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## PREFACE

The papers in these Proceedings are an outgrowth of lectures delivered at the eighteenth Summer Research Institute of the American Mathematical Society. The topic of the institute was partial differential equations, and it was held at the University of California at Berkeley from August 9 to August 27, 1971. The institute was financed by the National Science Foundation.

Notes of lectures were distributed during the conference (and remaining notes shortly afterward) to the participants, and many of the papers appearing in this volume are revised versions of the informal notes. Although some of the papers are expositions of known material, many contain new results. The papers are arranged under the headings of the five seminars of the conference: linear problems, nonlinear problems, geometry, mathematical physics, and singular integral operators.

The organizing committee for the institute consisted of: Alberto P. Calderón, Lars Hörmander, Charles B. Morrey, Jr., Louis Nirenberg (Chairman), James B. Serrin, Isadore M. Singer and Donald C. Spencer.

The editor would like to thank the many persons who cooperated to make the institute and this volume possible. Of special direct help were Lillian R. Casey, Beth Clarke, Hope Daly (conference secretary), Carole Kohanski and Margaret Reynolds.

D. C. SPENCER



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*Italic numbers* refer to pages on which a complete reference to a work by the author is given.

Roman numbers refer to pages on which a reference is made to a work of the author. For example, under Shiffman would be the page on which a statement like the following occurs: "The following corollary generalizes a result of Shiffman..."

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