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TO

PROFESSOR M. G. KREIN

ON HIS EIGHTIETH BIRTHDAY

WITH RESPECT AND ADMIRATION
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Preface

Function theory, spectral decomposition of operators, probability, approximation, electrical and mechanical inverse problems, prediction of stochastic processes, and the design of algorithms for signal-processing VLSI chips are among a host of fundamental theoretical and applied topics illuminated by the classical moment problem — a problem of remarkable elegance in its own right. To survey some of these ramifications, and the current research which derives from them, the American Mathematical Society sponsored the Short Course *Moments in Mathematics*, given in conjunction with its January, 1987, meeting in San Antonio, Texas. This volume contains the lectures delivered in that course.

I have received so much support and encouragement — from R. P. Kurshan and the Short Course Subcommittee, from all the speakers, and from the staff of the A.M.S. — that I cannot adequately acknowledge it all. I owe an additional special debt to Susan Pope for tireless, imaginative, incomparably expert preparation of the text. They all have made every aspect of this project a pleasure.

The modern flowering of this subject has grown largely from the deep insights and monumental contributions of M. G. Krein, to whom this book is therefore gratefully dedicated.

*Henry J. Landau, Editor*
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