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PREFACE

The American Mathematical Society held its Seventeenth Summer Research Institute at the University of Wisconsin, Madison, Wisconsin, from June 29 to July 17, 1970. The topic for the institute was algebraic topology. The organizing committee for the institute consisted of William Browder, Edward R. Fadell, Edwin E. Floyd, Peter J. Hilton, Richard K. Lashof, Arunas L. Liulevicius (chairman), Mark E. Mahowald, R. James Milgram, Franklin P. Peterson, James D. Stasheff, and P. Emery Thomas.

The program of the institute was divided into four parts: (1) survey talks on recent developments in the field of algebraic topology; (2) invited one-hour talks on important recent work (these tended to be more technical than the survey talks); (3) sessions on problems; (4) seminars organized by the participants in the institute. The survey talks were given by J. Frank Adams, William Browder, Edgar H. Brown, Jr., Samuel Gitler, Richard K. Lashof, Franklin P. Peterson, Larry Smith, and James D. Stasheff. Lecture notes for these series appear in this volume, the only exception being William Browder's "Homotopy and manifolds" (to appear in the proceedings of the Amsterdam conference on topology of manifolds). The remainder of this volume consists of the texts of the invited one-hour talks and a list of research problems. There were four sessions on problems in algebraic topology: a general session (R. James Milgram), immersions and embeddings (Samuel Gitler), H -spaces (James D. Stasheff), homotopy theory (Franklin P. Peterson); the resulting list of research problems was edited by R. James Milgram. Seminars at the summer institute were organized by the participants themselves. A partial list of the more stable seminars included categorical methods, cobordism, cohomology of Postnikov systems, H -spaces, foliations, and group actions. No seminar talks are included in this volume.

The summer institute was sponsored by the National Science Foundation under Grant GP 19276. The organizing committee wishes to thank Lillian R. Casey of the American Mathematical Society for her help in organizing the conference. The institute was fortunate in having Edward R. Fadell as its administrative director who made sure that things ran smoothly. To him and the other friendly people at the University of Wisconsin go the thanks of all of the participants.

Arunas Liulevicius

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Roman numbers refer to pages on which a reference is made to an author or a work of an author.

Italic numbers refer to pages on which a complete reference to a work by the author is given.

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