Trends in Optimization

American Mathematical Society
Short Course
January 5–6, 2004
Phoenix, Arizona

Serkan Hoşten
Jon Lee
Rekha R. Thomas
Editors
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Preface

The American Mathematical Society sponsors a series of *Short Courses*, each of which consists of a set of survey lectures on a single theme of pure and applied mathematics. Initiated in 1973, these two-day courses occur immediately preceding the Joint Mathematics Meetings held in January as well as at some summer meetings.

On January 5-6, 2004, the AMS Short Course *Trends in Optimization 2004* took place in Phoenix, Arizona. Optimization has not been the focus of a recent AMS Short Course. Past Short Courses on related themes consist of: (i) *Operations Research*, Washington D.C., 1975, (ii) *Applied Combinatorics*, Kalamazoo, 1975, and (iii) *Operations Research*, Duluth, 1979. There have been impressive advances in optimization since then, and our goal was to showcase some of the exciting more recent work.

Our Short Course consisted of seven 75-minute lectures given by leaders in the field of optimization. It is impossible to give a thorough cross-section of research in optimization in a two-day course. Rather, we chose seven exciting areas to focus on, with a clear bias toward the discrete side of optimization, somewhat reflecting the interests of the organizers. This volume comprises edited notes prepared by our lecturers.

Optimization is concerned with the efficient computation of the supremum of an *objective function* *f* whose domain is restricted to some set of *feasible solutions* *S*. Assumptions about the function *f* (linear, convex, continuous, differentiable, etc.) and the set *S* (a hypercube, a convex set, a polyhedron, the integer lattice points in a polyhedron, the set of symmetric positive semidefinite matrices, etc.) lead to structural results and/or efficient algorithms. When the set *S* is a subset of the power set of a finite set, we are in the domain of combinatorial optimization.

No sophisticated background is needed in order to read the lecture notes in this volume. Many of them are self-contained and all provide extensive references. However, basic knowledge of linear programming duality (which appears in almost all chapters), the geometric combinatorics of polyhedral sets (extreme points and rays, faces, valid inequalities, etc.), and the geometry of integer programming would be useful. There are many nice books devoted to each topic above, but most cover more than the introductory material. A good starting point where the reader can find the needed material in a condensed but very readable form is Chapter 0 of Jon Lee’s textbook *A First Course in Combinatorial Optimization* (Cambridge University Press, 2004).

Karen Aardal (Georgia Institute of Technology) in her lecture *Lattice basis reduction in optimization*, describes Lovasz’s fundamental algorithm for producing a short vector in a lattice by lattice basis reduction, and H.W. Lenstra’s use (in the
early 1980's) of this idea in his polynomial-time algorithm for integer programming in fixed dimension. This lecture note is one of the most lucid presentations of the material, and it also contains pointers to the developments on the practical side with an eye towards computational tools.

In his lecture *Polyhedral methods in discrete optimization*, Alper Atamtürk (University of California at Berkeley) surveys some of the powerful cutting-plane techniques which, in conjunction with modern computational tools for linear programming, have enabled the solution of large mixed integer linear programs. Particular attention is paid to the so-called *lifting* techniques for producing valid inequalities from simpler polyhedra such as knapsack polyhedra. These ideas are showcased in an application in robust combinatorial optimization.

In Gérard Cornuéjols's (Carnegie Mellon University) lecture *Graphs and combinatorial optimization*, we see the rich interplay between linear programming duality and combinatorial optimization problems arising from graphs and related structures. Important ideas such as packing and covering properties, perfectness and total dual integrality, are presented in a compact and coherent way. Most importantly, this lecture note contains a very readable outline of the recent proof of the Strong Perfect Graph Theorem by Chudnovsky, Robertson, Seymour and Thomas.

Linear programming duality has been a powerful tool for proving combinatorial min-max theorems. However, there is no commonly agreed upon way of extending these results to an integer programming duality. Jean-Bernard Lasserre (LAAS-CNRS, Toulouse), in his lecture *Integer-programming duality*, describes his recent and very promising work on a natural duality for integer programming based on generating functions and counting formulas. Ideas from Fourier analysis (Laplace transform) and complex analysis (residues) play a prominent role in this new and exciting direction.

Many combinatorial optimization problems are NP-hard, and are thus unlikely to admit exact solutions in polynomial time. David Shmoys (Cornell University), in his lecture *The design and analysis of approximation algorithms: Facility location as a case study*, surveys methods for the efficient computation of near-optimal solutions. Specifically, he focuses on algorithmic paradigms that yield efficient algorithms for producing a solution whose objective value is guaranteed to be within a given factor of the optimal objective value. The facility location problem is used as a relatively simple example to illustrate the fundamental ideas and directions.

In his lecture *Algebraic recipes for integer programming*, Bernd Sturmfels (University of California at Berkeley), describes how methods of commutative algebra and algebraic combinatorics can be used to successfully attack integer programming problems. In particular, Gröbner bases play a central role in algorithmic theory and practice. Moreover, techniques based on short rational functions are bringing new insights, such as in computing the integer programming gap.

Steve Wright (University of Wisconsin) surveys the terrain of *Nonlinear and semidefinite programming*. The techniques used in this subfield of optimization tend to be very different than those typically used in discrete optimization. The area, though only represented by one lecture, is a broad and exciting field. The development of efficient interior-point algorithms for linear and semidefinite-programming problems, and connections with approximation algorithms for combinatorial optimization problems, have generated considerable interest within the discrete optimization community.
We are grateful to Annalisa Crannell who helped us with our proposal to the AMS, and Wayne Drady and the rest of the AMS staff who helped us every step of the way. We are also very thankful to everyone who helped us review the chapters: Jesús De Loera, Christian Haase, Tyrell McAllister, Alex Milowski, Edwin O’Shea, Will Traves, Andreas Wächter and Rudy Yoshida did excellent jobs under a tight schedule.

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