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Introduction to Actuarial Mathematics

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Actuarial mathematics is a collection of mathematical ideas that has been found useful in designing and managing financial security systems. The criterion for whether a mathematical topic belongs in actuarial science is not whether the topic is derived from some fundamental mathematical notion. Instead, the criterion is pragmatic; does the topic contribute to the construction of useful intellectual models for insurance systems? The consequences of this definition of actuarial mathematics will be apparent in this short course. Basic ideas from calculus, linear algebra, probability, statistics, demography, mathematical programming and economics will appear as building blocks in models for insurance systems.

In order to provide a structure for this short course, it is necessary to create an outline for insurance systems. With such an outline in place the various topics to be reviewed in the short course can be placed in perspective. Clearly there are alternative principles by which insurance systems can be classified. For the current purpose, our basic division will be between those systems that involve long-term commitments, with recognition of the long-term cost implications, and short-term commitments. Economic considerations, such as interest rates, rather than the probability distribution of claims payment often play a dominant role in long-term models.

A. Long-term Insurance

1. **Individual life insurance.** For this important type of insurance the fundamental model requires that a balance be struck between the expected present value of the income and expenditures of the insurance system for each insured life. The lecture Updating Life Contingencies wil develop the basis model.

   A key component of the model will be an appropriately selected life table which will be used to describe the distribution of the random variable time until death. Actuarial science shares with biostatistics, demography and

reliability engineering an interest in constructing life tables from data. One aspect of the construction process is the systematic revision of observations to conform with prior notions of smoothness. Smoothing, or graduation, is a process which is used in many fields in science. E.S. Shiu will review actuarial contributions to this field in his lecture "A Survey of Graduation Theory". To complete the mathematical model for individual life insurance, components for capital growth and expenses will be required. In this short course only basic compound interest models will be used. However, one of the most active areas of actuarial research intersects with financial theory. The issue is to match expected insurance cash flows with expected cash flow generated by invested assets with the goal of minimizing the inconvenient consequences of changes in interest rates.

2. **Private pensions.** Models to guide private pensions contain all of the elements of individual life insurance models. However, in many cases pension models may be more complex. The complexity arises because benefit amounts may depend on the cause of termination from active employment (retirement, death, disability, withdrawal, lay-off) and the performance of an economic index such as salaries or prices. Some elements of these elaboration will appear in the lecture "Updating Life Contingencies". In his lecture, "The Performance of Pension Plans", C.J. Nesbitt will review the history of benefits paid by Teachers Insurance and Annuity Association and College Retirement Equity Fund. These two related systems are of great interest to college and university teachers and benefit payments are related to the economic performance of the supporting pools of assets.

3. **Social insurance.** Almost exactly fifty years ago, August 14, 1935, President Roosevelt signed the Social Security Act. This act established in the United States a broad social insurance system in which long-term costs are recognized but in which the balance between income and expenditures is sought not on the individual level but in a global sense. The issue is whether the provisions for aggregate income are in balance with aggregate benefits defined in the law. In order to examine this balance demographic and economic projections, supported by an examination of experience, are required. J.A. Beekman's lecture, "Actuarial Assumptions and Models for Social Security Projections", will survey these topics.

B. **Short-term Insurance**

1. **Risk models.** Most property, liability, health and group insurance systems can be classified in this category because the insurance contracts are typically for a short-term period. In constructing mathematical models for such systems distributions for two basic random variables are needed. First, it is necessary to specify the distribution of the number of insurance claims in one period. In his lecture, "Models in Risk Theory", H.H. Panjer will review models for short-term insurance with emphasis on considerations in selecting the frequency of claims distribution.

   A second random variable that enters risk models is associated with the loss amount, given that a loss event has occurred. The estimation of the loss amount distribution from data is a special case of statistical estimation theory. The unique aspects arise because of special features of insurance data and the highly skewed nature of some of these distributions. In his lecture, "Loss Distributions", S.A. Klugman will survey this field.

2. **Credibility.** In order to provide for stability in premium rates, it is necessary to update these rates by combining prior and current insurance experience data and information from collateral sources. Credibility theory is a collection of ideas developed by actuaries to perform this process of summarizing information from multiple sources in a coherent fashion. P.M. Kahn's lecture, "Overview of Credibility Theory", summarizes this topic. Credibility theory can be built on several somewhat different foundations. Consequently, credibility theory has fascinating connections with issues in the foundations of statistics.

Clearly within the limits of this short course only a few of the many topics that comprise actuarial mathematics can be reviewed. As financial security systems evolve, so will actuarial mathematics. Likewise, new mathematical tools can promote the development of new types of financial security systems.