Stochastic Processes in Mathematical Physics and Engineering

PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS

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INTRODUCTION

In the study of the classification and recognition of mathematical processes, dichotomies can be introduced in several different fashions. Two of the most useful have been "deterministic vs. stochastic" and "descriptive vs. variational." In recent years, we have seen greater and greater emphasis laid upon stochastic and decision processes, and even a blending of the two in the form of modern control theory.

The gradual shift of interest from the classical deterministic descriptive process has been largely a consequence of the continuing challenges to the mathematician to explain phenomena, produce numbers, and guide research in such fields as physics, engineering, economics, biology, medicine and operations research. Combinations of complexity and uncertainty have forced a more frequent use of probabilistic concepts.

In this volume, the emphasis is upon stochastic processes, divided a bit unevenly between descriptive and decision aspects. In dealing with random effects, we can use deterministic or stochastic equations, just as in dealing with deterministic effects we can employ stochastic or deterministic equations or in treating descriptive processes, we can use either descriptive or variational equations. The choice of a particular analytic formulation is a tribute to the existing analytic techniques and the computational resources available, as well as a reflection of the underlying physical process and the prevalent scientific philosophy.

The papers by Adomian, Bharucha-Reid, Hoffman, Keller, Richardson, and Twersky deal with various and overlapping aspects of the general theory of stochastic equations, principally ordinary and partial differential equations. Much of this work is motivated by particular equations arising in wave propagation and scattering theory, as the papers by Hoffman, Keller, and Twersky indicate. The objective of obtaining specific analytic results leads to the development of perturbation techniques and the consideration of questions of closure, as, for example, in Richardson's paper.

The paper by Bellman deals with stochastic iteration, a topic which arises from the consideration of stochastic differential equations in very much the same way as classical iteration theory arises from deterministic differential equations. A particularly important type of stochastic iteration arises in the theory of stochastic approximation. This approach is used by Gray to study a design problem arising from circuits with random components. In their papers, Blackwell and Derman use dynamic programming, the first to obtain estimates in probability theory by identification with a stochastic decision process, and the second to treat an interesting class of sequential decision processes of Markovian type.

Classical estimation and identification problems arising from stochastic processes are treated by Parzen and Root using new and powerful approaches.

Finally, the papers by Montroll, Rota, and Wong consider deterministic equations associated respectively with random walk, turbulence, and diffusion theory.

Looking over the papers in this volume, we see many new, fascinating, and difficult classes of problems, ideal areas for the young analyst to test his talents, develop his techniques, and plant his flag.

RICHARD BELLMAN
Santa Monica, California
1964

AUTHOR INDEX

Italic numbers refer to pages on which a complete reference to a work by the author is given.

Roman numbers refer to pages on which a reference is made to a work of the author. For example, under Minkowski would be the page on which a statement like the following occurs: "This theorem was proved earlier by Minkowski [7, §2] in the following manner . . ."

Boldface numbers indicate the first page of articles in this volume.

SUBJECT INDEX

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