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Representation Theory of Finite Groups and Related Topics

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Preface

The symposium on Representation Theory of Finite Groups and Related Topics was held in Madison, Wisconsin, on April 14–16, 1970, in conjunction with a sectional meeting of the American Mathematical Society. The symposium was held in honor of Professor Richard Brauer, whose fundamental work in representation theory lies at the heart of most of the further developments in this topic.

These proceedings contain articles by the participants, based on their symposium lectures. The articles range from brief surveys of results to detailed outlines of proofs, and are intended to indicate the scope of current research in representation theory.

The organizing committee consisted of Professors Richard Brauer, Charles W. Curtis, Walter Feit, James A. Green, and Irving Reiner (chairman). The committee wishes to express its thanks to the National Science Foundation for its financial support of the symposium. We are also grateful to our colleagues at the University of Wisconsin for making us welcome in Madison. Finally, we thank the American Mathematical Society staff for helping with the arrangements of the symposium.

IRVING REINER

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