

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 225

**Moduli of Riemann Surfaces,
Real Algebraic Curves, and
Their Superanalogs**

S. M. Natanzon



American Mathematical Society

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S. M. Natanzon

Translated by
Sergei Lando



American Mathematical Society
Providence, Rhode Island

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С. М. Натанзон

МОДУЛИ РИМАНОВЫХ ПОВЕРХНОСТЕЙ И ВЕЩЕСТВЕННЫХ АЛГЕБРАИЧЕСКИХ КРИВЫХ И ИХ СУПЕРАНАЛОГИ

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To Irina with love and gratitude

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- [1] N. L. Alling and N. Greenleaf, *Foundations of the theory of Klein surfaces*. Berlin–Heidelberg–New York. Springer-Verlag, 1971. Lecture Notes in Mathematics, vol. 219.
- [2] C. Arf, *Untersuchungen über quadratische formen in Körpern der Charakteristik 2*. I, J. Reine Angew. Math. **183** (1941), 148–167.
- [3] V. I. Arnold, *Topological classification of complex trigonometric polynomials and the combinatorics of graphs with an identical number of vertices and edges*. Funct. Anal. Appl. **30** (1996), no. 1, 1–17.
- [4] M. F. Atiyah, *Riemann surfaces and spin structures*. Ann. Sci. École Norm. Sup. (4). **4** (1971), 47–62.
- [5] H. F. Baker, *Abel's theorem and the allied theory including the theory of theta functions*. Cambridge, 1897.
- [6] A. M. Baranov and A. S. Shvarts [Schwarz], *Multiloop contribution to string theory*. JETP Lett. **42** (1985), no. 8, 419–421.
- [7] A. M. Baranov, Yu. I. Manin, I. V. Frolov, and A. S. Schwarz, *A superanalog of the Selberg trace formula and multiloop contributions for fermionic strings*. Comm. Math. Phys. **111** (1987), 373–392.
- [8] L. Bers, *Quasiconformal mapping and Teichmüller's theorem*. in: Analytic Functions. Princeton Univ. Press, 1960. pp. 89–119. Princeton Math. Ser. vol. 24.
- [9] A. I. Bobenko, *Uniformization and finite-gap integration*, preprint LOMI, P10-86, Leningrad, 1986.
- [10] ———, *Schottky uniformization and finite-gap integration*. Soviet Math. Dokl. **36** (1988), no. 1, 38–42.
- [11] W. Burnside, *On a class of automorphic functions*. Proc. London Math. Soc. **23** (1892), 49–88.
- [12] E. Bujalance, A. F. Costa, S. Natanzon, and D. Singerman, *Involutions of compact Klein surfaces*. Mathematische Zeitschrift **211** (1992), no. 3, pp. 461–478.
- [13] A. L. Carey and K. C. Hannabuss, *Infinite dimensional groups and Riemann surface field theories*. Comm. Math. Phys. **176** (1996), no. 2, 321–351.
- [14] I. V. Cherednik, *On the conditions of reality in "finite-gap integration"*. Dokl. Akad. Nauk SSSR **252** (1980), no. 5, 1104–1108.
- [15] A. Clebsch, *Zur Theorie der Riemann Flächen*. Math. Ann. **6** (1873), 1–15.
- [16] A. Comessatti, *Sulle variata abeliane reali*. Ann. Math. Pura Appl. (4). **2** (1925), 67–102; **3** (1926), 27–71.
- [17] M. Dehn, *Über Abbildungen*. Math. Tidsskr. B. **1939**, 25–48.
- [18] B. A. Dubrovin, *Theory of operators and real algebraic geometry*. in: Global analysis—studies and applications, III. Berlin: Springer-Verlag, 1988. pp. 42–59. Lecture Notes in Mathematics, vol. 1334.
- [19] ———, *Geometry of 2D topological field theories*. in: Integrable Systems and Quantum Groups. Berlin: Springer-Verlag, 1996. pp. 120–348. Lecture Notes in Mathematics, vol. 1620.

- [20] ———, *Painlevé transcendents in two-dimensional topological field theory*. Preprint SISSA. 1998. 24/98/FM.
- [21] B. A. Dubrovin and S. M. Natanzon, *Real two-zone solutions of the sine-Gordon equation*. Funktsional. Anal. i Prilozhen. **16** (1982), no. 1, 27–43, 96.
- [22] ———, *Real theta-function solutions of the Kadomtsev-Petviashvili equation*. Math. USSR-Izv. **32** (1989), no. 2, 269–288.
- [23] B. A. Dubrovin, S. P. Novikov, and A. T. Fomenko, *Modern geometry—methods and applications*. Springer-Verlag, New York, 1992.
- [24] C. J. Earle, *On the moduli of closed Riemann surfaces with symmetries*. in: Advances in the theory of Riemann surfaces. Princeton Univ. Press, 1971. pp. 119–130. Ann. of Math. Stud. no. 66.
- [25] G. Falqui and C. Reina, *$N = 2$ super Riemann surfaces and algebraic geometry*. J. Math. Phys. **31** (1990), no. 4, 948–952.
- [26] J. Fay, *Theta-functions on Riemann surfaces*. Berlin: Springer-Verlag, 1973. Lecture Notes in Mathematics, vol. 352.
- [27] D. Friedan, *Notes on string theory and two-dimensional conformal field theory*. in: Proc. Workshop on unified string theories (Santa Barbara, Calif., 1985). Singapore: World Sci. Publishing, 1986.
- [28] F. Fricke, F. Klein, *Vorlesungen über die Theorie der automorphen Funktionen*. B. 1, 2. Leipzig: Teubner, 1897, 1912. — New York: Johnson Reprint Corp., and Stuttgart: Teubner Verlagsgesellschaft, 1965.
- [29] D. Grepner, *Space-time supersymmetry in compactified string theory and superconformal models*. Nuclear Phys. **296** (1988), 757–779.
- [30] B. H. Gross and J. Harris, *Real algebraic curves*. Ann. Sci. École Norm. Sup. (4) **14** (1981), no. 2, 157–182.
- [31] A. Harnack, *Über die Vieltheiligkeit der ebenen algebraischen Kurven*. Math. Ann. **10** (1876), 189–199.
- [32] A. Hurwitz, *Über Riemannsche Flächen mit gegebenen Verzweigungspunkten*. Math. Ann. **39** (1891), 1–61.
- [33] ———, *Über die Fourierschen Konstanten integrierbarer Funktionen*. Math. Ann. **57** (1903), 425–446.
- [34] A. Jaffe, S. Klimek, and L. Lesniewski, *Representations of the Heisenberg algebra on a Riemann surface*. Comm. Math. Phys. **126** (1990), no. 2, 421–433.
- [35] D. Johnson, *Spin structures and quadratic forms on surfaces*. J. London Math. Soc. (2) **22** (1980), no. 2, 365–373.
- [36] L. Keen, *Intrinsic moduli on Riemann surfaces*. Ann. of Math. (2) **84** (1966), no. 3, 404–420.
- [37] ———, *Canonical polygons for finitely generated Fuchsian groups*. Acta Math. **115** (1965), 1–16.
- [38] ———, *On Fricke moduli*. in: Advances in the Theory of Riemann Surfaces. Princeton Univ. Press, 1971, pp. 205–224. Ann. of Math. Studies. No. 66. *A correction to “On Fricke moduli”*, Proc. Amer. Math. Soc. **40** (1973), 60–62.
- [39] B. Kerekjarto, *Vorlesungen über Topologie. I. Flächentopologie*. Berlin: Springer-Verlag, 1923.
- [40] F. Klein, *Riemann Flächen*. Göttingen: Vorlesung. 1892, vol. 1, 2; Neuedruck, 1906.
- [41] M. Kontsevich and Yu. Manin, *Gromov–Witten classes, quantum cohomology and enumerative geometry*. Comm. Math. Phys. **164** (1994), no. 3, 525–562.
- [42] S. Kravetz, *On the geometry of Teichmüller spaces and the structure of their modular groups*. Ann. Acad. Sci. Fenn. Ser. A I. **278** (1959), 1–35.
- [43] I. M. Krichever and S. P. Novikov, *Virasoro–Gelfand–Fuks type algebras, Riemann surfaces, operator’s theory of closed strings*. J. Geom. Phys. **5** (1988), no. 4, 631–661.

- [44] A. M. Macbeath, *The classification of non-Euclidean plane crystallographic groups*. *Canad. J. Math.* **19** (1967), no. 6, 1192–1205.
- [45] Yu. I. Manin, *Superalgebraic curves and quantum strings*. in: *Proc. Steklov Inst. Math.* **1991**, no. 4, 149–162.
- [46] Yu. Manin and S. Merkulov, *Semisimple Frobenius (super) manifolds and quantum cohomology of P^r* . *Topol. Methods Nonlinear Anal.* **9** (1997), no. 1, 107–161.
- [47] A. D. Mednykh, *Determination of the number of nonequivalent coverings over a compact Riemann surface*. *Dokl. Akad. Nauk SSSR* **239** (1978), no. 2, 269–271.
- [48] M. I. Monastyrsky and S. M. Natanzon, *The moduli space of superconformal instantons in sigma models*. *Modern Phys. Lett. A.* **6** (1991), no. 19, 1787–1796; *ibid.*, no. 31, 2919.
- [49] ———, *The moduli space of instantons in $N=2$ supersymmetrical σ -models*. *Classical Quantum Gravity* **12** (1995), no. 9, 2149–2156; **13** (1996), 1277.
- [50] ———, *The moduli space of instantons in $N=2$ supersymmetrical σ -models*. in: *Topics in statistical and theoretical physics*. Providence, RI: Amer. Math. Soc., 1996, pp. 195–202. *Amer. Math. Soc. Transl. Ser. 2*, vol. 177.
- [51] D. Mumford, *Theta characteristics of an algebraic curve*. *Ann. Sci. École Norm. Sup. (4)* **4** (1971), 181–192.
- [52] S. M. Natanzon, *Invariant lines of Fuchsian groups*. *Uspekhi Mat. Nauk* **27** (1972), no. 4(166), 145–160.
- [53] ———, *Invariant lines of Fuchsian groups and moduli of real algebraic curves*. PhD Thesis, Moscow, TsEMI, 1974.
- [54] ———, *Moduli of real algebraic curves*. *Uspekhi Mat. Nauk* **30** (1975), no. 1(181), 251–252.
- [55] ———, *Moduli spaces of real algebraic curves*. in: *Proc. Moscow Math. Soc.* **37** (1978), 219–253.
- [56] ———, *Finite groups of homeomorphisms of surfaces and real forms of complex algebraic curves*. PhD Thesis, Yaroslavl, 1980.
- [57] ———, *Geometric description of the action of hyperelliptic involutions on the homology group of the surface*. in: *Constructive algebraic geometry*, no. 194, Yaroslavl State Pedagogical Institute, Yaroslavl, 1981, pp. 89–96.
- [58] ———, *On the number and topological types of real hyperelliptic curves isomorphic over \mathbb{C}* . in: *Constructive algebraic geometry*, no. 200, Yaroslavl State Pedagogical Institute, Yaroslavl, 1982, pp. 82–93.
- [59] Natanzon S. M., *Spaces of real meromorphic functions on real algebraic curves*. *Dokl. Akad. Nauk SSSR* **279** (1984), no. 4, 803–805.
- [60] ———, *Uniformization of spaces of meromorphic functions*. *Dokl. Akad. Nauk SSSR* **287** (1986), no. 5, 1058–1061.
- [61] ———, *Real meromorphic functions on real algebraic curves*. *Soviet Math. Dokl.* **36** (1988), no. 3, 425–427.
- [62] ———, *The Fricke space of super-Fuchsian groups*. *Funktional. Anal. i Prilozhen.* **21** (1987), no. 2, 80–81.
- [63] ———, *Topology of two-dimensional coverings, and meromorphic functions on real and complex algebraic curves. I*. In: *Trudy Sem. Vektor. Tenzor. Anal.* No. 23 (1988), 79–103.
- [64] ———, *The moduli space of Riemann supersurfaces*. *Math. Notes* **45** (1989), no. 3–4, 341–345.
- [65] ———, *Prymians of real curves and their applications to the effectivization of Schrödinger operators*. *Funct. Anal. Appl.* **23** (1989), no. 1, 33–45.
- [66] ———, *Klein surfaces*. *Russian Math. Surveys* **45** (1990), no. 6, 53–108.
- [67] ———, *Klein supersurfaces*. *Math. Notes* **48** (1990), no. 1–2, 766–772 (1991).

- [68] ———, *Topology of two-dimensional coverings, and meromorphic functions on real and complex algebraic curves. II.* In: Trudy Sem. Vektor. Tenzor. Anal. No. 24 (1991), 104–132.
- [69] ———, *Supercoverings, SNEC-groups and inner groups of Riemann and Klein supersurfaces.* Russian Math. Surveys **45** (1990), no. 2, 225–226.
- [70] ———, *Discrete subgroups of $GL(2, C)$ and spinor bundles on Riemann and Klein surfaces.* Funct. Anal. Appl. **25** (1991), no. 4, 293–294 (1992).
- [71] ———, *Differential equations for Prym theta functions. A criterion for two-dimensional finite-gap potential Schrodinger operators to be real.* Funct. Anal. Appl. **26** (1992), no. 1, 13–20.
- [72] ———, *Topological invariants and moduli of hyperbolic $N = 2$ Riemannian supersurfaces.* Acad. Sci. Sb. Math. **79** (1994), no. 1, 15–31.
- [73] ———, *Moduli spaces of Riemann and Klein supersurfaces.* in: Developments in Mathematics: The Moscow School. London: Chapman & Hall, 1993, pp. 100–130.
- [74] ———, *Moduli spaces of Riemann $N = 1$ and $N = 2$ supersurfaces.* J. Geom. Phys. **12** (1993), no. 1, 35–54.
- [75] ———, *Topology of 2-dimensional coverings and meromorphic functions on real and complex algebraic curves.* Selecta Math. Soviet. **12** (1993), no. 3, 251–291.
- [76] ———, *Classification of pairs of Arf functions on orientable and nonorientable surfaces.* Funct. Anal. Appl. **28** (1994), no. 3, 178–186.
- [77] ———, *On quadratic forms over the field Z_2 .* Russian Math. Surveys **50** (1995), no. 5, 1090–1091.
- [78] ———, *Real nonsingular finite zone solutions of soliton equations.* in: Topics in topology and mathematical physics. Providence, RI: Amer. Math. Soc., 1995, pp. 153–183. Amer. Math. Soc. Transl. Ser. 2, vol. 170.
- [79] ———, *Trigonometric tensors on algebraic curves of arbitrary genus. An analogue of the Sturm–Hurwitz theorem.* Russian Math. Surveys **50** (1995), no. 6, 1286–1287.
- [80] ———, *Spinors and differentials of real algebraic curves.* in: Topology of real algebraic varieties and related topics. Providence, RI: Amer. Math. Soc., 1996, pp. 179–186. Amer. Math. Soc. Transl. Ser. 2, vol. 173.
- [81] ———, *Differential equations for Riemann and Prym theta-functions.* J. Math. Sci. **82** (1996), no. 6, 3821–3823.
- [82] ———, *Moduli spaces of real algebraic supercurves with $N = 2$.* Funct. Anal. Appl. **30** (1996), no. 4, 237–245 (1997).
- [83] ———, *Spaces of meromorphic functions on Riemann surfaces.* in: Topics in singularity theory. Providence, RI: Amer. Math. Soc., 1997, pp. 175–180. Amer. Math. Soc. Transl. Ser. 2, vol. 180.
- [84] ———, *The topological structure of the space of holomorphic morphisms of Riemann surfaces.* Russian Math. Surveys **53** (1998), no. 2, 398–400.
- [85] ———, *Moduli of Riemann surfaces of a Hurwitz-type space and their superanalogues.* Russian Math. Surveys **54** (1999), no. 1, 61–117.
- [86] ———, *Moduli of real algebraic curves and their superanalogues. Spinors and Jacobians of real curves.* Russian Math. Surveys **54** (1999), no. 6, 1091–1147.
- [87] A. M. Polyakov, *Quantum geometry of bosonic string.* Phys. Lett. **103** (1981), 207.
- [88] A. A. Rosly, A. S. Schwarz, and A. A. Voronov, *Geometry of superconformal manifolds.* Comm. Math. Phys. **119** (1988), no. 4, 129–152.
- [89] Y. Ruan and G. Tian, *A mathematical theory of quantum cohomology.* J. Differential Geom. **42** (1995), no. 2, 259–367.
- [90] M. Seppälä, *Teichmüller spaces of Klein surfaces.* Ann. Acad. Sci. Fenn. Ser. A I Math. Dissertationes. 1978, no. 15.
- [91] L. Vajsburd and A. Radul, *Non-orientable strings.* Comm. Math. Phys. **135** (1991), 413–420.

- [92] V. Vinnikov, *Self-adjoint determinantal representations of real plane curves*. Math. Ann. **296** (1993), no. 3, 453–479.
- [93] G. Weichold, *Über symmetrische Riemannsche Flächen und die Periodizitätsmoduln der zugehörigen Abelschen Normalintegrale erster Gatt.* Zeitschrift für Math. und Phys. **28** (1883), 321–351.
- [94] S. Wolpert, *The length spectra as moduli for compact Riemann surface*. Ann. of Math. (2). **109** (1979), no. 2, pp. 323–351.
- [95] H. Zieschang, E. Vogt, and H. D. Coldewey, *Surfaces and planar discontinuous groups*. Berlin: Springer-Verlag, 1980. Lecture Notes in Mathematics, vol. 835.
- [96] S. Zdravkovska, *The topological classification of polynomial mappings*. Uspekhi Mat. Nauk **25** (1970), no. 4(154), 179–180.
- [97] A. Zhivkov, *Finite-gap matrix potential with one and two involutions*. Bull. Sci. Math. **118** (1994), no. 5, pp. 403–440.

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The space of all Riemann surfaces (the so-called moduli space) plays an important role in algebraic geometry and its applications to quantum field theory. The present book is devoted to the study of topological properties of this space and of similar moduli spaces, such as the space of real algebraic curves, the space of mappings, and also superanalogs of all these spaces.

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