

GAME THEORY and its Applications

Volume 24

**PROCEEDINGS OF
SYMPOSIA IN
APPLIED MATHEMATICS**

AMERICAN MATHEMATICAL SOCIETY

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IN APPLIED MATHEMATICS**

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GAME THEORY AND ITS APPLICATIONS

**AMERICAN MATHEMATICAL SOCIETY
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AMERICAN MATHEMATICAL SOCIETY SHORT COURSE

GAME THEORY
AND ITS APPLICATIONS

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PREFACE

This volume contains the lecture notes prepared by the four speakers in the American Mathematical Society Short Course on Game Theory and its Applications given in Biloxi, Mississippi on January 22-23, 1979. The Short Course Advisory Subcommittee of the AMS selected this topic, assisted in the arrangements for the course, and recommended the publication of the lecture notes.

Game theory has been a topic of broad interest as a purely theoretical subject which has relationships to many other mathematical areas, and also as a subject widely used in applications over a large variety of problem areas. It is concerned with mathematical models for situations involving conflict and/or cooperation. These arise in a fundamental way throughout the behavioral and decision sciences. Game theory has become a basic modeling technique in much of modern economic theory, political science, sociology, and operations research, and it has frequently been applied to many other fields. It is a subject highly suitable for joint research of an interdisciplinary nature.

This volume is concerned mostly with the n -person theory ($n \geq 3$), although chapter 6 also describes several basic two-person models. The first five chapters deal for the most part with the multiperson cooperative games in the characteristic function (coalitional) form. The normal (strategic) form and the extensive (tree) form of a noncooperative game are stressed in chapter 6, although some basic definitions for the normal form do appear in an earlier chapter. Selected applications of the theory which are covered here in some detail include economic market games, measuring power in political systems, equitable allocation of costs, and auctions. Many of the important recent uses of game theory have involved the n -person cooperative models.

These lectures were presented to an audience of mature mathematicians. Nevertheless, this volume could also serve as a textbook for a general course in game theory at the upper division or graduate levels. The instructor may wish to add supplemental problem sets, and perhaps expand the coverage of the noncooperative games presented in chapter 6. Alternatively, the first five chapters provide material suitable for a course on the multiperson cooperative theory. Additional topics on bargaining theory and arbitration schemes could be included, as well as other cooperative models and solution concepts such as the nucleolus, kernel and bargaining sets.

The authors wish to express their appreciation to the Short Course Advisory Subcommittee of the Society's Committee on Employment and Educational Policy, to the various staff members of the AMS, and the individual typists who contributed to making the volume possible. They also gratefully acknowledge support for their research in game theory which they have received through various projects supported in part by the National Science Foundation and the Office of Naval Research.

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