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Short Course

January 4–5, 2011

New Orleans, Louisiana

Karl Sigmund

Editor



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Preface

Evolutionary game theory studies basic types of social interactions in populations of players. It is the ideal mathematical tool for methodological individualism, i.e., the reduction of social phenomena to the level of individual actions. Evolutionary game dynamics combines the strategic viewpoint of classical game theory (independent, rational players trying to outguess each other) with population dynamics (successful strategies increase their frequencies).

A substantial part of the appeal of evolutionary game theory comes from its highly diverse applications, such as social dilemmas, the evolution of language, or mating behavior in animals. Moreover, its methods are becoming increasingly popular in computer science, engineering, and control theory. They help to design and control multi-agent systems, often with large number of agents (for instance, when routing drivers over highway networks, or data packets over the Internet). While traditionally these fields have used a top down approach, by directly controlling the behavior of each agent in the system, attention has recently turned to an indirect approach: allowing the agents to function independently, while providing incentives that lead them to behave in the desired way. Instead of the traditional assumption of equilibrium behavior, researchers opt increasingly for the evolutionary paradigm, and consider the dynamics of behavior in populations of agents employing simple, myopic decision rules. The methods of evolutionary game theory are used in disciplines as diverse as microbiology, genetics, animal behavior, evolutionary psychology, route planning, e-auctions, common resources management or micro-economics.

The present volume is based on a mini-course held at the AMS meeting in New Orleans in January 2011. The lectures deal mostly with the mathematical aspects of evolutionary game theory, i.e., with the deterministic and stochastic dynamics describing the evolution of frequencies of behavioral types.

An introductory part of the course is devoted to a brief sketch of the origins of the field, and in particular to the examples that motivated evolutionary biologists to introduce a population dynamical viewpoint into game theory. This leads to some of the main concepts: evolutionary stability, replicator dynamics, invasion fitness, etc. Much of it can be explained by means of simple examples such as the Rock-Paper-Scissors game. It came as a surprise when childish games of that sort, intended only for the clarification of concepts, were found to actually lurk behind important classes of real-life social and biological interactions. The transmission of successful strategies by genetic and cultural means results in a rich variety of stochastic processes and, in the limit of very large populations, deterministic adjustment dynamics including differential inclusions and reaction-diffusion equations.

Some economists view these types of dynamics merely as tools for so-called equilibrium refinement and equilibrium selection concepts. (Indeed, most games have so many equilibria that it is hard to select the ‘right one’). However, evolutionary games have also permitted us to move away from the equilibrium-centered viewpoint. Today, we understand that it is often premature to assume that behavior converges to an equilibrium. In particular, an evolutionarily stable strategy need not be reachable. A homogeneous population using that strategy cannot be invaded by a minority of dissidents, but a homogeneous population with a slightly different strategy can evolve away from it. Limit phenomena such as periodic or heteroclinic cycles, or chaotic attractors, may be considered, perhaps not as ‘solutions of the game’, but as predictions of play. On the other hand, large classes of games leading to global convergence are presently much better understood.

This book offers a succinct state-of-the-art introduction to the increasingly sophisticated mathematical techniques behind evolutionary game theory.

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