

# Applications of Nonlinear Partial Differential Equations in Mathematical Physics

**Volume 17**

**PROCEEDINGS OF  
SYMPOSIA IN  
APPLIED MATHEMATICS**

**AMERICAN MATHEMATICAL SOCIETY**

Applications of Nonlinear  
Partial Differential Equations  
in Mathematical Physics

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Volume 17

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Partial Differential Equations  
in Mathematical Physics

R. Finn, Editor

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## PREFACE

The contributions to this volume arose from talks presented at a symposium on the nonlinear partial differential equations of mathematical physics, which took place in New York City, April 20–23, 1964. The organizational work and invitations were the responsibility of a committee, consisting of C. B. Morrey, W. Noll, J. B. Serrin, A. H. Taub and myself as chairman.

It was inevitable in view of the broad scope of the subject matter and the severe limitations of time that many important and original contributions could not be included in the program. An attempt was made, however, to organize the meeting in such a way that participants would gain acquaintance with some of the principal lines of modern research in a number of differing but interrelated subjects. Accordingly, the symposium was divided into four sessions of invited addresses, as follows:

1. General Nonlinear Theory
2. Finite Elasticity, Compressible Fluids
3. Viscous Fluids, Magnetohydrodynamics
4. General Relativity, Quantum Field Theory.

In addition, a fifth session was devoted to discussion of the invited talks and to the presentation of selected contributed papers.

The present volume is organized along similar lines, except that the abstracts of contributed papers have been placed in the sections to which they correspond. The underlying cohesive spirit which appeared in the diverse talks at the meeting will, it is hoped, be felt also by the reader who peruses the papers presented here. The volume will have served its purpose if an occasional reader is stimulated to probe more deeply into some of the questions that are discussed, or to discover some unifying principle which unites results that may at first seem to have little connection.

R. FINN  
Stanford University

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