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Volume 24

# Analytic Number Theory

Symposium on
Analytic Number Theory
March 27–30, 1972
St. Louis, Missouri

Harold G. Diamond
Editor

American Mathematical Society

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# FOREWORD

A symposium on Analytic Number Theory and Related Parts of Analysis was held at St. Louis University, St. Louis, Missouri, on March 27–30, 1972, in conjunction with the six hundred ninety-third meeting of the American Mathematical Society. The Organizing Committee for the symposium consisted of Harold G. Diamond (chairman), Patrick X. Gallagher, Hugh L. Montgomery, Wolfgang M. Schmidt, and Harold M. Stark.

Twenty-nine number theorists were invited to lecture on their recent research, which covers a broad spectrum of contemporary work in number theory. The program was arranged in seven half day sessions, chaired by Paul T. Bateman, Paul Erdös, Lowell Schoenfeld, and the organizers.

This volume contains accounts of all the lectures presented at the symposium. Paul Erdös, who could attend only the first hours of the symposium, also contributed an article to the volume. The articles are arranged alphabetically (according to the name of the speaker in the case of joint work).

The conference participants are indebted to a number of individuals and organizations for their good planning and administration. In particular, mention should be made of the work of AMS Associate Secretary Paul T. Bateman, Mrs. Lillian Casey of the AMS, and Lawrence W. Conlon of St. Louis University. Financial support for the symposium was provided by a grant from the National Science Foundation.

It is hoped that the lively ideas presented at the symposium will be further disseminated by this volume and will spawn new number theoretic research in the years to come.

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P. T. BATEMAN

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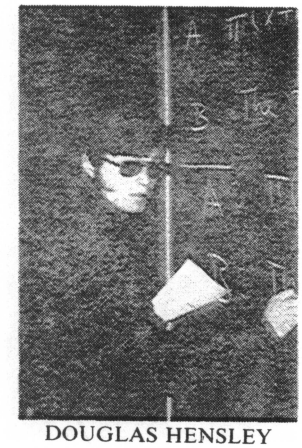

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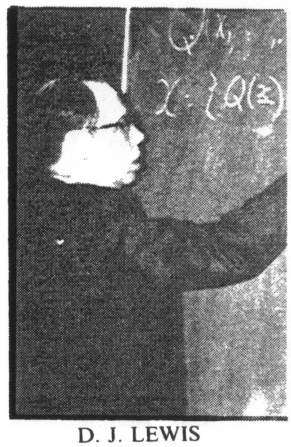

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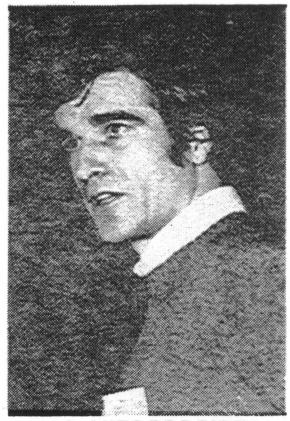

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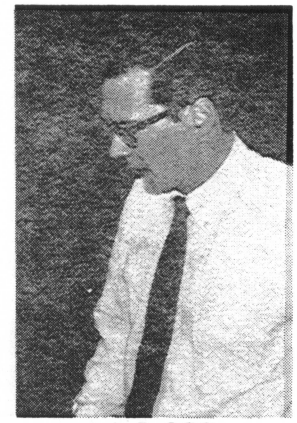

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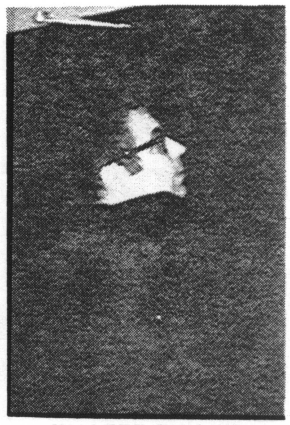

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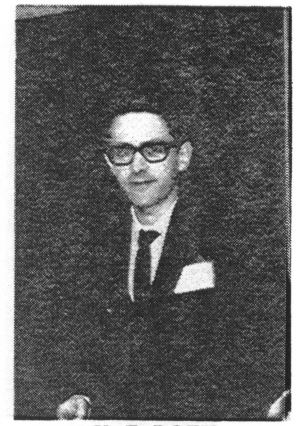

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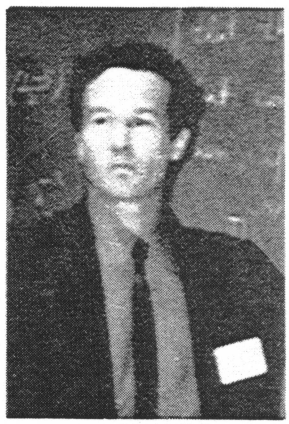

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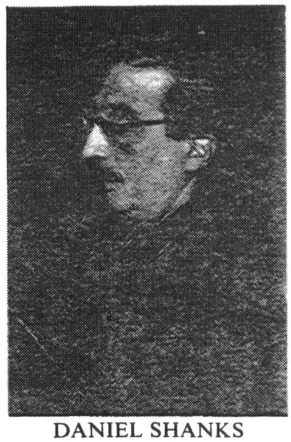

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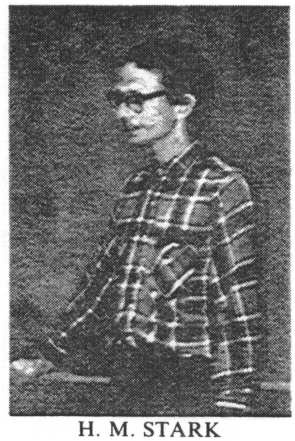

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*Italic numbers* refer to pages on which a complete reference to a work by the author is given.

Roman numbers refer to pages on which a reference is made to a work of the author. For example, under Oppenheim would be the page on which a statement like the following occurs: "This result had been conjectured by Oppenheim [**13**] in 1929,..."

**Boldface numbers** indicate the first page of the articles in this volume.


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