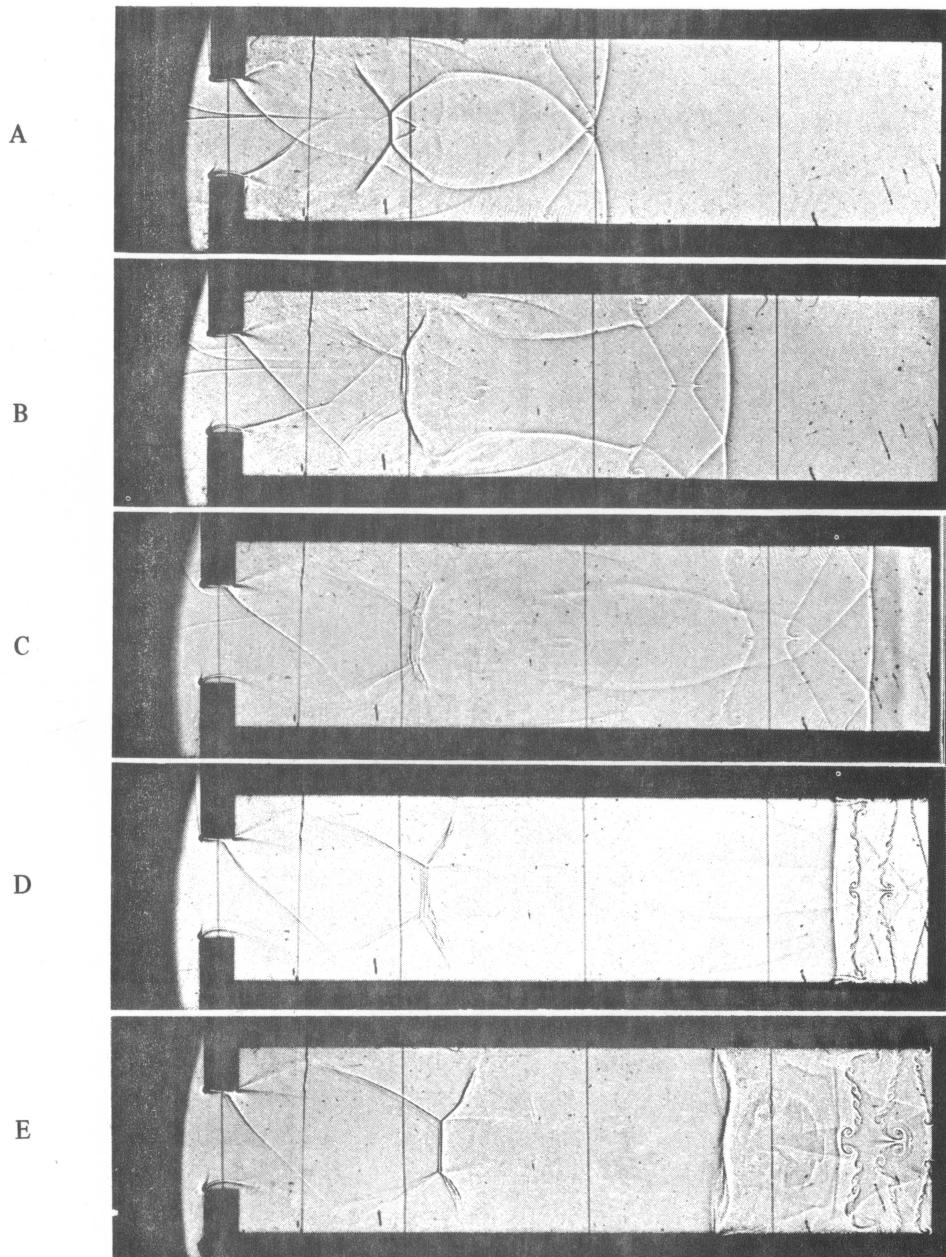


Hydrodynamic Instability



**PROCEEDINGS OF
SYMPOSIA IN
APPLIED MATHEMATICS**

VOLUME XIII



Diffraction of a Mach 2.1 shock wave, showing Helmholtz instability of slip lines (see p. 77)

PROCEEDINGS OF
SYMPOSIA IN APPLIED MATHEMATICS
VOLUME XIII

HYDRODYNAMIC
INSTABILITY

AMERICAN MATHEMATICAL SOCIETY
PROVIDENCE, RHODE ISLAND
1962

PROCEEDINGS OF THE
THIRTEENTH SYMPOSIUM IN APPLIED MATHEMATICS
OF THE AMERICAN MATHEMATICAL SOCIETY

COSPONSORED BY
THE OFFICE OF ORDNANCE RESEARCH

Held at the Hotel New Yorker,
New York, New York on April 14-15, 1960

Garrett Birkhoff, Richard Bellman and C. C. Lin
EDITORS

ISSN 0160-7634
ISBN 0-8218-1313-7
Library of Congress Catalog Number 50-1183

Prepared by the American Mathematical Society under Contract
No. DA-19-020-ORD-5060 with the Ordnance Corps, U. S. Army

Copyright © 1962 by the American Mathematical Society
Printed in the United States of America
All rights reserved except those granted to the United States Government

CONTENTS

FOREWORD	vii
On the instability of shear flows	1
By C. C. LIN and D. J. BENNEY	
Hydrodynamic stability and the initial value problem	25
By K. M. CASE	
Stability theory in plasma physics	35
By MARSHALL N. ROSENBLUTH	
Experimental determination of stability limits	41
By R. J. DONNELLY	
Helmholtz and Taylor instability	55
By GARRETT BIRKHOFF	
Slip line instability	77
By RUSSELL E. DUFF	
Generation of surface waves by shear flows	79
By JOHN W. MILES	
Resonance phenomena in gravity waves	91
By O. M. PHILLIPS	
The statistical geometry of random surfaces	105
By M. S. LONGUET-HIGGINS	
On the oscillations of ships in confused seas	145
By M. ST. DENIS	
Remarks on the functional-analytic approach to turbulence	157
By EBERHARD HOPF	
Statistical mechanics of continuous media	165
By J. KAMPÉ DE FÉRIET	
The closure problem of turbulence theory	199
By ROBERT H. KRAICHNAN	

CONTENTS

Wave propagation in random media	227
By JOSEPH B. KELLER	
Stability of many-body computations	247
By S. ULAM	
Control in linear systems	259
By BROCKWAY McMILLAN	
On the foundations of a theory of stochastic variational processes	275
By RICHARD BELLMAN	
Control of randomly varying linear dynamical systems	287
By R. E. KALMAN	
Asymptotic stability criteria	299
By J. P. LASALLE	
Flow between rotating cylinders in hydrodynamics and hydromagnetics	309
By S. CHANDRASEKHAR	
AUTHOR INDEX	311
SUBJECT INDEX	315

FOREWORD

This volume deals with the following loosely related subjects: hydrodynamic instability, the statistical theory of turbulence, water waves, waves in a medium with random properties, and the theory of guidance and control. These subjects all use similar mathematical methods and ideas, and it seemed impossible to give an adequate picture of contemporary views about hydrodynamic instability without touching on them all. The fact that such a diversified group of problems are exhibited in hydrodynamics alone can only make one admire the inexhaustibility of fascinating problems that occur in nature. Yet we have hardly tapped on the range of problems that arise in astrophysics.

Even a casual perusal of the papers below will bring out a steady trend in theories of hydrodynamic instability, from the linear deterministic processes analyzed so successfully by Kelvin and Rayleigh, to *nonlinear random processes* whose theory must be regarded as the primary goal of future research efforts. We hope this book will help to stimulate and guide such efforts.

The editors wish to thank the authors for providing an excellent collection of papers dealing with such a diversified group of topics and for their cooperation in the publication of this volume.

G. BIRKHOFF

R. BELLMAN

C. C. LIN

AUTHOR INDEX

Italic numbers refer to pages on which a complete reference to a work by the author is given.

Roman numbers refer to pages on which a reference is made to a work of the author. For example, under Minkowski would be the page on which a statement like the following occurs: "This theorem was proved earlier by Minkowski [7, § 2] in the following manner. . . ."

Boldface numbers indicate the first page of articles in this volume.

- Alfven, H., 40
Arrow, K., 275, 285
Atkins, K. R., 53

Balaczs, N. L., 168, 190, 197
Barbasin, E. A., 300, 303, 307
Batchelor, G. K., 143, 196, 208, 213, 224
Bateman, H., 278, 285
Bazer, J., 229, 246
Beckwith, R., 283, 286
Bellman, R., 63, 275, 275, 276, 280, 283,
 285–286, 297, 307
Benjamin, T. B., 79–81, 89
Benney, D. J., 1
Bergeron, T., 57
Berkovitz, L. D., 285, 286
Bernstein, I. B., 40
Bertram, J. E., 295, 297–298
Betchov, R., 211, 224
Bethe, H., 224
Betz, A., 76
Bharucha-Reid, A. T., 167, 196, 297
Birkhoff, G., 55, 63, 70, 76, 143, 165, 168,
 177, 183, 197, 258
Bishop, E., 72
Bjerknes, J., 57
Bjerknes, V., 57
Blair, A., 72
Blanc-Lapiere, A., 196
Bochner, S., 176, 197
Bode, H. W., 259
Born, M., 229, 246
Brard, R., 155
Bromberg, E., 258
Bürgers, J. M., 165, 197

Caldwell, D., 48, 53
Carrier, G. F., 63
Carter, D. C., 70
Cartwright, D. E., 108, 142–143, 154

Case, K. M., 1, 3, 24, 25, 33
Catton, D. B., 136, 143
Chandrasekhar, S., 40, 44–47, 49–51, 53,
 163, 208, 224, 275, 285, 309, 309
Chandrasekharan, K., 176, 197
Chang, C. T., 63
Chang, Y. P., 57
Chernov, L. A., 229, 235, 246
Chew, G., 40
Cole, R. H., 57
Conte, S. D., 89
Corrsin, S., 124–125, 142
Cote, L. J., 154
Couette, M. M., 41, 53
Cox, C., 141, 143

Deissler, R. G., 199, 208, 224
Dombrowski, N., 57
Donnelly, R. J., 41, 44–52, 53
Doob, J. L., 106, 142, 180, 183, 196, 275,
 285
Dorfman, R., 275, 285
Dreyfus, S., 280, 286
Duff, R. E., 77
Dunford, N., 196
Dyson, F. J., 29, 33

Eckart, C., 79, 89, 143
Ehrenfeld, S., 142
Eisenklam, P., 57
Eskinazi, S., 65
Evans, M., 253

Fales, E. N., 76
Feller, W., 275, 285, 297
Fermi, E., 249
Fisher, J., 67, 76
Fleishmann, B. E., 196
Fleming, W. H., 281, 285, 286
Foldy, L. L., 229, 246

- Fortet, R., 196
Francis, J. R. D., 89
Franklin, G. F., 297
Fraser, R. P., 57
Fréchet, M., 177, 179, 196
Freimer, M., 283, 286
Friedman, E. A., 40
Fultz, D., 44–47, 53
Furstenberg, H., 296, 298

Gibson, A. H., 73
Gillis, J., 72
Glicksberg, I., 280, 286
Goldberger, M., 40
Goldstein, S., 45, 53, 76
Goldstine, H. H., 72
Görtler, H., 1, 24
Grim, O., 155
Gross, O., 280, 286

Hagerty, J. C., 76
Hahn, W., 307
Halmoos, P., 177, 196
Hama, F. R., 65, 68, 76
Hanaoka, T., 155
Harlow, F., 72, 253
Harris, T. E., 275, 285
Harrison, W. J., 63
Haskind, M. D., 155
Hasson, D., 57
Havelock, T. H., 155
Heisenberg, W., 79, 89, 163, 206, 208, 224
von Helmholtz, H. L. F., 57, 76
Hille, E., 172, 175–176, 196, 276, 285
Hinze, J. O., 196
Hoffman, W. C., 229, 246
de Hoffmann, F., 224
Holland, J., 280, 286
Hooker, S. G., 65
Hopf, E., 157, 159, 161–162, 163, 165, 197,
 199, 205, 208, 224

Ingram, R. H., 63, 72
Isaacs, R. P., 285, 286
Ishikawa, K., 154

Jacobs, W. R., 155
James, R. W., 143
Jasper, N. H., 143

Kac, M., 275, 285
Kalaba, R., 276, 280, 283, 285–286
Kalman, R. E., 287, 297–298

Kampé de Fériet, J., 165, 168, 196–197
Kaplan, P., 155
Karlin, S., 275, 285
Karman, 65
Kato, M., 154
Kaufman, A., 40
Kay, I., 229, 246
Keller, J. B., 64, 227
Kelvin, Lord, 42, 57, 61–62, 65, 76, 79, 89
Kesten, R., 296, 298
Khinchin, A., 165, 179, 196
Kikuchi, R., 275, 283, 285
Kirchoff, G., 68
Klebanoff, P. S., 1, 24
Kolmogoroff, A. N. [Kolmogorov], 165,
 178, 186, 196–197, 206, 224
Kolodner, I., 64
Kopal, Z., 229, 246
Korvin-Kroukovsky, B. V., 155
Kotik, J., 143
Kraichnan, R. H., 199, 224
Kramer, J. D. R., Jr., 283, 286
Krasovskii, N. N., 300, 303, 307
Kruskal, M. D., 40
Kulsrud, R. M., 40
Kutateladze, S. S., 57

Lagally, M., 149, 155
Lamb, Sir Horace, 62, 76, 155
Landau, L., 26, 33
Landweber, L., 155
Langer, R. E., 1, 5, 24
LaSalle, J. P., 280, 286, 299, 307
Laufer, J., 199, 224
Lax, M., 229, 246
Lax, P., 258
van Lear, G. A., Jr., 168, 188, 190, 197
Lefschetz, S., 307
Lehman, R. S., 286
Lehman, S., 283, 286
Leray, J., 66
Lévy, P., 196, 198
Lewis, D. J., 60, 63, 76
Lewis, F. M., 155
Liapunov, A. M., 307
Lichtenstein, L., 66
Lighthill, M. J., 83, 89
Lin, C. C., 1, 5, 24, 28, 33, 61, 79–80, 82,
 89, 224
Lions, J. L., 66
Lock, R. C., 89
Loève, M., 196
Long, J. D., 76

AUTHOR INDEX

313

- Longmire, C. L., 40
Longuet-Higgins, M. S., 105, 108–109, 112,
 142–143, 154
Low, F., 40
Lynch, R. E., 258
- McFadden, J. A., 109, 142
McMillan, B., 259
de Macagno, M. C., 155
Machlup, S., 275, 283, 285
Malkin, I. G., 307
Malkus, W. V. R., 206, 224
Mallock, A., 42, 53
Martin, J. C., 71
Maruyama, G., 194, 198
Meksyn, D., 1, 24
Mercier, R. P., 143
Mergelyan, S. N., 72
Metropolis, N., 72
Miles, J. W., 79, 80, 89, 91, 96, 103
Millionshtchikov, M., 161, 163, 208, 224
Millis, B. G., 136, 143
Milne-Thomson, L. M., 64
Morgenstern, D., 248
Morgenstern, O., 285
Morse, P. M., 191, 198
Motora, S., 154
Mourier, E., 177, 179–180, 196
Moyce, W. J., 71
Mullikin, T., 177, 197
Munk, W., 141, 143
- Neumann, G., 143
von Neumann, J., 72, 285
Newman, N., 155
Niblett, E. R., 50–51, 53
- Oberman, C. R., 40
Onsager, L., 275, 283, 285
Osborn, H., 281, 286
Ozima, M., 50–51, 53
- Paley, R. E. A. C., 186, 198
Palmer, D. S., 109, 142
Pasta, J. R., 72, 249, 251
Penney, W. G., 71
Pennington, R. H., 63
Penrose, R., 297
Phillips, O. M., 79, 89, 91, 94–95, 100–101,
 103, 143
Phillips, R. S., 172, 175–176, 196, 276, 285
Pierson, W. J., 143, 154–155
Poincaré, H., 68
- Prandtl, L., 60, 65, 72, 76, 80, 89
Price, A. T., 71
Prohaska, C. W., 155
Proudman, I., 161, 163, 199, 208–209, 211,
 224
- Rabenstein, A. L., 1, 5–6, 24
Ragazzini, J. R., 297
Ramakrishnan, A., 275, 285
Rayleigh, Lord, 42, 53, 57, 63–64, 76, 111,
 142, 190, 198
Reid, W. H., 161, 163, 199, 208–209, 211,
 219, 224
Reynolds, O., 199, 224
Rice, S. O., 105–106, 108–109, 111–113,
 140, 142, 154
Richardson, J. M., 280, 286
Richtmyer, R. D., 76
Roberts, P. H., 219, 224
Rosenblatt, M., 154
Rosenbluth, M. N., 35, 40
Rosenhead, L., 65, 76
Rostoker, N., 40
- St. Denis, M., 143, 145
Samuelson, P., 275, 285
Sarachik, P. E., 295, 298
Savart, F., 72
Scarf, H., 275, 285
Schaefer, J. W., 65
Schooley, A. H., 143
Schubauer, G. B., 1, 24, 76
Schwartz, J. T., 196
Schweber, S., 224
Silverman, R. A., 229, 246
Simon, N. J., 49, 53
Skramstad, H. K., 76
Smith, N. D., 143
Solberg, H., 57
Solodovnikov, V. V., 155
Solow, R., 275, 285
Spitzer, L., 40
Squire, H. B., 57
Sternberg, S., 67
Stoker, J. J., 66, 155
Stuart, J. T., 1, 24, 49, 53
- Tasai, F., 155
Tatarski, V. I., 229, 246
Tatsumi, T., 161, 163, 199, 208–209, 224
Taub, A. H., 72, 258
Taylor, G. I. (Sir Geoffrey), 29–30, 33,
 43–44, 53, 76, 80, 89, 199, 213, 225

AUTHOR INDEX

- Tchen, C.-M., 63
Thornhill, C. K., 71
Tick, L., 155
Tidstrom, K. D., 1, 24
Tietjens, O. M., 65
Titchmarsh, E. C., 173, 198
Titt, E. W., 162, 163
Tolman, R. C., 225
Townsend, A. A., 199, 208, 225
Truesdell, C., 248
Tsingou, M., 72
Twersky, V., 229, 246
Tychonoff, A., 173, 198
Tyndall, J., 57
- Uhlenbeck, G. E., 168, 188, 190, 197
Ulam, S., 72, 247, 249, 251
Ursell, F., 80, 89, 155
- Van Kampen, N. G., 26, 33
- Wald, A., 275, 285
Walsh, J. L., 72
- Wasow, W., 1, 5, 24
Watson, G. N., 33
Watson, W., 40
Wendel, K., 155
Westwater, F. L., 74
Whittaker, E. L., 29, 33, 58
Wiener, N., 165, 186–187, 197–198, 275, 285
Winzer, A., 155
Witting, H., 1, 24
Wolf, E., 229, 246
Wood, A. B., 72
Wuest, W., 89
- Yih, C. S., 155
Yoshizawa, T., 304, 307
Young, L. C., 276, 285
- Zahm, A. F., 155
Zarantonello, E. H., 76
Zuber, N., 57

SUBJECT INDEX

- Adiabatic motion, 37
Ambiguity, 27
Amplitude, 238
 fluctuations, 240
 functions, 16
Approximations
 formal relations among classic, 221
 quasi-normality, 208
 stochastic model, 211, 219
 Tamm-Danoff, 222
 (See also Cumulant-discard)
Asymptotic expansion, 236
Asymptotic stability
 in the large, 300
 region of, 299, 302
 with probability 1, 289
Asymptotically stable, 299
 in the mean square, 289
 in the norm, 289
Attenuation coefficient, 245
Atomization, 57
Autonomous equation, 289

Banach spaces
 as phase spaces, 173
 random elements in, 177
Barbasin-Krasovskii, theorem of, 301
Basic reference equation, 6
Boltzmann equation, collision free, 38
Boltzmann transport equation, 35
Boundary condition, hidden, 30
Boundedness, 301

Capillary waves, 88–89
Cauchy problem, abstract, 176
Characteristic function, 209
Charlier (See Gram)
Classic approximations, formal relations among, 221
Closure
 application of the —— approximations, 213
 problem, 203
 schemes for approximate, 205
Comparison
 theorem, 38
 with Gibbsian statistical mechanics of Hamiltonian systems, 205

Components
 secondary, 101
 tertiary, 102
Computational aspects, 280
Confused seaway, spectral representations.
 145
Conservation laws, 68
Continuous games, 284
Contours, 138
Control, 287
 “bang-bang”, 280
 feedback, 275, 278
 vector, 277
Control law, 289–290, 292
 optimal, 292
Correlation, 283
 coefficient, 232, 244
 distance, 232, 234–236
 function, 232
 lengths, 244
Corrsin’s theorem, 124
Couette viscometer, 47
Countable Schauder basis, 173
Cumulant-discard approximations, 209,
 216
 and negative probabilities, 211
 convergence of, 210
 convergence of —— in random convection, 218
 relation to iteration expansion, 210
Cumulants, expansion in, 208
Curves, generalized, 276

Danoff (See Tamm)
Degrees of freedom
 continuous number of, 192
 countable number of, 174
Distribution functions, description by, 201
Difference equation, 278
 random linear —— (discrete-time
 Markov process), 288
Dishonest method, 228, 235, 237
Disturbances, three dimensional, 9
Drift $E \times B/B^2$, 37
Duration, critical, 97
Dynamic programming, 275, 279

Economy of numerical work, 255

- Effective index, 245
Eiconal equation, 240
Energy
 equipartition of, 188
 spectrum, 188
 transfer, 85
Envelope, distribution of, 111
Equilibrium, statistical, 180
Expansion
 in cumulants, 208
 in powers of inverse Reynolds number, 208
Expansion in powers of Reynolds number, 207
 convergence of, 208

Feedback
 amplifier, 259
 control, 275, 278
 principle, 259
Field, 238
Fixed point, 292–293
Flow
 between rotating cylinders, 41
 of helium II, 50
 secondary, 13
 shear, 79
 spiral, 45
Fokker–Planck equation, 235
Formal relations among classic approximations, 221
Fourier series, 249
Functionals of the motion, 247, 249

Gain margin, 268
Games, continuous, 284
Generalized curves, 276
Geometrical optics, 229
Gram–Charlier expansion, 209
Gravity waves, 86, 91
 generation of, 91

Hamiltonian systems, comparison with Gibbsian statistical mechanics of, 205
Helmholtz instability, 55
Honest method, 228, 231, 235, 237–238
Huygens principle, 174
Hydrodynamic stability, 2, 79

IBM STRETCH, 258
Implicit criteria, 279

Index
 effective, 245
 of refraction, 230
Information pattern, 278
Instability, 199
 dynamic, 80
 Helmholtz, 55
 interfacial, 55
 Kelvin–Helmholtz, 88
 sinuous, 64
 slip line, 77
 static, 80, 88
Taylor, 55, 252
theory in wave generation, 91
varicose, 63

Intensity
 coherent, 246
 incoherent, 246
Interaction
 Mach, 77
 time scale, 103
Interactions
 second order, 102
 tertiary, 102
 wave, 100
Intervals between successive zeros, 109
Invariant measure, 179
Inviscid
 equation, 83
 limit, 1
Irreversible phenomena, 275
Iteration expansion, 215
 convergence properties of —— in random convection, 216

Karman vortex street, 60, 64
Krasovskii (*See* Barbasin)

L-measures, 177
Lagrange stability, 301
Lagrangian particles, abstract, 249
Liénard's equation, 301
Light rays, 231
Limit, passage to, 280
Linear
 random —— difference equation (discrete-time Markov process), 288
 theory, 1
Liouville's theorem, 205
Lyapunov [Liapunov]
 direct or second method of, 300
 functions, 295

SUBJECT INDEX

317

- Mach interaction, 77
Mackerel clouds, 57
Magnetohydrodynamic system, 39
Magneto hydrodynamics, 49
 fluid, 38
Margin
 gain, 268
 phase, 268
Markoff [Markov] process, 235
 continuous, 235–236,
 discrete-time, 288
Markov property, 288
Matrix, controllability, 295
Maxima, 123
 heights of, 108, 133
Mean square
 asymptotically stable in, 289
 fluctuation, 232
 transverse displacement, 238
Mean value, 245
 theorem, 244
Measure
 admissible regular, 193
 invariant, 179
 probability, 177
 regular, 192
Minima, 123
Mixing of two fluids, 252
Mode, normal, 5, 58, 66
Moments, description by, 201
Momentum transfer, 24
Motion, 289
 adiabatic, 37
 functionals of, 247, 249
 of C_0 type, 175
 of well-set type, 172
Moving surfaces, 136
Navier-Stokes equation, nonlinearity of, 204
Neighbors of a given point, 252
Nonlinear effects, 1
Norm, asymptotically stable in, 289
Normal
 congruences of rays, 238
 form, 7
 mode, 5, 58, 66
Numerical work, economy of, 255
Nyquist diagram, 266
Omissions, 28
Optics, geometrical, 229
Optimability, principle of, 279
Orr-Sommerfeld equation, 3, 32, 81
Oscillations
 finite amplitude, 9
 overstable, 46
 primary, 2
 ship, 145
 two dimensional, 9
Partial differential, random, 240
Performance index, 290, 292
Phase, 238–240
 fluctuations, 238
 margin, 268
Planck (*See* Fokker)
Plant, 287
Plasma
 physics, 35
 stable —— system, 39
Poincaré's Recurrence Theorem, 73
Positive-real, 264
Potential energy
 spectrum, 95
 transfer of, 100
Probability
 asymptotic stability with —— 1, 289
 measures, 177
 negative, 218
 paths of maximum, 283
Programming, dynamic, 275, 279
Propagation, 227
 constant, 238
Quadratic criteria, 282
Quantum fluids, 50
Quasi-normality approximation, 208
Rademacher functions, 249
Radii, principal, 238
Random
 linear difference equation (discrete-time
 Markov process), 288
 medium, 230–231
 moving —— curve, 115
 partial differential, 240
 stationary —— function, 180
 wave notion, 227
Random convection, 213
 classic problem in, 213
 formal relation to quantum-field theory,
 222
Random elements
 in Banach spaces, 177
 normal, 179

SUBJECT INDEX

- Random surfaces, 105
reflexions in, 133
- Ray diffusion coefficient, 235
- Rayleigh's analogy, 59
stability criterion, 43–44
- Rays, 230–231, 235, 240
normal congruences of, 238
- Reference equation, basic, 6
- Refraction, index of, 230
- Refractive index, complex, 245
- Regular measures, 192
admissible, 193
- Resonance
band, 93
condition, 93, 96
mechanism, 97
mechanism in wave generation, 91
- Resonant wave-number pairs, 103
- Reversibility, 175
- Reynolds number, 200, 207
(*See also* Expansion)
- Reynolds stresses, 9
- Saddle-points, 123
- Schauder basis, countable, 173
- Semi-group property, 174
- Semi-groups, 276, 282
- Sensitive gas jet, 57
- Sequential machines, 280
- Shear flow, 79
parallel, 79
- Ship
oscillations, 145
wave excitation of, 147
- Ship response to waves
coherency, 151
co-spectrum, 151
differential equation, 151
quadrature spectrum, 151
spectrum, 150, 154
- Slip line
instability, 77
vortex sheet, 77
- Slopes, 129
- Sommerfeld (*See* Orr)
- Sound waves, plane, 191
- Space average, 196
- Spectrum
energy, 188
potential energy, 95
pressure fluctuation, 95
surface elevation, 95, 97
- Specular points, 134, 138
life-time of, 140
paths of, 140
- Stability, 265
hydrodynamic, 2, 79
Lagrange, 301
of periodic systems, 305
of the water surface, 96
practical, 304
under persistent disturbances, 305
(*See also* Asymptotic)
- Stable, 299
conditionally, 267
unconditionally, 265
(*See also* Asymptotically)
- Stationarity with respect to time and to space, 193
- Stationary points, densities of, 131
- Statistical
average, 195
comparison with Gibbsian — mechanics of Hamiltonian systems, 205
- Stochastic
continuous — process, 276
differential equations, 227
equations, 227
equilibrium, 180
medium, 227
variational processes, 275
- Stochastic model
approximations, 211, 219
relation to iteration expansion, 212
- Superfluid, 50
- Surfaces, moving, 136
- Tamm-Danoff approximations, 222
- Taylor instability, 55, 252
- Tietjens function, 82
- Time average, 195
- Transition, 1
region, 201
- Transitivity, metric, 194
- Transmission medium, 227
- Transverse
displacement, 234
gradient, 231
- Turbulence
dynamical processes of, 206
fully-developed, 201
functional-analytic approach to, 157
homogeneous, 165

- Turbulence *continued*
isotropic, 165, 206
laminar stability theory, 206
physical nature of, 199
- Turbulence theory
closure problem of, 199
non-simultaneous averages in, 205
- Turbulent velocity field
definition of, 200
description by probability distributions, 200
- Turning point, 6
- Twinkling, 117, 139
of a star, 229
- Variance, 240, 242, 246
- Vibrating string with fixed ends, 181
- Viscometer, Couette, 47
- Viscosity, effective, 42
- Viscous
liquid, 88
solution, 82
- Viscous layer
inner, 80
outer, 80
- Vortex, 23
Karman — street, 60, 64
slip line — sheet, 77
street, 65
- Wave, 227
capillary, 88-89
coherent, 246
excitation of ships, 147
gravity, 86, 91
initial stage of — growth, 94
interactions, 100
plane sound, 191
propagation, 227
reduced — equation, 243
resonant — -number pairs, 103
- Wavefront, 238, 241
- Wavelength, 238
- Wave generation
water, 79
instability theory in, 91
resonance mechanism in, 91
- Well
posed problem, 227, 243
-set problem, 171

