

# Axiomatic Set Theory



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## TABLE OF CONTENTS

|  |          |
|--|----------|
| <b>Foreword .....</b>  | <b>v</b> |
| <b>Sets constructible using <math>L_{\kappa\kappa}</math></b>                |          |
| By C. C. CHANG .....   | 1        |
| <b>Comments on the foundations of set theory</b>                             |          |
| By PAUL J. COHEN .....   | 9        |
| <b>Unsolved problems in set theory</b>                                       |          |
| By P. ERDÖS and A. HAJNAL .....  | 17       |
| <b>A more explicit set theory</b>  |          |
| By HARVEY FRIEDMAN .....   | 49       |
| <b>Sets, semisets, models</b>  |          |
| By PETR HÁJEK .....  | 67       |
| <b>The Boolean prime ideal theorem does not imply the axiom of choice</b>    |          |
| By J. D. HALPERN and A. LEVY .....   | 83       |
| <b>On models for set theory without AC</b>                                   |          |
| By THOMÁŠ JECH .....   | 135      |
| <b>Primitive recursive set functions</b>                                     |          |
| By RONALD B. JENSEN and CAROL KARP .....                                     | 143      |
| <b>End extensions of models of set theory</b>                                |          |
| By H. JEROME KEISLER and JACK H. SILVER .....                                | 177      |
| <b>Observations on popular discussions of foundations</b>                    |          |
| By G. KREISEL .....  | 189      |
| <b>Indescribability and the continuum</b>                                    |          |
| By KENNETH KUNEN .....   | 199      |
| <b>The sizes of the indescribable cardinals</b>                              |          |
| By AZRIEL LÉVY .....   | 205      |
| <b>On the logical complexity of several axioms of set theory</b>             |          |
| By AZRIEL LÉVY .....   | 219      |
| <b>Categorical algebra and set-theoretic foundations</b>                     |          |
| By SAUNDERS MAC LANE .....   | 231      |
| <b>The solution of one of Ulam's problems concerning analytic rectangles</b> |          |
| By R. MANSFIELD .....  | 241      |
| <b>Predicative classes</b>   |          |
| By YIANNIS N. MOSCHOVAKIS .....  | 247      |

|   |     |
|---|-----|
| <b>On some consequences of the axiom of determinateness</b>                                 |     |
| By JAN MYCIELSKI .....  | 265 |
| <b>Embedding classical type theory in ‘intuitionistic’ type theory</b>                      |     |
| By JOHN MYHILL .....  | 267 |
| <b>Ordinal definability</b>   |     |
| By JOHN MYHILL and DANA SCOTT .....   | 271 |
| <b>An axiom of strong infinity and analytic hierarchy of ordinal numbers</b>                |     |
| By KANJI NAMBA .....  | 279 |
| <b>Liberal intuitionism as a basis for set theory</b>                                       |     |
| By LAWRENCE POZSGAY .....   | 321 |
| <b>Forcing with perfect closed sets</b>   |     |
| By GERALD E. SACKS .....  | 331 |
| <b>Unramified forcing</b>   |     |
| By J. R. SHOENFIELD .....   | 357 |
| <b>The independence of Kurepa’s conjecture and two-cardinal conjectures in model theory</b> |     |
| By JACK SILVER .....  | 383 |
| <b>The consistency of the GCH with the existence of a measurable cardinal</b>               |     |
| By JACK SILVER .....  | 391 |
| <b>Real-valued measurable cardinals</b>   |     |
| By ROBERT M. SOLOVAY .....  | 397 |
| <b>Transfinite sequences of axiom systems for set theory</b>                                |     |
| By G. L. SWARD .....  | 429 |
| <b>Hypotheses on power set</b>  |     |
| By GAISI TAKEUTI .....  | 439 |
| <b>Multiple choice axioms</b>   |     |
| By MARTIN M. ZUCKERMAN .....  | 447 |
| <b>Author Index</b>   | 467 |
| <b>Subject Index</b>  | 471 |

## **FOREWORD**

The Fourteenth Annual Summer Research Institute, sponsored by the American Mathematical Society and the Association for Symbolic Logic, was devoted to Axiomatic Set Theory. Financial support was provided by a grant from the National Science Foundation. The institute was held at the University of California, Los Angeles, from July 10 to August 5, 1967, and was attended by more than 125 participants. The Organizing Committee consisted of Paul J. Cohen, Abraham Robinson (chairman), and Dana S. Scott (editor). Special thanks are due to the Department of Mathematics of UCLA for providing facilities and assistance which contributed in large measure to the excellent success of the meeting.

The program for the four weeks of the institute was organized into two ten-lecture series, given by Dana S. Scott and Joseph R. Shoenfield, plus individual contributions generally in one-hour sessions at the rate of four lectures per day. By the last week this was reduced to three per day, as the strength of the participants had noticeably weakened. Nevertheless, most of the success of the institute was due to the fact that nearly everyone attended all of the sessions.

The papers in this volume of the proceedings represent revised and generally more detailed versions of the lectures presented at the institute. In view of the large number of papers, which resulted in delaying the receipt of papers from some authors, it was felt advisable to divide the proceedings into two volumes so as not to delay the publication of these papers any longer.

DANA S. SCOTT

## Author Index

Roman numbers refer to pages on which a reference is made to an author or a work of an author.

*Italic numbers* refer to pages on which a complete reference to a work by the author is given.

**Boldface numbers** indicate the first page of the articles in the book.

- Balcar, B., 74, 80, 81  
Bar-Hillel, Y., 219, 229  
Barker, S., 324, 330  
Barwise, J., 158, 175, 248, 264  
Bateman, P. T., 456, 466  
Benecerraf, P., 330  
Bernays, P., 321, 323, 324, 327, 330  
Bing, R. H., 193, 197, 321, 330  
Birkhoff, Garrett, 231, 232, 234, 240  
Bleicher, M. N., 447, 449, 452, 453, 454,  
    455, 466  
Brauer, A. T., 456, 466  
Bukovský, L., 73, 74, 79, 80, 81  
  
Cantor, G., 322, 330  
Chang, C. C., 1, 1, 8, 388, 390  
Church, C., 265, 266  
Cohen, Paul J., 4, 7, 8, 9, 133, 136, 141,  
    197, 219, 221, 224, 226, 228, 229, 229,  
    319, 325, 329, 330, 331, 335, 354, 361,  
    380, 384, 386, 390, 439, 446  
Czipszer, J., 17, 48  
  
Dulmage, A. L., 456, 466  
  
Easton, W., 4, 8, 101, 133, 229, 229, 380,  
    380, 441, 446  
Eilenberg, S., 233, 239  
Erdős, P., 17, 17, 19, 20, 21, 23, 24, 25,  
    26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37,  
    38, 39, 40, 42, 43, 44, 45, 46, 47, 48  
  
Feferman, S., 226, 229, 229, 341, 343, 345,  
    349, 350, 354, 354, 364, 370, 380, 429,  
    438  
Finsler, P., 321, 322, 323, 324, 330  
Födör, G., 17, 48, 398, 418, 428  
Fraenkel, A., 219, 229, 321, 330  
Fraissé, R., 275, 278  
Freyd, Peter, 232, 237, 238, 239  
  
Gabriel, Pierre, 234, 239  
Gaifman, H., 5, 8, 177, 187, 319  
Gandy, R.), 144, 175, 243, 245, 248, 264  
    332, 335, 343, 344, 345, 346, 348, 354  
  
Garland, S., 199, 203  
Gladysz, S., 35, 48  
Gödel, K., 1, 8, 106, 133, 171, 175, 181,  
    187, 189, 190, 197, 221, 227, 229, 230,  
    241, 245, 277, 278, 278, 306, 308, 319,  
    321, 330, 332, 335, 339, 355, 359, 381,  
    407, 426, 428, 440, 441, 446, 448, 465,  
    466  
Gray, J. W., 239, 239  
Grothendieck, Alexander, 239  
  
Hájek, P., 67, 67, 70, 73, 74, 78, 79, 80,  
    81, 141  
Hajnal, A., 17, 17, 19, 20, 21, 22, 23, 24,  
    25, 26, 27, 28, 29, 31, 32, 33, 34, 36, 37,  
    38, 39, 40, 42, 43, 44, 45, 46, 47, 48,  
    384, 387, 390  
Halmos, P. R., 397, 399, 411, 414, 428  
Halpern, J. D., 83, 133, 224, 230  
Hanf, W., 187, 205, 206, 218, 406, 428  
Hausdorff, F., 440, 446  
Hrbaček, K., 75, 79, 80, 81  
  
Isbell, J. R., 235, 239, 239  
  
Jaegermann, M., 84, 133  
Jech, T., 75, 79, 80, 81, 135  
Jensen, R. B., 143, 230, 348, 355, 392, 394  
    395  
Johnson, S. M., 456, 466  
Juhász, I., 36, 46, 48  
  
Kan, D. M., 233, 239  
Karp, C., 143, 158, 175  
Keisler, H. J., 4, 8, 36, 48, 177, 178, 187,  
    205, 206, 212, 214, 215, 216, 218, 389,  
    390, 392, 395  
Kelley, J., 332, 355  
Kinna, W., 84, 133  
Kino, A., 144, 158, 176  
Kleene, S. C., 162, 175, 258, 264, 342, 347,  
    352, 353, 355  
Kochen, Simon, 319, 421, 428  
Kreisel, G., 157, 158, 176, 189, 190, 192,  
    194, 195, 196, 197, 197, 235, 239, 334,  
    345, 346, 347, 355

- Kripke, S., 157, 176, 248, 249, 264, 351, 355  
 Krivine, J. L., 190, 192, 196, 197, 235, 239  
 Kunen, K., 8, 199, 200, 202, 203, 409, 420, 428  
 Kuratowski, K., 241, 242, 245
- Läuchli, L., 133  
 Lawvere, F. W., 235, 239, 240  
 Lévy, A., 83, 133, 144, 162, 168, 176, 205, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 277, 278, 319, 366, 370, 381, 384, 390, 391, 392, 393, 395, 441, 446, 447, 449, 450, 451, 452, 457, 462, 465, 466  
 Linden, T., 176  
 Lopez-Escobar, E. G. K., 158, 176  
 Loé, J., 133  
 Luschei, E., 325, 330
- MacDowell, R., 177, 187  
 Machover, M., 157, 176  
 Mac Lane, Saunders, 231, 231, 232, 233, 234, 235, 237, 239, 240  
 Mahlo, P., 212, 218  
 Makkai, M., 45, 48  
 Mansfield, R., 241  
 Martin, D. A., 406, 428  
 Maté, A., 35, 48  
 Mathias, A. R. D., 335, 342, 343, 355  
 McAloon, K., 277, 278  
 McNaughton, R., 321, 327, 330  
 Mendelsohn, N. S., 456, 466  
 Mendelson, E., 450, 466  
 Milner, F. C., 17, 27, 39, 48  
 Mitchell, Barry, 232, 240  
 Montague, R., 183, 187, 223, 227, 230, 253, 264  
 Morley, M., 178, 187, 388, 390  
 Moschovakis, Y. N., 247, 248, 264  
 Mostowski, Andrzej, 133, 321, 330, 358, 381, 409, 428, 447, 449, 451, 466  
 Mycielski, Jan, 265, 265, 266, 266, 446  
 Myhill, John, 267, 271
- Namba, Kanji, 279, 319  
 Novák, J., 319
- Platek, R., 144, 158, 176, 222, 230, 351, 355  
 Poincaré, H., 324, 330  
 de Possel, R., 275, 278  
 Post, E. L., 277, 278, 352, 353, 355
- Pozsgay, Lawrence, 321  
 Příkrý, K., 80, 342, 355, 411, 428  
 Putnam, H., 330, 347, 353, 354, 355
- Quine, W. V., 84, 133, 273, 278
- Rado, R., 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 48  
 Ramsey, F. P., 19, 48, 327, 330  
 Ricabarra, R. A., 385, 388, 390  
 Rieger, L., 67  
 Roberts, J. B., 456, 466  
 Robinson, A., 190, 195, 198, 228, 230  
 Rödding, D., 176  
 Rogers, H., Jr. 349, 355  
 Rosser, J. B., 430, 438  
 Rowbottom, F., 4, 5, 6, 8, 241, 245, 385, 389, 390, 393, 395, 406, 428  
 Rubin, H., 83, 134  
 Rubin, J. E., 83, 134, 233, 240  
 Russell, Bertrand, 327, 330  
 Ryłl-Nardewski, C., 133
- Sacks, G. E., 157, 176, 331, 332, 335, 343, 345, 346, 348, 351, 352, 354, 355  
 Samuel, Pierre, 233, 240  
 Scarpellini, B., 84, 134, 229, 230  
 Scott, Dana, 1, 4, 8, 101, 134, 135, 141, 158, 176, 187, 200, 203, 205, 206, 215, 218, 244, 245, 271, 273, 278, 384, 390, 410, 411, 412, 413, 414, 417, 421, 422, 428  
 Seelbinder, B. M., 456, 466  
 Shepherdson, J. C., 228, 230, 359, 381  
 Shockley, J. E., 456, 466  
 Shoenveld, J., 162, 176, 242, 245, 335, 355, 357, 359, 366, 381, 441, 446  
 Sierpiński, W., 32, 43, 48, 84, 134, 449, 450, 452, 466  
 Sikorski, R., 129, 134, 401, 428  
 Silver, J. H., 5, 6, 8, 28, 48, 177, 178, 180, 187, 245, 319, 383, 389, 390, 391, 394, 395, 407, 409, 427, 428, 445, 446  
 Skolem, T., 322, 330  
 Sochor, A., 79, 80, 81  
 Solovay, R., 5, 8, 101, 134, 135, 141, 200, 203, 241, 244, 245, 335, 355, 384, 390, 392, 393, 394, 395, 406, 410, 411, 412, 413, 414, 417, 421, 428  
 Sonner, Johann, 234, 240  
 Specker, E., 21, 48, 177, 187  
 Spector, C., 331, 337, 343, 347, 350, 353, 354, 355

- Stark, H. M.,** 198  
**Steinhaus, H.,** 446  
**Stenius, Erik,** 321, 323, 325, 330  
**Štěpánek, P.,** 77, 81  
**Sward, G. L.,** 429  
**Szmielew, W.,** 449, 466
- Takeuti, G.,** 144, 155, 157, 158, 176, 277,  
278, 280, 319, 430, 438, 439, 439, 440,  
446  
**Tarski, A.,** 20, 36, 48, 158, 176, 177, 178,  
187, 205, 206, 212, 214, 215, 216, 218,  
392, 395, 397, 400, 428  
**Tharp, L.,** 332, 355  
**Turing, A. M.,** 430, 438
- Ulam, S. M.,** 241, 245, 397, 399, 428  
**Vaught, R. L.,** 177, 183, 187, 215, 216,  
218, 253, 264, 388, 390  
**von Heijenoort, J.,** 247, 264  
**Vopěnka, P.,** 73, 74, 75, 77, 78, 79, 79, 80,  
81, 135, 141
- Wagner, K.,** 84, 133  
**Wang, Hao,** 321, 325, 330  
**Wiśniewski, K.,** 449, 466
- Zermelo, E.,** 190, 198  
**Zuckerman, M. M.,** 447, 447, 449, 453, 466

# Subject Index

- $\forall$ -formulas, 228
- Absolute
  - formula, 112, 113, 115, 116, 122
  - function, 120
  - intervals, 90
  - relation symbol, 359
  - term, 121
- Absoluteness, 120
- Abstraction term, 441
- $\text{AC}_\alpha$ , 442
- $\text{AC}_0$ , 221
- Adequacy conditions, 190
- Adequate reduction, 192
- Adjoint functor, 233
- Admissible class, 160
- $\aleph_1$ -saturated  $k$ -complete ideal, 202
- Analytic basis, 297
- Antihomogeneous cardinal, 292
- Arithmetical ( $\Delta_1^1$ ) transcendency of cardinals, 280
- Assignment, 87
- Axiom
  - DC of dependent choice, 226
  - comprehension, 50
  - of choice (AC), 83, 84, 219, 326, 448
  - of choice, (WAC) weak term of the, 448
  - of choice for finite sets, 449
  - of constructibility, 101, 219, 359
  - of determinateness, 265
  - of extensionality, 106, 232
  - of infinity, 110, 115, 325
  - of  $\kappa$ -constructibility, 2
  - of pairing, 323
  - of replacement, 327
  - of separation, 326
  - of union(s), 107, 323
  - of the power set, 101, 107, 326, 439
  - of ZF, 106
  - multiple choice, 462, 447
  - parallel, 195, 196
  - schema
    - of continuity, 90
    - of foundation, 111
    - of replacement, 110
  - systems for set theory, 429
- Bernays, Gödel-, set theory, 233
- Boolean
  - algebra, 95
  - prime ideal theorem, 83, 84, 95
  - valued
    - ultraproduct, 420
- model ( $\nabla$ -model), 135
  - symmetric submodels of, 135
- Cantor's paradox, 323, 324, 325
- Cardinal(s)
  - arithmetical ( $\Delta_1^1$ ) transcendency of, 280
  - antihomogeneous, 292
  - inaccessible, 211, 216
  - indescribable, 420
    - $\Pi_m^n$ , 207
  - measurable, 4, 392
    - normally, 286
  - Ramsey, 279
  - regular, 358
  - singular, 358
    - $\Sigma_1^1$ -transcendency of, 305
    - weak-measurable, 399
    - weakly compact, 178
- Category, 232
  - complete, 238
  - fibered, 239
  - large, 233
  - small, 233
    - locally, 233
- CH, 221
- Chain condition, 371
- Chang's conjecture, 389
- Characterizable
  - $\nabla_2^1$ , 199
- Choice, (AC) axiom of, 83, 84, 219, 326, 448
- Class, 359
- Cleavage, 239
- Codomain, 232
- Cohen's method of forcing, 7, 219, 383
- Collapsing function, 386
- Colouring number of a graph  $G$ , 37
- Commutative diagram, 232
- Compatible elements, 371
- Composite, 232
- Comprehension axiom, 50
- Condition(s), 224, 360
- Consistency of SP, 101
- Constructibility, axiom of, 101, 219, 359
  - generalized, 219
  - simple, 219
- Continuum hypothesis, 321, 329
- Covariant hom functor, 237, 239
- $C(n)$ , 449
- (DAC), 457
- Dedekind finite set, 84, 91
- Definable set, 271
- Degree of constructibility, 331

- $\Delta_3^1$  well-ordering, 394  
 Dependent choices (DC), axiom of, 226  
 DC, 442  
 Denotation operator, 441  
 Denumerable set, 448  
 Dense set, 360  
 Describable ordinal, 206  
     weakly  $\Omega$ -, 226  
 Determinateness, axiom of, 265  
 Direct-limits, 308  
 Domain, 232  
 $\exists$ -formulas, 228  
 $\exists \forall$ -formulas, 228  
 Enforceable class, 206  
     weakly  $\Omega$ -, 226  
     in  $A$ , 214  
     at  $\theta$ , 206  
 Equalizer, 238  
 Extension, 178, 360  
     elementary, 177  
     end, 177  
 Existential-universal, 222  
 Extensionality, axiom of, 106, 232  
  
 $F$ -symmetric, 136  
     hereditarily, 136  
 Fiber over  $C$ , 239  
 Fibered categories, 239  
 Filter, 136  
 Finistic trees, 97  
 First-order arithmetic, 50  
     predicate calculus, 103  
 $FL$ , 86  
 Forcing, 84, 362, 384  
     Cohen's method of, 219  
     language, 362  
     notion of, 360  
 Formal independence, 190, 194, 196  
     language, 85  
 Formalism, 191  
 Formalist conception, 189  
 Formula, 178  
 Foundation  
     and organization, 189  
     axiom schema of, 111  
 $FS_m$ , 449  
 Functional, 359  
 Functor, 232  
     adjoint, 233  
     left, 233  
     category, 234  
     covariant hom, 237, 239  
  
 Fraenkel-Mostowski-type models, 447  
 Fraenkel, Zermelo-  
     axioms, 321, 323  
     set theory, 234, 235  
 GCH, 221, 392  
 Generative function, 295  
 Generic subset, 360  
 Gödel  
     ramified hierarchy, 3  
     -Bernays set theory, 233  
  
 Hereditarily ordinal-definable set, 276  
 Hierarchy of formula in set theory, 220  
 Hilbert's program, 190, 191  
 HOD over, 369  
 Homogeneous, 368  
 Hyperprojective set, 248  
  
 Inaccessible cardinals, 211, 216  
 Incompatible elements, 371  
 Incompleteness theorem, 190, 191  
 Identity, 232  
 Indescribable, 180, 206  
      $\bigvee_n^1 (\bigwedge_n^1 -)$ , 201  
      $\bigwedge_n^1$ , 202  
     cardinal, 420  
      $\Pi_m^n$ , 207  
 Indiscernibles, 394  
 Infinity, axiom of, 110, 115, 325  
 Insertion, 232  
 Intersection, diagonal, 211  
 Interval designators, 90  
 Intuitionistic type theory, 267  
 Inverse limits, 238  
 Jónsson algebra, 4  
 $\kappa$ -  
     closed, 181  
     constructible sets, 2  
     constructibility, axiom of, 2  
     definable subsets of  $A$ , 2  
     well founded, 181  
 Kurepa's conjecture, 385  
  
 $\mathcal{L}$ , 85  
 $L_\alpha$ , 85  
 $L_{\kappa\kappa}$ , 2  
 $L_{\lambda\kappa}$ , 5  
 $\lambda$ -saturated ideal, 400  
 $\Lambda_n$ , 220  
 Large category, 233  
 Law of the excluded middle, 328  
 $LC_n$ , 455

- Left adjoint functor, 233
- Liberal intuitionism, 322
- Limited formula, 441
- Lin Comb( $m_1, \dots, m_2, \dots, m_n$ ), 455
- Locally small category, 233
- Logical complexity, 219
- Lowenheim-Skolem theorem for  $L_{\infty\infty}$ , downward, 4
- Mahlo's theorem, 212
- $MC_\Phi$ , 464
- Measurable, 215
  - cardinal, 4, 392
- Measure of complexity, 219
- Model-class, 135
- Morphisms, 232
- Mostowski-, Fraenkel-, type models, 447
- Multiple choice axiom, 447
  - ordinal characterizations of, 462
- $n$ -element set, 448
- Name, 362
- Natural transformations, 232
- Nontrivial set, 448
- Normal, 136
  - class, 215
  - ultrafilter, 392
- Normally measurable cardinal, 286
- Notion of forcing, 360
- Objects, 232
  - of type, 205
- OD form, 369
  - rule, 429
- Ordering theorem, 84
- Ordinal, 121
  - definable set, 272
  - number, 448
  - characterizations of multiple choice axioms, 462
  - regular, 207
- Ordinary partition symbol, 19
- Organization and foundation, 191
- Pairing, axiom of, 323
- Paradox, 323
  - Cantor's, 323, 324, 325
- Parallel axiom, 195, 196
- Partition relations, 5
  - $\phi$ -commutative, 296
    - invariant, 296
  - $\Pi_m^n$ , 205
    - indescribable cardinals, 207
- transcendency of measurable cardinal numbers, 317
  - universal formula for, 209
- Pointed, 351
- Pointedly generic, 352
- Polarized partition relations, 29
- Power set
  - axiom of, 101, 107, 326, 439
  - reflection principle on, 440
- $PP_n$ , 461
- Predicate calculus, first order, 103
- Predicatively definable class, 248
- Prime set for  $n$ , 456
- Primitive recursive set function, 145
  - ordinally, 146
- Product, 238
  - theorem, 367
- Proper classes, 324
- $R$ -function, 269
- Ramified hierarchy, 441
- Ramsey
  - cardinals, 279
  - theorem, 19
- Rank, 358
- Real-measurable cardinals, 399
- Regular
  - cardinal, 358
  - ordinal, 207
- Relative
  - consistency, 389
  - constructibility, 84
  - constructible, 391
- Replacement, axiom of, 327
- Reflection
  - principle, 272, 430, 439
    - on power set, 440
  - property, 179
  - $\bigwedge_n^1 (\bigvee_n^1)$ , 199
- Relativizations to  $M$ , 112
- Restricted
  - formula, 220
  - quantifier, 220
  - $\rho_0$ -numbers, 212
  - $\rho_1$ -numbers, 212
- Satisfaction class, 210
- Second order, 196
  - arithmetic, 50
- Section, 372
- Semisets, 71
- Semiuniversal class, 444

- Separation, axiom of, 326
- Set mappings, 34
- Set theory, 49, 83
  - Gödel-Bernays, 233
  - axiom systems for, 429
  - hierarchy of formulas in, 220
  - $SP$ , 90
  - Zermelo-Fraenkel, 234, 235
- $\Sigma_n$ , 220
- $\Sigma_m^n$ , 205
- $\Sigma_1^1$ , transcendency of cardinals, 303
- Singular cardinal, 358
- Skolem's hypothesis, 329
- Stable sets, 268
- Standard sets, 85
- Strongly normal class, 315
- Substitution function, 88
- Support, 76
- Syntactic model, 68
  
- T*-
- absolute, 144
- definable, 144
- persistent, 148
- TL*, 86
- Transfinite sequences, 429
- Transitive substructure, 228
- Tree, 97, 242
  
- finistic, 97
- Triangular matrix, 47
- Two-cardinal conjecture, 388
  
- U*-invariant, 286
- Ultraproduct, 286
- Union(s), axiom of, 107, 323
- Universal, 222
- Universal-existential, 222
- Universe, 234
- Urelemente, 447
  
- $V_\Phi$ , 462
- $V_\Phi^*$ , 464
- $V = L$ , 101
- Valuation function  $\text{val}_P(u)$ , 225
- Variables of type, 205
  
- WAC, 448
  
- Yoneda lemma, 237
  
- ZF, 83
  - axiom(s) of, 106
- ZFM, 222, 226
- $Z(n)$ , 457
- Zermelo-Fraenkel
  - axioms, 321, 323
  - set theory, 234, 235

