinate triplets of predictable characteristics were studied in [**226**] (the case of discontinuous characteristics was also treated in [**226**]).

4.3. An extensive literature is devoted to the problem of estimation of parameters for diffusion processes. The Gaussian processes were the first class to be studied in the literature; see, for example, [**3**], [**39**], [**50**], [**96**], [**123**], [**167**] and the references in [**40**], [**57**], [**96**]. This case was treated in detail in Arató's book [**2**]. The problem becomes complicate if the diffusion process is not Gaussian. Nevertheless, the asymptotic behavior of statistical estimates was also studied for a linear parameter by using limit theorems for stochastic integrals over the Wiener process [**52**], [**61**], [**62**], [**103**], [**116**], [**123**], [**133**]. The next step is to study the case of a nonlinear parameter, and this requires a method based on the LAN condition [**40**]. The main problem arising in this approach is to prove condition **K3** for $\beta > k$ and **K4**. To prove condition **K4**, some authors (see [**5**], [**57**], [**76**]) pose restrictions on the shift in the following form:

$$\int_0^t (a(s,x,y) - a(s,x,\theta))^2 \, ds \geq \kappa |y-\theta|^2, \qquad \kappa > 0.$$

Note that for a wide class of processes no conditions of this kind are satisfied, and this does not allow one to study such processes. The truncation method for the space of trajectories used in the proof of Theorem 4.2.5 solves this problem (this method was proposed in [**64**]; see also [**68**]). The proof of condition **K3** for $\beta > k$ is based on the idea of the proof of Lemma 3.3.2 in [**40**]. This idea instead of Lemma 3.5.2 in [**40**] works in the Gaussian case (see Lemma 4 in [**76**] and Theorem 4.2.6). We mention also [**111**], where LAN condition was established for diffusion processes with periodic coefficients depending on the process. Methods of estimation were described in [**229**], [**230**], [**183**], [**238**], [**239**] for diffusion processes.

4.4. The parameter estimation for counting processes was treated in a rather great number of papers. Most of these papers deal only with the case of Poisson processes; see [**47**], [**57**], [**72**], [**134**], [**173**], [**190**]. The application of the general asymptotic method based on the LAN property [**4**] encounters the same problems as in the case of diffusion processes. The LAN property was obtained almost simultaneously for Poisson processes [**55**], Poisson type processes [**56**], and general counting processes [**71**]. The proof of condition **K3** for $\beta > k$ is simpler in this case due to an idea used for the first time in the proof of Lemma 4 in [**76**], where the case of diffusion type processes was considered; the proof for counting processes can be found in [**81**]. Condition **K4** was also proved in [**81**] by using the truncation method introduced in [**64**] for diffusion processes. Further results on properties of the likelihood ratio and parameter estimation can be found in [**229**], [**230**], [**183**], [**238**], [**239**].

Chapter 5

5.1. Theorems 5.1.1–5.1.3 are taken from [**74**] and [**79**]. Theorem 5.1.1 was proved in [**68**] for $k = 1$. Relation (5.1.3) was obtained earlier for sequences of independent random variables in [**68**] and [**117**], purely discontinuous processes with independent increments in [**73**], and Markov processes in [**77**]. Relation (5.1.32) for sequences of independent random variables was proved in [**63**]; with terms of higher orders in the asymptotic expansion it was also proved in [**187**] and [**193**] for $p = 1$ and Θ' finite. Relation (5.1.32) with terms of higher orders in the asymptotic expansion was obtained in [**242**] for $p = 1$ and the scheme of general statistical observations ξ^t. Upper and lower bounds as well as the asymptotic behavior of the information $I(\xi^t, \theta)$ up to terms of order $O(1)$ can be found in [**246**] for the case of a continuous parameter θ and observations of independent identically distributed random variables.

5.2. Theorems 5.2.1–5.2.4 are taken from [**65**], [**6**], [**74**] and given here in a somewhat different form. The idea of the proof of Lemma 5.2.1 was used for the first time in [**243**]. Inequality (5.2.29) was obtained in [**34**] and [**69**] for the case $l(v) = |v|^2$ and sequences of independent random variables. The rate of convergence in asymptotic relation (5.2.51) was studied in the case of $p = 1$ and a finite Θ' in [**187**] and [**245**] for observations of independent random variables, and in [**242**] and [**244**] for the scheme of general statistical observations.

5.3. Corollary 5.3.1 and Theorem 5.3.3 are taken from [**74**]. Theorem 5.3.2 was proved in [**65**] and [**69**] for special classes of functions $L(y, z)$ and sequences of independent random variables. The equivalence between the sufficiency and information sufficiency was proved in Linnik's book [**241**] for the Shannon information and in Kullback's book [**53**] for the Kullback information. More details on the asymptotic information sufficiency were given in [**107**], [**65**], [**69**], and [**74**].

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