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N. C. Metropolis
A. H. Taub
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EDITORS

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PREFACE

This volume contains all but two of the papers which were presented at two symposia sponsored by the American Mathematical Society and other co-sponsors in the spring of 1962.

The first symposium was held in Chicago, Illinois, April 12–14, on the subject of

**Experimental Arithmetic.**

This symposium was sponsored by the Society and the Association for Computing Machinery, and it was supported financially by the Institute for Defense Analyses. The organizing and invitations committee consisted of N. Metropolis (Chairman), Marshall Hall, Jr., Peter Henrici, Mark Kac, R. D. Richtmyer, and A. H. Taub.

The objective of this symposium was to examine ways in which the arithmetical potential of modern high-speed computers can furnish experience which sheds light on outstanding problems in mathematics and other sciences.

The second symposium was held at Atlantic City, New Jersey, April 16–18 on the subject of

**Interactions between Mathematical Research and High-Speed Computing.**

This symposium was sponsored by the Society and the Association for Computing Machinery and was supported financially by the Army Research Office and the National Science Foundation. Its organizing and invitations committee consisted of John Todd (Chairman), G. E. Forsythe, P. D. Lax, D. H. Lehmer, H. H. Goldstine, C. B. Tompkins, and D. M. Young, Jr.

The objective of this symposium was to enable mathematicians to become familiar with the potentialities of computers of types currently available and with the problems involved in the proper and effective exploitation of these computers.

The close relationship between the subject matters of the two symposia prompted the organizing committees to merge the proceedings into this single volume. Generally speaking, papers in this volume have been edited by one of the four members of the editorial committee in accordance with subject matter; no attention has been paid in ordering the papers to the particular symposium in which they were presented and various borderline cases have been decided on the basis of editorial convenience. The editors have not tried to settle conflicting opinions expressed in different papers in the series.

N. **Metropolis**

University of Chicago

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University of Illinois

**John Todd**

California Institute of Technology

C. B. **Tompkins**

University of California at Los Angeles
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*Italic numbers* refer to pages on which a complete reference to a work by the author is given. Roman numbers refer to pages on which a reference is made to a work of the author. For example, under Minkowski would be the page on which a statement like the following occurs: "This theorem was proved earlier by Minkowski [7, § 2] in the following manner. . . ."

*Boldface numbers* indicate the first page of articles in this volume.

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