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## EDITOR'S PREFACE

This volume contains the papers which were presented at the Fourth Symposium in Applied Mathematics of the American Mathematical Society held at the University of Maryland on June 22 and 23, 1951. The subject of the Symposium was *Fluid Dynamics*, and the four sessions were devoted to *Turbulence*, *Compressible Flow*, *Foundations*, and *Incompressible Flow*. The Symposium was cosponsored by the U.S. Naval Ordnance Laboratory, White Oak, Maryland.

One of the papers appears as an abstract, due to prior arrangements for publication, and carries a reference to the complete paper. Another appears in this volume although circumstances beyond our control prevented the author from delivering it at the time of the Symposium.

All who participated in the Symposium are indebted to the McGraw-Hill Book Company, Inc., which, beginning with the Proceedings of the Symposium on Elasticity, has undertaken in these uncertain times the task of bringing the Proceedings of these Symposia on Applied Mathematics to the scientific public in book form.

The Editor gratefully acknowledges the invaluable help afforded by the Committee on Arrangements consisting of W. Leutert, H. Polachek, R. J. Seeger, J. H. Taylor, J. L. Vanderslice, A. Weinstein, and W. M. Whyburn. He is also indebted to R. V. Churchill, J. B. Diaz, and R. M. Davis for help in the editorial work.

Particular thanks are also due to the Mathematical Sciences Division of the Office of Naval Research for their aid in bringing a number of our colleagues, who participated on the program, to the Symposium.

M. H. MARTIN

*Editor*

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