1960 Institute on Finite Groups

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INTRODUCTION

A Summer Institute on Finite Groups was held at the California Institute of Technology from August 1 to 28, 1960. This Institute was sponsored by the American Mathematical Society and supported by the National Science Foundation.

The notes in this volume cover the material presented at four seminars, each of which held two sessions of an hour and a half each week. The first of these on "p-length theorems" was conducted entirely by Professor Graham Higman. The second on "Arithmetical and normal subgroup structure of finite groups" was conducted for the first two weeks by Dr. Bertram Huppert and for the second two weeks by Professor Helmut Wielandt. The third on "Combinatorial analysis" was conducted by Professor Herbert Ryser and in this seminar sessions were also given by Dr. Daniel Hughes, Dr. Ernest Parker and Professor Marshall Hall. The fourth seminar on "Group characters" was to have been conducted by Professor Richard Brauer, and although he was prevented from coming by illness in his family, there was a considerable amount of his work presented. In this seminar Professor William Reynolds presented work of his own and of Professor Brauer's. Other speakers in this seminar were Professors Walter Feit, J. S. Frame and Michio Suzuki. The last paper in this volume was the subject of a special lecture given by Rimhak Ree.

These notes were originally distributed in mimeographed form to the participants in the Summer Institute and it was expected that the individual articles would be published separately. But the number of requests for the notes was sufficient to warrant the publication of these proceedings even though some of these articles have appeared elsewhere.

MARSHALL HALL, JR.
California Institute of Technology

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