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Principal Structures and Methods of Representation Theory

D. Zhelobenko

American Mathematical Society

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D. Zhelobenko

Translated by Alex Martsinkovsky

American Mathematical Society
Providence, Rhode Island

Д. П. Желобенко

ОСНОВНЫЕ СТРУКТУРЫ И МЕТОДЫ
ТЕОРИИ ПРЕДСТАВЛЕНИЙ

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Preface

The title of this book admits two interpretations, with emphasis on either the "principal structures" or the "representation theory". The latter is more preferable, as it is difficult to identify what the basic structures of modern mathematics are. Nevertheless, in a sense, the two interpretations agree.

Indeed, representation theory deals with fundamental aspects of mathematics, beginning with algebraic structures like semigroups, groups, rings, associative algebras, Lie algebras, etc. Eventually topology enters the play by way of algebro-topological and algebro-analytical structures like topological groups, manifolds, Lie groups, etc. Formally speaking, the subject of representation theory is the study of homomorphisms (representations) of abstract structures into linear structures consisting, as a rule, of linear operators on vector spaces. But in fact representation theory is tied up with structure theory. Very early the students of mathematics learn that "ring theory is inseparably linked with module theory". An important feature of this setting is that the above structures are either linear of have suitable linearizations (linear hulls of semigroups, tangent Lie algebras of Lie groups, etc.).

Here we come to the question of the role representation theory plays in modern mathematics. Originally (in the beginning of the 20th century) that role was rather modest and was confined to the representation theory of finite groups and, eventually, finite-dimensional (associative) algebras. We should mention the connections of that theory with problems of symmetry in algebra and geometry, including Galois theory (the symmetries of algebraic equations), and with problems of crystallography. Eventually the subject of representation theory significantly expanded in response to general questions from analysis, geometry, and physics. Fundamental discoveries in theoretical physics, such as the theory of relativity and quantum mechanics, played a significant role in that process. For example, it turned out that logical foundations of quantum mechanics can be adequately expressed in terms of automorphisms of certain algebras (the algebras of observables). The process of describing observables reduces to representation theory of certain Lie groups and algebras. Among classical results of that period we specifically mention the works of E. Cartan and H. Weyl on the general aspects of the theory of Lie groups and on harmonic analysis on compact groups.

The underlying idea of harmonic analysis on groups is based on the connection between a group G and the "dual object" $\hat{G}$ consisting, roughly speaking, of irreducible representations of G. Usually G can be recovered, up to isomorphism, from its dual object $\hat{G}$. A remarkable feature of harmonic analysis is that numerical functions on G can be recovered from their (operator) "Fourier images", where the role of elementary harmonics is played by irreducible representations of G. A meaningful definition of Fourier images on locally compact groups is possible because of the fundamental results of A. Haar, J. von Neumann, and A. Weil on the existence (and uniqueness) of invariant measures on such groups. In that sense, the classical Fourier analysis (Fourier series and integrals) is subsumed into an impressive development program of harmonic analysis on topological groups.

Logical foundations of Fourier analysis can be significantly clarified within the framework of "abstract harmonic analysis", where the group G is replaced by a C^*-algebra. Fundamental results in that direction are due to I. M. Gelfand and M. A. Naimark (in the 1940s). Beginning with the 1950s, the theory of C^*-algebras develops very rapidly and, to a large extent, characterizes the functional analysis of the 20th century. It is important to observe that that theory has fundamental applications to operator algebras, Hopf algebras, dynamical systems, statistical mechanics, quantum field theory, etc.

Modern representation theory deals with a wide variety of associative algebras, including structure algebras of manifolds and Lie groups, universal enveloping algebras of Lie algebras, group (convolution) algebras, Hopf algebras, quantum groups, etc. Notice that the theory of Lie groups, born within the context of differential geometry, is now included in the framework of functional analysis by way of bialgebras and formal groups associated with Lie groups.

One may also expand the definition of representation theory to include, if desired, such neighboring disciplines as abstract theory of differential equations, theory of sheaves on homogenous spaces, microanalysis, quantum field theory, etc.

There is a known thesis according to which "mathematics is representation theory". The corresponding antithesis can be stated as "mathematics does not reduce to representation theory". It is worthwhile to note the nature of the question. Whatever is true, it appears that the scope of representation theory is already comparable with that of the entire mathematics.

It may be that the desire to systematize mathematics in the spirit of representation theory made N. Bourbaki write the multi-volume set "Elements of mathematics". Despite certain shortcomings of that titanic work (excessive formalism, unfinished parts) one finds original treatment of several fundamental issues, including general aspects of algebra, topology, the theory of integration, the theory of Lie groups and Lie algebras, etc.

At present, there is a large number of monographs dealing with various aspects of representation theory, including Lie groups and Lie algebras ([**4**], [**10**], [**14**], [**31**], [**35**], [**61**]). Banach algebras ([**6**], [**8**], [**13**], [**22**], [**49**], [**58**]), algebraic groups ([**3**], [**29**], [**64**], [**73**]), infinite-dimensional groups ([**53**]), general representation theory ([**40**]). The author's monograph [**75**] can be used as an easily accessible source of information on representations of Lie groups, especially suitable for physicists. However, there is still no monograph which would put together all of those aspects of representation theory.

To fill the gap, this book was conceived as a compilation of canonical texts on representation theory. It provides a systematic description of a wide spectrum of algebro-topological structures. On one hand, the concept of such a book is appealing because it allows us to compare ideas and methods from different parts of representation theory. On the other hand, it is also risky just because of the sheer volume of the material to be covered. Nevertheless, the author thinks that a partial resolution of this dilemma is possible because the offered texts have been carefully worked upon and refined.

The contents of the book split into three parts. Part I (Introduction) contains general facts for beginners, including linear algebra and functional analysis. The survey-type sections on topology, theory of integration, etc. (see [**23**], [**24**], [**26**], [**31**]) as well as Appendices A, B, C, and D are written in the same spirit. In the main Part II (General theory) we consider associative algebras, Lie algebras, topological groups, and Lie groups. We also mention some aspects of ring theory and the theory of algebraic groups. We provide a detailed account of classical results in those branches of mathematics, including invariant integration and Lie's theory of connections between Lie groups and Lie algebras. In Part III (Special topics) we consider semisimple Lie algebra and Lie groups, Banach algebras, and quantum groups.

The book brings the reader close to the modern aspects of "noncommutative analysis", including harmonic analysis on locally compact groups. The author regards the contents of this book as a prerequisite for those who want to seriously study representation theory.

The style of the book allows the author to choose the depth of the exposition to his taste. For example, we prove the theorem on the conjugacy of Cartan subalgebras (in complex Lie algebras) but omit a similar result for Borel subalgebras (in semisimple Lie algebras). Yet the author hopes that the reader will see a detailed enough panoramic description of representation theory.

The diverse nature of the compiled material unavoidably leads to discrepancies in traditions, which sometimes cause certain redundancy in the definitions and notation. For example, the notation $\operatorname{End} X$ in the category of vector spaces is sometimes replaced by $\mathrm{L}(X)$ where $\dim X < \infty$.

The exercises included in the book, as a rule, are designed as tests for beginners. Sometimes (in moderation) the results of the exercises are used to shorten certain proofs. Only the exercises marked with an asterisk can be viewed as more or less serious problems.

While working on the book the author felt himself a chronicler. Indeed, the book covers a century in the development of mathematics, a period which is probably not yet fully appreciated.

The contents of the book are, to a large extent, based on two elective courses the author gave at the Independent University of Moscow in 1996–1998. The lecture notes of one of those courses were published in 2001 ([**78**]). The work on this book was partially supported by the RFFI Grant 01-01-00490 and NWO 047-008-009.

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The book is divided into three parts. Part 1 surveys general facts for beginners, including linear algebra and functional analysis. Part 2 considers associative algebras, Lie algebras, topological groups, and Lie groups, along with some aspects of ring theory and the theory of algebraic groups. The author provides a detailed account of classical results in related branches of mathematics, such as invariant integration and Lie's theory of connections between Lie groups and Lie algebras. Part 3 discusses semisimple Lie algebras and Lie groups, Banach algebras, and quantum groups.

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