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MATHEMATICAL MONOGRAPHS

Volume 203

Mathematics of Information and Coding

Te Sun Han
Kingo Kobayashi

American Mathematical Society

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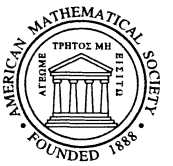

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Dedicated to Claude E. Shannon

Research rather than exposition is the keynote.
— Claude E. Shannon

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Preface to the English Edition

This book is intended to provide engineering and/or statistic students, communication engineers, and relevant mathematicians with a firm theoretic basis in *source coding* (or *data compression*) in information theory. Usually, standard information theory textbooks contain not only *source coding* but also *channel coding*. In this book, however, we have focused only on *source coding*. The reason is first that *source coding* is generally more basic than *channel coding* in elementary information theory textbooks, and second that the advancement of theories and practices in source coding has indeed been prominent, particularly in the 1990's, and so we have preferred to systematically describe this recent advancement of source coding from the very basics to the top frontiers in an intuitively transparent but mathematically sound manner. We believe that this systematic manner is possibly one of the best methods to help the reader clearly and deeply understand step-by-step the probabilistic and/or combinatorial essences of *source coding*.

As a consequence, however, we regret that we have no space to discuss *channel coding*. We just hope to be fortunate enough to have an opportunity in the near future to write another book about *channel coding*.

The main modern trend of source coding is featured by the *universality* of coding schemes, where by universality we mean that an encoder and a decoder can achieve the asymptotically optimal compression ratio without even knowing the source statistic. In view of this trend, we have attempted as much as possible in this book to highlight the *universality*.

This book consists of eight chapters. Chapter 1 provides a short overview about several typical areas in the very wide spectrum of information theory. This chapter will enable the reader to intuitively grasp what information theory is to deal with. Chapters 2 and 3 are devoted to the basic notions of information theory and the fundamental mechanism of source coding, respectively. By reading only these three chapters, the reader could obtain clear insight into the fundamental notions of information theory. We give a comprehensive description of *arithmetic code* in Chapter 4. Although arithmetic code itself is *not* originally universal, it is straightforward to modify the original version of arithmetic code so as to be *universal*. In fact, various kinds of universal codes in this direction have actually been devised. Chapters 5, 6, and 7 are devoted to *universal coding* of integers, *universal coding* of texts, and *universal coding* of compound sources, respectively. These three chapters elucidate, from a respective point of view, the relevant theoretical details of the modern and standard *universal codings*. Finally, Chapter 8 is incorporated to demonstrate that the universal coding as stated in Chapter 7 is directly applicable, as a general principle (called the MDL principle), also to a wide range of data analysis problems, including problems of pattern recognition, self-organization of data, machine learning, artificial intelligence, and so on.

Thus, this book can be said to be an attempt to summarize as compactly as possible a modern typical trend of universal source codings as well as its applications to statistical data analysis in general. This clearly reveals a new intrinsic linkage between information theory and statistics, although these two research fields have been regarded for a long time as being separate.

Furthermore, in this English version, at the end of each chapter (except for Chapter 1), a considerable number of exercises are appended (approximately 60 pages in total). This will help the reader not only to understand more deeply the essence of those arguments as developed in the text, but also to broaden their own views about the many interesting information-theoretic topics. The reader may enjoy reading and proving these exercises.

We would like to thank all the people who, in one way or another, helped us publish this book. First, we should thank Masao Iri and Shun-ichi Amari who gave us the chance to write this book. We are grateful also to Suguru Arimoto who stimulated our fascination with information theory. Our sincere gratitude goes also to our colleagues Shojiro Sakata, Mamoru Hoshi, and Hiroyoshi Morita for useful discussions. We are very thankful to Joe Suzuki who prepared the first English draft of this book, without which the present English version could not have appeared. Last, we would like to thank all members of I&C (a small research group for Information and Coding in Tokyo). In particular, we are very pleased to thank Hiroshi Sato who not only organized this group but has continued to take the leading role in research activities at I&C.

Te Sun Han
Kingo Kobayashi
April 24, 2001

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