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LATTICE THEORY

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LATTICE THEORY

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PREFACE

This volume contains the papers presented at the Symposium on Partially Ordered Sets and Lattice Theory held in conjunction with the Monterey meeting of the American Mathematical Society in April 1959. The Symposium was sponsored by the American Mathematical Society and supported by a grant from the National Science Foundation. The interest and support of these organizations is gratefully acknowledged.

Some twenty-one years earlier, on April 15, 1938, the first general symposium on lattice theory was held in Charlottesville in conjunction with a regular meeting of the American Mathematical Society. The three principal addresses on that occasion were entitled: Lattices and their Applications, On the Application of Structure Theory to Groups, and The Representation of Boolean Algebras. It is interesting to observe that the first and last of these titles appear again as section titles for the present Symposium. Furthermore the second title is still of current interest as evidenced by the paper of Marshall Hall. Nevertheless there have been major changes in emphasis and interest during the intervening years and thus some general comments concerning the present state of the subject and its relationship to other areas of mathematics appear to be appropriate.

The theory of groups provided much of the motivation and many of the technical ideas in the early development of lattice theory. Indeed it was the hope of many of the early researchers that lattice-theoretic methods would lead to the solution of some of the important problems in group theory. Two decades later, it seems to be a fair judgment that, while this hope has not been realized, lattice theory has provided a useful framework for the formulation of certain topics in the theory of groups (for example, generalizations of the Jordan-Hölder theorem) and has produced some interesting and difficult group-theoretic problems (cf. the excellent monograph of M. Suzuki). On the other hand, the fundamental problems of lattice theory have, for the most part, not come from this source but have arisen from attempts to answer intrinsically natural questions concerning lattices and partially ordered sets; namely, questions concerning the decompositions, representations, imbedding, and free structure, of such systems. It should be pointed out that group theory and other areas of mathematics have furnished concepts and methods which have proved to be useful in the study of these questions. Thus the techniques associated with the study of composition series and chief series in group have been successfully applied to the structure of modular and semi-modular lattices. Set topology and ring theory have been the source of many fruitful ideas in the study of Boolean algebras. Also the theory of linear vector spaces and projective

geometries have contributed some of the basic methods for the development of the theory of complemented modular lattices and, in particular, continuous geometries. Nevertheless, as the study of these basic questions has progressed, there has come into being a sizable body of technical ideas and methods which are peculiarly lattice-theoretic in nature. These conceptual tools are intimately related to the underlying order relation and are particularly appropriate for the study of general lattice structure.

At the 1938 Symposium, lattice theory was described as a "vigorous and promising younger brother of group theory". In the intervening years it has developed into a full-fledged member of the algebraic family with an extensive body of knowledge and a collection of exciting problems all of its own. Such outstanding problems as the construction of a set of structure invariants for certain classes of Boolean algebras, the characterization of the lattice of congruence relations of a lattice, the imbedding of finite lattices in finite partitions lattices, the word problem for free modular lattices, and the construction of a dimension theory for continuous, non-complemented, modular lattices, have an intrinsic interest independent of the problems associated with other algebraic systems. Furthermore, these and other current problems are sufficiently difficult that imaginative and ingenious methods will be required in their solutions. A vigorous group of mathematicians are attacking these problems and the results of some recent progress may be found in the papers included in this volume.

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