Numerical Analysis

PROCEEDINGS OF
SYMPOSIA IN
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NUMERICAL ANALYSIS

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Preface

This volume contains lecture notes prepared by the speakers for the American Mathematical Society Short Course on Numerical Analysis given in Atlanta, Georgia, 3-4 January 1978.

We were very pleased that the Short Course Advisory Subcommittee decided to hold a short course on Numerical Analysis, and even more pleased by the large attendance. We are indebted to our colleagues for their enthusiastic cooperation and efforts which made the Short Course and these published proceedings possible.

The choice of topics was influenced rather strongly by the subcommittee's objectives for these short courses. These objectives are that the short courses provide an entree to an area and lead up to current research problems in that area. We have tried to emphasize those areas where research activity is greatest and the present state of understanding is not satisfactory. Consequently, many classical problem areas are hardly mentioned, or ignored.

The term numerical analysis is too narrow as a description of the area as it is generally viewed today, since the construction of algorithms is an important aspect of the subject as well. In understanding why a particular algorithm "works" or "does not work" one often is led to better algorithms. Thus, the constructive and analytical aspects are not independent.

It is often said that the "computations are ahead of the analysis." This means that known algorithms perform in an inexplicable manner when they are tested on problems with known solutions. They often work better than we can guarantee them to work--our error estimates are not sharp enough or don't exist. This illustrates the strong influence that computations performed in the physical sciences and engineering have on the subject.

The papers given here are mainly of a mathematical nature. The results presented describe properties of computational methods that are only relevant in the context of that computation. It is the need to perform the computation which presents the problems to the subject and justifies it. For example, in the emerging field of Computational Physics methods are developed as they are <u>needed</u> for various problems. These methods are usually constructed via physical reasoning, experience, and intuition. They are often tested on problems with known solutions, but their validity is often judged on their behavior in physical terms. It is then the numerical analyst who attempts to give error estimates and describe the numerical behavior of these methods. The convergence results needed here differ from those of classical constructive analysis. Error estimates which hold for finite values of the discretization parameters

are what are really needed, as opposed to asymptotic estimates as these parameters tend to zero. The effect of rounding errors is a central issue in numerical analysis and is a unique aspect of the subject. Algorithms which are otherwise exact may be useless because of rounding errors.

Though applications are discussed here, the important relationships between the problems, the algorithms, and the machines used for the computations which are vital to the spirit of the field cannot be found here. Numerical analysis is not a textbook subject; computational experience is essential.

We hope that these manuscripts and their bibliographies will prove useful to those who wish to learn something of the nature of numerical analysis and what some of the current problems of interest are.

Gene H. Golub
Joseph Oliger

Stanford University
July 1978