Matrix Theory and Applications

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Charles R. Johnson, Editor

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Table of Contents

Preface

As a subfield of mathematics, matrix theory continues to enjoy a renaissance that has accelerated during the past decade, though its roots may be traced much further back. This is due in part to stimulation from a variety of applications and to the considerable interplay with other parts of mathematics, but also to a great increase in the number and vitality of specialists in the field. As a result, the once popular misconception that the subject has been fully researched has been largely dispelled. The interest on the part of the American Mathematical Society and the approximately 140 participants in the Short Course (at the January 1989 Phoenix Meeting) on which this volume is based is a reflection of this change. The steady growth in quality and volume of the subject's three principal journals, Linear Algebra and its Applications, Linear and Multilinear Algebra, and the SIAM Journal on Matrix Analysis and Applications is another. Approximately 500 different authors have published in one of these three journals in the last two years. Geographically, strong research centers in matrix theory have developed recently in Portugal and Spain, Israel, the Netherlands, Belgium, and Hong Kong.

The purpose of the Short Course was to present a sample of the ways in which modern matrix theory is stimulated by its interplay with other subjects. Though the course was limited to seven speakers, the "other subjects" represented included combinatorics, probability theory, statistics, operator theory and control theory, algebraic coding theory, partial differential equations, and analytic function theory. Among other important examples, numerical analysis, optimization, physics and economics are, unfortunately, at most lightly touched. There is no limit to the specific examples that might be cited.

One of the ingredients in the recent vitality of matrix theory is the variety of points of view and tools brought to the subject by researchers in different areas. This is responsible for a number of important trends in current research. For example, the notion of majorization (mentioned in the talk by Olkin) has become pervasive in a historically brief period of time. The trend away from the "basis-free" point of view is illustrated by work in combinatorial matrix theory (Brualdi, Johnson), the Hadamard product

(Horn) and nonnegative matrices (Mukherjea). There are many quite worthy issues that are at least excruciatingly difficult to view in a basis-free way, and freedom from the basis-free view has opened many exciting avenues of research. On the other hand, recognition of the "right" problem dependant symmetries can provide vital insight (Diaconis). The synergy between matrix theory and systems theory has had a tremendous impact on both, and on the now highly mathematically driven field of electrical engineering (Gohberg). The immense variety of tools and problems illustrates a reason for use of the term "matrix theory" or "matrix analysis" in place of "linear algebra". A large portion of current work is neither primarily linear nor primarily algebraic in nature. No point of view on what the subject is or where it is going could, or should, be without substantial disagreement. This only reflects the remarkable breadth of interest enjoyed by the subject. For an historical perspective on the nature and role of the subject the reader might enjoy the prefaces to each of the following: *Recent Advances in Matrix Theory* (Schneider 1964); *A Survey of Matrix Theory and Matrix Inequalities* (Marcus and Minc, 1964); *Linear Algebra and its Applications*, volume 1 (Alan Hoffman, 1968); and *Matrix Analysis* (Horn and Johnson, 1985). A glimpse of the contagious appeal of the subject is communicated by Olga Taussky in her November 1988 *Monthly* article "How I Became a Torchbearer for Matrix Theory".

As organizer, I would like to again thank each of the speakers for a contribution that will advance both the subject and the general understanding of it. The significant time necessary to prepare both a talk and then subsequent paper is much appreciated. The community would also like to thank the American Mathematical Society for recognizing, and providing a forum for, the subject.

Charles R. Johnson
College of William and Mary

Subject Index