

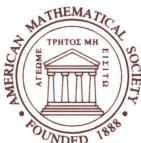
Proceedings of Symposia in PURE MATHEMATICS

Volume 16

Global Analysis

American Mathematical Society
Summer Institute
Held at the University of California
Berkeley, California
July 1-26, 1968

Shiing-Shen Chern
Stephen Smale
Editors



American Mathematical Society

Proceedings of Symposia in PURE MATHEMATICS

Volume 16

Global Analysis

American Mathematical Society
Summer Institute
Held at the University of California
Berkeley, California
July 1-26, 1968

Shiing-Shen Chern
Stephen Smale
Editors



American Mathematical Society
Providence, Rhode Island

*Prepared by the American Mathematical Society
under National Science Foundation Grant GP-8410*

AMS 1968 Primary Subject Classification 5750

Standard Book Number 0-8218-1416-8
Library of Congress Catalog Number 70-95271

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Manager of Editorial Services, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@math.ams.org.

Copyright © 1970 by the American Mathematical Society

Reprinted 1983.

Printed in the United States of America.

The American Mathematical Society retains all rights
except those granted to the United States Government.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

10 9 8 7 6 5 4 3 01 00 99

CONTENTS

Preface	v
Existence Theorems for Nonlinear Partial Differential Equations	1
BY FELIX E. BROWDER	
A Fixed Point Theorem for Holomorphic Mappings	61
BY CLIFFORD J. EARLE AND RICHARD S. HAMILTON	
Variation Integrals in Fiber Bundles	67
BY HALLDOR I. ELIASSON	
Note on the Differentiability of Nonlinear Semigroups	91
BY TOSIO KATO	
Scattering Theory as the Analysis of Elliptic Fixed Points	95
BY I. E. SEGAL	
Topology of Elliptic Operators	101
BY MICHAEL F. ATIYAH	
Eigenvalues of the Laplacian	121
BY MARCEL BERGER	
On a Topological Obstruction to Integrability	127
BY RAOUL BOTT	
An Asymptotic Expansion for the Heat Equation	133
BY PETER GREINER	
An Analytic Proof of the Classical Riemann-Roch Theorem	137
BY TAKESHI KOTAKE	
The Eigenvalues of the Laplace Operator for the Exterior Problem	147
BY PETER D. LAX AND RALPH S. PHILLIPS	
Pseudo-differential Operators	149
BY L. NIRENBERG	
On the Calculus of Symbols for Pseudo-differential Operators	169
BY BENT E. PETERSEN	
Une Remarque sur les Operateurs Integraux	175
BY WEISHU SHIH	
A Differential Geometric Approach to Characteristics	179
BY ROBERT B. GARDNER	
Formal Theory of Overdetermined Linear Partial Differential Equations	187
BY HUBERT GOLDSCHMIDT	
Algebraic Results of Characteristics	195
BY V. W. GUILLEMIN	
On Estimate $\ (A(D) + B(x))u(x)\ \geq c\ u\ $	199
BY MASATAKE KURANISHI	
Quantization of the De Rham Complex	205
BY I. E. SEGAL	
Uncoupled Overdetermined Systems	211
BY D. C. SPENCER	

Triangular Operators	221
BY W. J. SWEENEY	
Regularity Theorems for Solutions of Elliptic Polynomial Equations	225
BY K. UHLENBECK	
Problems in Nonlinear Hyperbolic Equations	233
BY J. GLIMM AND P. LAX	
Hamiltonian Mechanics on Lie Groups and Hydrodynamics	237
BY J. MARSDEN AND R. ABRAHAM	
Author Index	245
Subject Index	247

PREFACE

The papers in these Proceedings grew out of lectures given at the fifteenth Summer Mathematical Institute of the American Mathematical Society, whose topic was global analysis. The Institute was held at the University of California at Berkeley from July 1 to July 26, 1968, and was partially financed by the National Science Foundation.

Notes of lectures were distributed at the time of the conference and some of the papers here are just as in those notes. These volumes, however, can be distinguished from the notes in the sense that in general the papers here are not just expositions of material that has or will appear elsewhere; most of the articles could just as well have appeared in Journals.

The unity given by the subject matter makes it desirable to collect them here. It is hoped that the volumes will provide an important start to the scientist who wishes to learn what is going on in that part of mathematics called global analysis.

The organizing committee for the institute consisted of: F. Browder, S.-S. Chern, L. Hörmander, I. Singer, and S. Smale, with the co-editors serving as co-chairmen.

Seminar organizers were: F. Browder, E. Calabi, H. Goldschmidt, R. Hermann, C. Morrey, R. Palais, C. Pugh, I. Singer, and D. Spencer.

Finally the editors would like to thank the many people who made the institute and volumes possible. Of especially direct help to ourselves were Celeste Andrade, Ann Harrington, Gordon and Jacqueline Walker.

S.-S. Chern

December 1968

S. Smale

AUTHOR INDEX

Roman numbers refer to pages on which a reference is made to an author or a work of an author.
Italic numbers refer to pages on which a complete reference to a work by the author is given.
Boldface numbers indicate the first page of the articles in the book.

- Abraham, R., **237**, 243
 Adams, J. F., 102, 106
 Agmon, S., *146*, 229, 231
 Arima, R., *134*
 Arnold, V., 237, 241, 244
 Atiyah, Michael F., **101**, *108*, *118*, 124, *146*, 149,
 159, *166*, *176*, *177*, 200
 Aubin, T., 123
- Baum, P., 130, 131
 Bergandal, G., *144*, *146*
 Berger, Marcel, **121**, *125*
 Bernstein, Sergei, 1, 2
 Blancheton, E., 237, 244
 Bott, Raoul, **127**, 131, *131*
 Brezis, H., 25
 Browder, Felix E., **1**, 25, *42*, *45*, *55*, *57*, 238
 Brouwer, L. E. J., 2
- Calabi, 220
 Calderón, A. P., 89, 161, 163, *166*, 169, *173*
 Carathéodory, C., 61, 64, 65, 65
 Carleman, T., 121
 Cartan, Elie, 181, *186*, 191
 Cheeger, J., 123
 Chern, S. S., *65*, *131*, *146*
 Courant, 40
 Crandall, M. G., 94
- Douglis, A., 229, *231*
 Duhamel, 143
- Earle, Clifford J., **61**
 Ebin, D., 241, *244*
 Eells, J., Jr., 89
 Ehrenpreis, 219
 Eidelman, S. D., *146*
 Eliasson, Halldor I., **67**, 89, 225
 Eskin, 163
 Euler, 239
- Faber, 121
 Fisher, 40
 Folk, 123
 Fredholm, 176
 Friedrichs, K. O., 155, 166, *166*
- Gårding, L., 144, *146*
 Gardner, Robert B., **179**
 Glimm, J., **233**, 234, 235
 Godunov, *234*
 Goldschmidt, Hubert, **187**, *194*, 197, 219
 Goursat, E., *186*
 Greiner, Peter, **133**
 Grothendieck, A., 105, 113
 Guillemin, V. W., *194*, **195**, 211, 219, 220
- Hamilton, Richard S., **61**
 Harris, L. A., 64, 65
 Hartman, 25, 25
 Henrich, C. J., 187
 Hervé, M., 65
 Hille, E., 65
 Hirzebruch, F., 111, *119*, *146*, *177*
 Hopf, 233
 Hörmander, L., *119*, 134, *135*, 145, 156, 162,
 163, *166*, *167*, 169, 173, *173*, 199, *199*, 200
 Huber, H., *125*
- Kac, M., 121, *125*, 134, *135*
 Kachurovski, 4
 Kato, Tosio, 55, *57*, *57*, **91**, 238, *244*
 Kneser, M., *124*, *125*
 Kobayashi, S., 65
 Kohn, J. J., *167*, 216, 217, 219, *220*, 222, *224*
 Kōmura, Y., 55, *57*, 91, *94*
 Koschorke, U., 101
 Kotake, Takeshi, **137**, *146*, 163
 Krahn, 121
 Krasnolselskiĭ, 9
 Kumano-go, H., 162
 Kuranishi, Masatake, 155, **199**, 219
- Ladyzhenskaya, O., *244*
 Lavine, R., *210*
 Lax, Peter D., **147**, *148*, 166, *167*, **233**, 233, *234*,
 234, 235
 Leray, 2
 Leslie, J., 241, *244*
 Levine, H. I., 65
 Lewittes, J., 64, 65, 65
 Liapounov, 2
 Lichnerowicz, A., 123, *125*
 Ludwig, D., *147*, *148*
 Lusternik, 40, *40*, 41

- MacKichan, B. B., 211, 214, 218, 220
 McKean, H. P., Jr., 121, 125, 134, 135, 145, 146
 Marsden, J., 237, 243, 244
 Milnor, J., 122, 125
 Minakshisundaram, S., 123, 125, 146
 Minty, George, 9
 Mizohata, S., 135
 Morawetz, C. S., 147, 148
 Morrey, C. B., 219, 220, 228, 230, 231, 242

 Narasimhan, M. S., 146
 Nirenberg, L., 65, 77, 89, 101, 118, 118, 149, 167, 216, 220, 222, 224, 229, 231

 Obata, M., 123, 125
 Oleinik, 233, 234, 235
 Omori, H., 241, 244

 Palais, R. S., 40, 67, 72, 80, 89, 119, 167, 173, 177, 225, 231
 Pasternack, Joel, 129
 Pazy, A., 94
 Peetre, J., 149
 Petersen, Bent E., 169, 173
 Phillips, Ralph S., 65, 147, 148
 Pleijel, 123, 125, 146
 Pohojhayey, 19
 Poincaré, 40

 Quillen, D. G., 193, 194, 194

 Reiffen, H.-J., 61, 65
 de Rham, G., 89, 177

 Saber, J. C., 67, 89
 Sampson, J. H., 89
 Schauder, 2
 Schmidt, Erhard, 2
 Schnirelman, 40
 Seeley, R. T., 146, 159, 163, 167
 Segal, I. E., 95, 100, 205, 210
 Serre, 191
 Shih, Weishu, 169, 175, 177
 Singer, I. M., 118, 121, 125, 134, 135, 145, 146, 159, 166, 169, 176, 211
 Smale, S., 67, 89
 Smithies, F., 177
 Smoller, 234
 Spencer, D. C., 189, 194, 194, 199, 211, 219, 220, 222
 Stampacchia, 25, 25
 Sternberg, S., 186, 194
 Strauss, W., 100
 Sweeney, W. J., 149, 212, 219, 220, 221

 Tanaka, S., 125
 Tondeur, P., 244

 Uhlenbeck, K. K., 67, 72, 89, 225, 231

 Vainberg, 4
 Višik (Vishik), 6, 163

 Weyl, Hermann, 40, 41, 221, 146

 Zygmund, A., 166

SUBJECT INDEX

- Adjoint operator, 70
- Adjoint, 213
- Arithmetic genus, 111
- Asymptotic expansion, 133

- Banach algebra
 - Gleason parts, 64
- Banach manifold
 - Manifold model, 74, 80
- Boundary problems, 133

- Carathéodory metric, 65
- Cauchy characteristic system, 181
- Cauchy problem, 182
- Cauchy–Riemann operator, 106
- Chern-classes, 128
- Chern number, 138
- Clifford algebra, 106
- Cohomology, 205
- Commutation, 213
- Compatible, 239
- Compatibility conditions, 192
- Completely regular, 192
- Complex, de Rham, 205
- Condition (C), 67, 68, 86
- Connection map, 68, 78
- Connectivity invariants, 205
- Conservation of energy, 238
- Conservation form, 233
 - Nonlinear motions, 233
- Continuous symbol, 160
- Contraction, 91
- Covariant derivative, 68
- Covariant differentiation, 69
- Curvature tensor, 70

- δ -estimate, 212
- de Rham's theorem, 110
- Diffeomorphism, volume preserving, 241
- Differentiable p -forms, 106
- Differential equation, 188
 - Boundary problems, 133
 - Conservation form, 233
 - Entropy condition, 233
 - Euler-Lagrange equation, 68
 - Formally integrable, 189
 - Formal solutions, 189
 - Hyperbolic differential equations, 163
 - Initial value problem, 163, 164
 - Parabolic Monge-Ampere equation, 179
 - Regularity theorem, 68
 - Riemann problem, 234
- Differential forms, 205
 - Poincaré lemma, 209
- Differential operator, 101, 187
 - Adjoint operator, 70
 - Cauchy-Riemann operator, 106
 - Completely regular, 192
 - δ -estimate, 212
 - Dirac operator, 106
 - Dirichlet problem, nonlinear, 67
 - Elliptic, 70, 101, 157, 158, 162, 194
 - Energy operator, 67
 - Existence of local solutions, 211
 - Fourier transform, 150, 151, 155
 - Heat operator, 133
 - Hypoelliptic differential operators, 153
 - Index, 102, 158, 159
 - Laplace operator, 105, 106
 - Laplacian, 70
 - Leibniz' formula, 153
 - Linear differential operators, 69, 71
 - Neumann problem, 215, 216, 221
 - Quasi-polynomial differential operators, 74, 84
 - Of order k and weight w , 67
 - Regularity, 77
 - Riemann-Roch Theorem, 111, 137
 - Symbol, 70, 150, 153, 155–160, 169, 189
 - Wave operator, 98
- Dirac operator, 106
- Dirichlet problem, nonlinear, 67
- Dolbeault complex, 110

- Eigenfunction
 - Monotonicity, 148
 - Radiation condition, 147
- Eigenvalue
 - Spectrum, 121
- Elliptic, 70, 101, 157, 158, 162, 194
 - Strongly, 70, 133
- Elliptic complex, 109, 194
- Elliptic operators, 157, 158
 - Inverse, 158
 - Invertible, 157
- Elliptic pseudodifferential operator, 157, 158, 159
- Elliptic symbols, 159
- Energy, 98
 - Positivity, 99
- Energy function, 68, 86
- Energy integral, 67, 86
- Energy operator, 67
- Entropy condition, 233

- Euler characteristic, 108
- Euler equations, 239
- Euler–Lagrange equation, 68
- Existence of local solutions, 211
- Exterior derivation, 106
- Exterior problem, 147

- Fibre bundle
 - Local trivialization, 78, 82
- Fixed point, 91
- Elliptic, 95
- Flow, geodesic, 239
- Formally integrable, 189
 - Spencer sequence, 193
- Formal solutions, 189
- Fourier transform, 150, 151, 155
- Fredholm operator, 158
- Functionals, invariant, 234

- Gårding's inequality, 160, 161
- Gauss–Bonnet formula, 145
- Generalized Neumann problem, 199
- Generator, 91
- Geodesic flow, 239
- Gleason parts, 64
- Global Bott theorem, 112
- Green's matrix, 133
- Guillemin's local decomposition, 213

- Half-estimate, 199
- Hamiltonian vectorfield, 238
- Harmonic forms, 108
- Heat operator, 133
 - Parametric, 133
- Hodge theory, 108, 110
 - de Rham's theorem, 110
- Holomorphic, 61
- Hörmander's Green's formula, 134
- Hydrodynamics, 237
 - Kelvin's circulation theorem, 243
 - Pressure, 242
- Hyperbolic differential equations, 163
- Hypoelliptic, 153, 158
- Hypoelliptic differential operators, 153

- Index, 102, 158, 159
 - Signature, 108
- Index theorem, 113
- Infinitely smoothing operators, 153
- Initial value problem, 163, 164
- Invariant functionals, 234
- Inverse, 158
- Invertible, 157
- Involutive, 190
- Isospectral deformation, 124
- Isospectrality, 121

- J -characteristic system, 182
 - Monge characteristic curves, 185
- J -focal system, 182

- k -stable vector field X , 184
- K -theory, 112
- Kelvin's circulation theorem, 243
- Kernel, 151

- Laplace operator, 105, 106
 - Exterior problem, 147
 - Harmonic forms, 108
- Laplacian, 70
 - Spectrum, 121
- Leading symbol, 101
- Leibniz' formula, 153
- Levi–Civita connection, 69
- Lie group, 237
 - Regular, 239
- Linear differential operators, 69, 71
- Local, 149
- Local Bott theorem, 106
- Local connectors, 79
- Local operator, 149
- Local trivialization, 78, 82
- Locality, 99

- Manifold, 127
 - Arithmetic genus, 111
 - Asymptotic expansion, 133
 - Cauchy characteristic system, 181
 - Cohomology, 205
 - Connectivity invariants, 205
 - Dolbeault complex, 110
 - Elliptic complex, 109, 194
 - Global Bott theorem, 112
 - Signature, 108
 - Symplectic structure, 98
 - Volume bundle, 172
- Manifold model, 74, 80
- Map
 - Condition (C), 67, 68, 86
 - Connection map, 68, 78
 - Contraction, 91
 - Energy function, 68, 86
 - Exterior derivation, 106
 - Fixed point, 91
 - Half-estimate, 199
 - Holomorphic, 61
 - Involutive, 190
 - Symbol, 70, 150, 153, 155–160, 169, 189
 - Topological index, 113
 - Translation, 82
 - 2-acyclic, 190
- Matrix
 - Elliptic, 157
 - Green's matrix, 133
 - Monge characteristic curves, 185
- Monotonicity, 148

- Neumann problem, 215, 216, 221
- Noncharacteristic Cauchy problem, 195
- Nonexpansive (nonlinear) operators, 91
- Nonlinear motions, 233
- Operator
 - Adjoint, 213
 - Elliptic, 157, 158
 - Fredholm, 158
 - Hypoelliptic, 153, 158
 - Index, 102, 158, 159
 - Infinitely smoothing operators, 153
 - Kernel, 151
 - Local, 149
 - Nonexpansive (nonlinear) operators, 91
 - Order, 150
 - Overdetermined system, 158
 - Pseudolocal, 149, 150, 152, 153, 158, 162, 163
 - Quantization, 205
 - Singular integral operators, 151
 - Symbol, 70, 150, 153, 155–160, 169, 189
- Order, 150
- Overdetermined system, 158
 - Elliptic, 158
- Parabolic Monge-Ampere equation, 179
- Parallel to the boundary, 241
- Parametrix, 133
- Poincaré lemma, 209
- Pontryagin classes, 127
- Pressure, 242
- Prolongation, 188
- Pseudodifferential operators, 102, 149, 150, 152–158, 160, 162–165, 169
 - Cauchy problem, 182
 - Elliptic pseudodifferential operator, 157, 158, 159
 - Gårding's inequality, 160, 161
 - Noncharacteristic Cauchy problem, 195
- Pseudolocal, 149, 150, 152, 153, 158, 162, 163
- Quantization, 205
- Quasi-polynomial differential operators, 74, 84
- Quasi-polynomial operators of order k and weight w , 67
- Radiation condition, 147
- Rarefaction waves, 236
- Regularity, 77
- Regularity theorem, 68
- Regular Lie group, 239
- Relativistic equations, 98
- Relativistic invariance, 99
- Riemann problem, 234
- Riemann surface, 104
 - Gauss-Bonnet formula, 145
- Riemann-Roch Theorem, 111, 137
- Riemannian connection, 69
- Riemannian manifold, 108
- Riemannian metric, 68, 69
 - Compatible, 239
 - Weak, 237
- RMC structures, 68, 69, 78
- Scattering, 96
- Section functors, 73, 74
- Semigroup, 91
 - Generator, 91
- Set
 - Sing supp v , 149, 152, 153
- Signature, 108
- Sing supp v , 149, 152, 153
- Singular integral operators, 151
- Sobolev embedding theorems, 71
- Sobolev spaces, 71, 109
- Spaces of sections, 70
- Spectrum, 121
 - Isospectral deformation, 124
 - Isospectrality, 121
- Spencer sequence, 193
- Submanifold
 - Locality, 99
- Symbol, 70, 150, 153, 155–160, 169, 189
 - Commutation, 213
 - Continuous symbol, 160
 - Elliptic, 159
 - Leading symbol, 101
 - Local Bott theorem, 106
 - Of order m , 150
- Symbol cohomology groups, 190
- Symplectic form, 237
 - Weak, 237
- Symplectic structure, 98
- Tangent bundle, 127
 - Chern-classes, 128
 - Pontryagin classes, 127
- Tangent trivializations, 78
- Topological index, 113
 - Index theorem, 113
- Translation map, 82
- 2-acyclic, 190
- Variation integrals, 67, 84
 - Energy integral, 67, 86
- Vector bundle
 - Compatibility conditions, 192
 - Levi-Civita connection, 69
 - Local connectors, 79
 - Relativistic invariance, 99
 - Riemann-Roch theorem, 137
 - Riemannian connection, 69
 - Riemannian metric, 68, 69
 - RMC structures, 68, 69, 78
- Vector field
 - Euler equations, 239

- Hamiltonian vectorfield, 238
- k-stable vector field X , 184
- Parallel to the boundary, 241
- Vector field system
 - J -characteristic system, 182
 - J -focal system, 182
- Volume bundle, 172
- Volume preserving diffeomorphisms, 241
- Wave operator, 98

