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CONTENTS

Preface ............................................. ix

Part I: Invited Lectures
Representations of solvable and nilpotent groups and harmonic analysis on nil and solvmanifolds .................................. 3
   BY CALVIN C. MOORE
The theory of characters and the discrete series for semisimple Lie groups . 45
   BY V. S. VARADARAJAN
Functions on symmetric spaces ...................................... 101
   BY SIGURDUR HELGASON
Quantization and representation theory .................................. 147
   BY ROBERT J. BLATTNER
Harmonic analysis on reductive p-adic groups .............................. 167
   BY HARISH-CHANDRA
Boundary theory and stochastic processes on homogeneous spaces ...... 193
   BY HARRY FURSTENBERG

Part II: Seminars
Seminar on Representation Theory of Solvable Groups and Harmonic Analysis on Solvmanifolds (Chairman, L. Auslander)
The primitive ideal space of solvable Lie groups ............................ 233
   BY L. PUKANZSKY
Group extensions and the Plancherel formula .................................. 235
   BY ADAM KLEPPNER AND RONALD L. LIPSMAN
Square-integrable representations of nilpotent groups .................... 239
   BY JOSEPH A. WOLF AND CALVIN C. MOORE
Mackey's little group method and $L^2$ of compact homogeneous spaces 245
   BY J. BREZIN
On the spectrum of a compact solvmanifold ................................ 249
   BY ROGER HOWE
Study of some holomorphically induced representations of solvable groups and restriction of the holomorphic discrete series of a semisimple group $G$ to the minimal parabolic of $G$  

By H. Rossi and M. Vergne

Seminar on Irreducibility and Realization of Various Series of Representations of Semisimple Groups (Chairman, B. Kostant)

Determination of intertwining operators 

By A. W. Knapp

Kostant's $P^c$ and $R^c$ matrices and intertwining integrals 

By Nolan R. Wallach

On an exceptional series of representations 

By Kenneth Johnson

Explicit form of the characters of discrete series representations of semisimple Lie groups 

By Takeshi Hirai

The subquotient theorem and some applications 

By J. Lepowsky

On the continuous spectrum for a semisimple Lie group 

By P. C. Trombi

The spectrum of a reductive Lie group 

By Joseph A. Wolf

Geometric realizations of representations of reductive Lie groups 

By Joseph A. Wolf

The use of partial differential equations for the study of group representations 

By Leon Ehrenpreis

Seminar on Boundary Behavior, Special Functions, and Integral Transforms in Group Representations (Chairman, E. Stein)

Boundary behavior of harmonic and holomorphic functions

Some remarks on boundary behavior of analytic functions 

By Charles Fefferman

Boundary behavior of Poisson integrals 

By Lars-Åke Lindahl

Generalized Poisson integrals and their boundary behavior 

By H. Lee Michelson

Special functions

Refinements of Abel summability for Jacobi series 

By Richard Askey
Spherical functions on rank one symmetric spaces and generalizations
BY M. FLENSTED-JENSEN

Bessel functions, representation theory, and automorphic functions
BY STEPHEN GELBART

Generalized Bessel transforms and unitary representations
BY KENNETH I. GROSS AND RAY A. KUNZE

Explicit formulas for special functions related to symmetric spaces
BY TOM KOORNWINDER

Lie algebras and generalizations of hypergeometric functions
BY WILLARD MILLER, JR.

A noncompact analogue of spherical harmonics
BY R. STRICHARTZ

Applications of singular integrals on nilpotent groups

Subelliptic operators on the Heisenberg group
BY G. B. FOLLAND

Singular integrals related to nilpotent groups and $\mathcal{D}$ estimates
BY E. M. STEIN

Amenable groups in harmonic analysis

Operators transferred by representations of an amenable group
BY RONALD R. COIFMAN AND GUIDO WEISS

Invariant means and unitary representations
BY PIERRE EYMARD

Seminar on Representations of $p$-adic Groups (Chairman, Harish-Chandra)

Two conjectures about reductive $p$-adic groups
BY ROGER HOWE

Zeta functions of simple algebras (Local theory)
BY HERVÉ JACQUET

On work of Macdonald and $L^2(G/B)$ for a $p$-adic group
BY ALLAN J. SILBERGER

Character formulas for $SL_2$
BY P. J. SALLY, JR.

Characters of finite Chevalley groups
BY T. A. SPRINGER

On the discrete series for Chevalley groups
BY PAUL GERARDIN

The Steinberg character as a true character
BY W. CASSELMAN
Harmonic analysis on trees ........................................ 419
BY P. CARTIER
Whittaker models for admissible representations of reductive p-adic split
groups ............................................................... 425
BY FRANÇOIS RODIER
Seminar on Automorphic Forms and $L^2(G/\Gamma)$ (Chairman, R. Godement)
Spectra of discrete subgroups ....................................... 431
BY RAMESH GANGOLLI
An example in the theory of automorphic forms .................. 437
BY STEPHEN GELBART
Spectral analysis of automorphic forms on rank one groups by perturbation
methods .............................................................. 441
BY GILLES LACHAUD

Indexes
Authors ..................................................................... 453
Referenced Authors .................................................. 457
Subjects ................................................................... 461
PREFACE

This book is an outgrowth of the nineteenth Summer Research Institute of the American Mathematical Society which was devoted to the topic Harmonic Analysis on Homogeneous Spaces. The Institute was held at Williams College in Williamstown, Massachusetts from July 31 to August 18, 1972, and was supported by a grant from the National Science Foundation. It was the aim of the Institute to concentrate mainly on the more analytic aspects of the subject and, of course, the theory of unitary group representations for groups not only over the reals and complex numbers but also for $p$-adic groups plays the fundamental role. Part of the program was devoted to automorphic functions, but it was decided not to go exhaustively into the very rich and deep connections of representation theory with automorphic functions nor to explore the arithmetic and algebraic geometric aspects of the theory of automorphic forms. This is certainly a most fruitful direction of research, but this is more properly the subject of another conference or future Summer Institute organized with this as its major goal.

The scientific program of the Institute consisted first of all of six major lecture series, each devoted to relatively broad areas within the subject and consisting of from four to six lectures. The lecturers and their topics were: C. C. Moore, Representations of solvable and nilpotent groups and harmonic analysis on nil and solvmanifolds; V. S. Varadarajan, Character theory and the Plancherel formula for semisimple groups; S. Helgason, Function theory on symmetric spaces; B. Kostant (jointly with R. Blattner), Quantization and representation theory; Harish-Chandra, Representation theory of $p$-adic groups; and H. Furstenberg, Boundary theory and stochastic processes on homogeneous spaces. The first part of this volume contains articles prepared by the lecturers based on these lecture series. The manuscript for the lectures on Quantization and representation theory was prepared by R. Blattner. These are intended to be surveys of various aspects of the subject of the Institute and it is hoped that they would provide material not only for advanced graduate students and postdoctoral mathematicians who want to get into the subject, but also for more senior mathe-
maticians who work in related areas and who need a survey of what is known in the subject, and what techniques are available.

The second part of the scientific program of the Institute consisted of five research seminars each under the leadership of an invited chairman. These seminars and chairmen are as follows: Representation theory of solvable groups and harmonic analysis on solvmanifolds (Chairman, L. Auslander); Irreducibility and realization of various series of representations of semisimple groups (Chairman, B. Kostant); Boundary behavior, special functions, and integral transforms in group representations (Chairman, E. Stein); Representations of $p$-adic groups (Chairman, Harish-Chandra); and a seminar devoted to $L^2(G/F)$ and automorphic functions (Chairman, R. Godement). These seminars were devoted to the presentation of talks by participants in the Institute on current research in the various fields. The second part of this volume contains short summaries of these talks in the format of Research Announcements. Formal papers containing complete proofs will appear in the regular journals. These short notes are arranged under the five seminar headings. It should be noted that Varadarajan’s survey lectures contain a description of the discrete part of the spectrum of a semisimple group; a description of the continuous spectrum seemed more properly to fit into one of the seminars, and is in fact contained in the two articles by Trombi and Wolf listed under Kostant’s seminar. It is hoped that this combination of lecture series devoted to broad areas and short summaries of current research results and problems will facilitate access to the subject and foster further progress.

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INDEXES
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REFERENCED AUTHORS

Roman numbers refer to pages on which a reference is made to an author or work of an author.

Italic numbers refer to pages on which a complete reference to a work by the author is given.

Boldface numbers indicate the first page of the articles in the book.

Askey, Richard, 335, 338
Auslander, L., 4, 24, 42, 164, 233, 261
Azencott, R., 229
Bailey, W. N., 336, 338
Bargmann, V., 11, 42, 47, 98
Behnke, H., 262
Benson, C. T., 406
Berezin, F. A., 143
Bernat, P., 4, 15, 42, 259, 261
Bhanu-Murthy, T. S., 115, 143
Blattner, Robert J., 16, 43, 147, 257, 261
Bochner, S., 261, 344, 345
Borel, A., 142, 143, 141, 406, 410, 423, 424
Bourbaki, N., 392, 410
Braaksma, B. L. J., 353, 353
Brezin, J., 42, 245, 253

Brinkman, H. C., 353, 354
Bruhat, F., 167, 192, 264, 268, 387, 392, 419, 424, 428, 430
Burkholder, D. L., 323
Calderón, A. P., 370, 371, 372
Cartan, È., 102, 143
Cartier, P., 419, 423, 424
Casselman, W., 397, 399, 413, 423, 424, 439
Coifman, Ronald R., 361, 367, 369, 372
Cotlar, M., 370, 372
Curtis, C. W., 406
Demazure, M., 410
Dixmier, J., 4, 8, 21, 43, 290, 293
Duflo, M., 9, 10, 17, 31, 43, 148, 164
Eaton, T., 435, 436
REFERENCES

Eguchi, M., 133, 144
Ehrenpreis, L., 98, 144, 317, 320, 358
Erdélyi, A., 354
Eymard, Pierre, 373, 376
Faddeev, L. D., 442, 449
Faraut, J., 110, 144
Fefferman, Charles, 321, 323
Fejer, L., 336, 338
Fell, J. M. G., 6, 8, 43
Fife, D., 371, 372
Flensted-Jensen, M., 114, 144, 339, 342, 353, 354
Folland, G. B., 359, 361, 366
Furstenberg, Harry, 110, 140, 144, 193, 229

Gangolli, Ramesh, 121, 144, 338, 351, 354, 431, 436
Garland, H., 442, 449
Gasper, G., 353, 354
Gelbart, Stephen, 343, 345, 348, 350, 434, 436, 437
Gel'fand, I. M., 98, 108, 109, 128, 144, 358, 397, 399, 403, 406, 427, 430, 435, 436
Gérardin, Paul, 399, 407, 407, 410, 411
Gindikin, S. G., 114, 144, 256, 261
Glimm, J., 30, 43
Godement, R., 108, 109, 111, 139, 144, 256, 261, 290, 293, 345, 381, 385, 443, 449
Graev, M. I., 98, 128, 144, 358, 397, 399, 403, 406
Grauert, H., 262, 363, 364, 367
Greenleaf, F. P., 229, 376
Gross, Kenneth I., 345, 345, 347, 347, 350
Grothendieck, A., 410
Gundy, R. F., 323
Harzallah, K., 110, 144
Henkin, G. M., 363, 364, 365, 367
Herz, C., 344, 345, 370, 371, 372
Hirai, Takeshi, 99, 281, 287
Hirzebruch, F., 435, 436
Hörmander, L., 135, 145, 155, 164
Hotta, R., 433, 436
van Hove, L., 162, 165
Howe, Roger, 14, 41, 43, 188, 192, 249, 253, 377, 395, 399, 411
Hua, L. K., 142, 143, 145
Hulanicki, A., 376
Ikeda, M., 352, 354
Iwahori, N., 285, 287, 390, 393, 406
Jacquet, Hervé, 167, 168, 177, 192, 381, 381, 385, 395, 399, 406, 417, 437, 439
John, F., 123, 145
Johnson, Kenneth, 109, 145, 270, 273, 273, 275, 279, 280, 442, 449
Kajdan, D. A., 427, 430
Karpelevič, F. I., 110, 114, 140, 144, 145
Kayama, T., 352, 354
Kerzman, N., 363, 364, 367
Kilmoyer, R., 406
Kirillov, A. A., 8, 11, 43, 261
Kleppner, Adam, 37, 43, 235, 238
Knapp, A. W., 141, 145, 162, 164, 263, 263, 268, 270, 273, 326, 328, 361, 367
Kogbetliantz, E., 338
Kohn, J. J., 359, 361, 365, 366, 367
Knapp, A. W., 141, 145, 162, 164, 263, 268, 270, 273, 275, 278, 280, 292, 293, 345, 346
Kunze, R., 263, 268, 270, 273, 345, 345, 347, 347, 350
Lachaud, Gilles, 441
Langlands, R. P., 146, 167, 192, 406, 433, 436, 437, 439, 441, 449
de Leeuw, K., 371, 372
Lepowsky, J., 289, 293
Lieb, I., 363, 364, 367
Limić, N., 358
Lindahl, Lars-Åke, 141, 146, 325, 328
Lipsman, Ronald L., 37, 43, 235, 238
Lohoué, N., 372, 372
Lojasiewicz, S., 135, 146
Lowdenslager, D., 142, 146
Maass, H., 437, 439, 439
Macdonald, I. G., 387, 393
Mackey, G. W., 7, 25, 43, 103, 146, 235, 238
Malgrange, B., 136, 146
Martin, W. T., 261
Matsumoto, H., 417, 417
Mautner, F. I., 98, 111, 144, 146, 395, 399, 423, 424
McCollum, G. W., 293
McKean, H. P., 435, 436
Meulenbeld, B., 353, 353
Michel, R., 126, 146
Michelson, H. Lee, 329, 333
Miller, Willard, Jr., 355, 356
Moore, Calvin C., 3, 42, 43, 44, 146, 165, 229, 234, 239, 243, 253
Mostow, G. D., 250, 253
Naïmark, M. A., 108, 109, 144
Narasimhan, M. S., 99, 436
Nelson, E., 44
Niederle, J., 358
Nirenberg, L., 359, 361
Okamoto, K., 99, 133, 144, 436
Parthasarathy, K. R., 99, 268, 433, 436
Peyrière, J., 372, 372
Pjateckiĭ-Šapiro, I. I., 256, 261, 262
Pollard, H., 338
Pólya, G., 338, 338
Pukanszky, L., 4, 8, 15, 16, 17, 31, 34, 39, 44, 233, 233
Quint, S. R., 5, 44
Raczka, R., 358
Rader, C., 290, 293
Radon, J., 123, 146
Ragunathan, M. S., 442, 449
Rallis, S., 270, 271, 273
Rao, R. Ranga, 268
Reiter, H., 376
Richardson, L., 41, 44, 253
Riesz, M., 320, 358
Rodier, François, 404, 425
Romanov, A. V., 365, 367
Rossi, H., 255
Sally, P. J., Jr., 395, 399
Satake, I., 388, 389, 393
Schiffman, G., 263, 268, 270, 273, 273
Schmid, W., 99, 143, 146, 433, 436
Schoenberg, I. J., 338
Schwartz, L., 99, 104, 146
Segal, I. E., 44, 162, 164, 165
Selberg, A., 432, 433, 436, 441, 449
Semjanisty, V. T., 146
Serre, J.-P., 419, 423, 424
Shale, D., 148, 162, 164, 165
Shalika, Joseph A., 378, 380, 397, 399, 400, 411, 417, 417, 437, 439
Shintani, T., 400, 411
Silberger, Allan J., 387, 400, 411, 423, 424
Silverstein, M. L., 323
Solomon, L., 406, 413, 417, 417
Springer, T. A., 401, 406
Steinberg, R., 99, 406, 411
Stern, A. I., 146
Sternberg, S., 261
Stinespring, W. F., 44
Streater, R. F., 44
Strichartz, R., 357, 358
Sugiura, M., 284, 287
Takahashi, R., 110, 146, 268, 278, 280
Takenouchi, O., 4, 15, 44
Tamagawa, T., 436
Tanaka, S., 397, 400, 411, 435, 436, 437, 439
Tate, J., 381, 385
Tatsumma, N., 44
Tits, J., 191, 387, 392, 411, 419, 424
Trombi, P. C., 99, 122, 146, 295, 298, 303, 433, 434, 436
Urakawa, H., 146
Vagi, I., 367
Varadarajan, V. S., 45, 99, 122, 146, 268, 298, 303, 433, 434, 436
Vergne, M., 9, 16, 43, 44, 255
Vilenkin, N., 144, 358
Vinberg, E. E., 256, 261
Wallach, Nolan R., 267, 268, 269, 270, 273, 273, 279, 280, 293, 302, 303
Warner, G., 146, 436
Weil, A., 44, 148, 162, 165, 252, 253, 345, 407, 411
Weisner, L., 356
Weiss, Guido, 361, 367, 369, 372
Weiss, N., 143, 146
Williamson, R. E., 141, 145, 328
Wolf, Joseph A., 44, 146, 239, 261, 305, 313
Żelobenko, D. P., 268
Zernike, F., 353, 354
Zygmund, A., 323, 326, 328, 364, 367
abstract symplectic group, 344
additional formula for Jacobi polynomials, 351, 352, 353
adele group, 14
admissible, 379, 410, 425, 432
admissible polarization, 24
admissible representation, 169, 378, 379, 380, 381
irreducible, 378, 423
admissible unrestricted convergence, 326
affine root structures on a $p$-adic group, 388
algebraic group over a $p$-adic field, 14
$\alpha$-densities, 150, 151
amenability, 196, 369, 373
antipodal manifold, 124
associated parabolic subgroup, 309, 402
asymptotic expansion, 378
Auslander-Kostant theory, 20, 243
automorphic forms, 437, 441
$ax + b$ group, 16, 376
Bergman kernel operator, 365
Bergman-Šilov boundary, 142
Bessel function, 343, 347, 348, 349
operator-valued, 343
Borel-Weil theorem, 316
boundary, 196
$\mu^*$, 211
universal, 198
boundary points, 419
boundary values, 329
bounded spherical function, 109, 422
Cartan decomposition, 48, 104
Cartan involution, 48, 104, 308
Cartan subgroup, 46, 313, 378
conjugacy classes of, 281
connected components of, 281
Casimir element, 307
CCR representation, 8, 17, 31
central character, 239
central extension, 307
Cesàro summability, 337
c-function, 301, 390, 449
character, 281, 285, 377, 378, 380, 395
central, 239
cusp, 402
distribution, 8, 307
global, 16, 31, 33
infinitesimal, 10, 241, 285, 291, 307
parabolic, 402
principle, 426
regular, 404
character of an induced representation, 237
Chevalley group, 320, 401, 407
coadjoint representation, 20
commutation relations, 6
compact root, 141
compact symmetric space of rank one, 351
compatible \((\mathcal{Q}, K)\)-module, 289, 290, 291, 292
complementary series, 263
completely invariant open set, 52
completely transverse polarization, 152
complex distance, 126
complex flag manifold, 313
complex projective space, 352
complex subordinate subalgebra, 19
condition \((A_o)\), 341
conditional distribution, 206
conjugacy class, 71
of Cartan subgroups, 281
connected components of Cartan subgroups, 281
connection, 148
connection form, 149
constant term, 174
along a parabolic subgroup, 86
continuous spectrum, 358
converge admissibly and restrictedly, 332
convolution structure, 339, 341
for Jacobi series, 351, 353
Curv\((L, \nabla)\), 149
cusp form, 402
cuspidal character, 402
cuspidal parabolic subgroup, 305, 308, 309, 313
principal series of representations associated to, 299
cuspidal representation, 402
differential recurrence relations, 355
direct integral, 36
discrete part of Plancherel formula, 94
discrete series, 46, 275, 278, 279, 306, 315, 407, 424
Harish-Chandra analysis of, 306 holomorphic, 256, 259, 344, 348, 349
representation, 345
relative, 305, 306, 307, 313
representation, 40, 281, 285, 320, 345
\(\mathfrak{s}\), 306
discrete spectrum, 358
discrete uniform subgroup, 40, 243
distal, 35
distribution character, 8, 307
distribution of a random variable, 202
distributions on a l.c.t.d. (locally compact, totally disconnected) space, 427
Dolbeault operator, 314
dominant, 267, 268
dual space, 6
dual transform, 127
eigenfunction of all \(G\)-invariant differential operators on \(X\), 329
Eisenstein integral, 298, 318
Eisenstein series, 318, 441, 448
\(E(L, \nabla)\), 149
SUBJECTS

\( \tilde{E}(L, \nabla) \), 161

elementary \( \tau \)-spherical function, 297, 318
eLL, 161

elliptic component, 60
elegant element, 60
eLL, 377, 380

regular, 377
equivalence, weak, 111
equivalent, 107
equivariant, 194

essential restricted root, 266

exception, 433

exponent, 175

exponential map, 5

exponential solvable, 14

exponential type, 119

extendible \( M \)-module, 292

extremal, 284

\( F^* \), 321

\( F \), 378, 380

Fatou theorem, 140, 326, 330

fixed point property, 373

flag manifold orbit, 313

formal degree, 40, 239, 241, 306

Fourier transform, 129, 317, 348, 379

formula, 350

fractional integral, 365

Frechet algebra, 69

fundamental Cartan subalgebra, 80

fundamental domain, 442

fundamental solution, 82

\( \tilde{\gamma}(X, \omega) \), 159

\( \tilde{\delta}(X, \omega) \), 159

\( G \)-amenable homogeneous space, 375
gamma integral, generalized, 349

\( \Gamma_\tau(\mathcal{D}) \), 364

\( \Gamma_\tau(X) \), 366
generalized gamma integral, 349
generalized Poisson integral, 329
generalized upper half-plane, 348
generating function, 356
geodesic, 419
germ, 378

(\( \mathcal{D}, K \))-module, 289

compatible, 289, 290, 291, 292

\( G_\mathbb{A} \), 320, 377-380
global character, 16, 31, 33, 50
grossencharacter, 437
group extension, 235

\( G \)-space, 194

minimal, 194

proximal, 194

half-density pairing, 152

half-form, 157

half-form pairing, 160, 161

Hankel transform, 348, 349

Harish-Chandra analysis of discrete series, 306

Harish-Chandra condition, 256, 260

Harish-Chandra transform, 444

harmonic, 139, 200

harmonic function, 423

Hecke algebra, 420

Hecke finite, 172

Hecke \( L \)-function, 438

Hecke subgroup, 420

height, 284

Heisenberg group, 6, 26, 238, 242, 250, 251, 360, 366, 408

Heisenberg related polarizations, 162

highest parts of \( \pi \), 284

\( H_f \)-series, 309

Hölder inequalities, 363

holomorphic arc component, 314

holomorphic discrete series, 256, 259, 344, 348, 349

representation, 345

holomorphic induction, 20, 24, 35

holomorphic line bundle, 316
homogeneous bundle, 314
homogeneous function, 319, 357
homogeneous Hermitian vector bundle, 313
horocycle, 420
$H^p$, 322
$H^p(X)$, 329
hyperboloid, 317, 318, 357
hypergeometric function, 355
imaginary root, 282
independence of polarization, 26, 28, 147
induced representation, 309, 375, 378
character of, 237
inessential restricted root, 266
infinitesimal character, 10, 241, 285, 291, 307
infinitesimal module, 290, 291
infinitesimal nonunitary principal series, 290, 292
integer-order Hankel transform, 348
integral form, 23
integral orbit, 30
intertwining form, 425
intertwining isomorphism, 152
intertwining operator, 107, 264, 363
invariant distribution, 378
invariant eigendistribution, 281
invariant mean, 373
isotropy subgroup, 9
Iwahori subgroup, 387
Jacobi function, 340
Jacobi polynomial, 335, 351
addition formula for, 351, 352, 353
Jacobi series, convolution structure for, 351, 353
Kirillov map, 29
Kirillov theory, 5, 240, 250
Lagrangian basis, 159
Lagrangian subspace, 159
Langlands decomposition, 49
Laplace operator, 283
Laplace transform, 349
Laplacian, 319, 357
lattice subgroup, 41, 320
Lauricella function, 356
Lebesgue point, 331
Leray spectral sequence, 316
$L/F$, 101
Lie algebra, 355
Lie algebra cohomology, 316
Lie group, solvable, 14
linear fractional transformation, 349
Lipschitz inequalities, 363
Lipschitz space, standard, 363
little group method, 245
Lusin area integral, 321
Maass-Selberg relations, 180, 301
Mackey little group, 19, 25
Mackey machine, 235
Mackey obstruction class, 30
Maslov bundle, 163
maximal function, 321
maximal parahoric subgroup, 389
mean-proximal, 225
metaplectic
automorphism, 161
bundle, 163
frame bundle, 158
group, 157
manifold, 158
representation, 162, 252, 347, 348
structure, 158
minimal G-space, 194
µ-boundary, 211
µ-process, 208
proper, 210
µ-proximality, 220
multiplicity, 105
multiplier, 372
multiplier function, 204
nilpotent group, 5
nilpotent Lie group, 239
nil-radical, 249
noncompact root, 141
nondegenerate series, 308, 309
characters, 310
nonunimodular group, 38
nonunitary principal series, 290
infinitesimal, 290, 292
norm map, 438
normal, 126
normal j-algebra, 256
O(n, N), 357
operator-valued Bessel function, 343
orbit, 240, 318
integral, 30
order <, 284
order <, 284
orthogonality relations, 236, 239
oscillator group, 19
Paley-Wiener theorem, 255, 257
parabolic, 378, 380
parabolic character, 402
parabolic subalgebra, 49
parabolic subgroup, 49, 309, 402
associated, 402
cuspidal, 305, 308, 309, 313
parahoric subgroup, 238, 389
maximal, 389
partial differential equation, 317, 358
partial Hodge-Kodaira-Laplace
operator, 314
partially harmonic form, 314
square-integrable, 315
partially smooth form, 314
perturbation method, 93
Pfaffian, 240
r-minimal, 177
Plancherel formula, 10, 34, 46, 305,
310, 320, 358
discrete part of, 94
for automorphic forms, 448
for group extensions, 237
for spherical functions, 422
Plancherel measure, 10, 37, 189,
235, 241, 300, 310, 421
Plancherel theorem, 305
plane wave, 123
Poincaré-Eisenstein series, 344
Poisson integral, 325
generalized, 329
Poisson kernel, 325, 336
Poisson representation, 216
polarization, 21, 147, 150, 255
admissible, 24
independence of, 26, 28, 147
positive, 22, 24, 26
tractable, 151
positive definite function, 338
positive polarization, 22, 24, 26
prequantization, 147
primary representation, 35
primitive ideal space, 14, 39
principal character, 426
principal series, 58, 263, 309
of representations associated to
the cuspidal parabolic subgroup, 299
representation of SL(2, R), 318,
437
spherical, 275, 389
principal spherical series, 422
projective representation, 236
proper holomorphic fibration, 316
proper $\mu$-process, 210
proximal $G$-space, 194
Pukanszky condition, 16, 22, 24
quasi-admissible, 173
quasi-simple, 111
Radon transform, 123, 124, 127, 423
random walk, 209
rank, 105
real, 105
rapidly decreasing, 119
rational subalgebra, 42
real quadratic field, 437
real rank, 105
real root, 282
real symplectic group, 252, 320, 343
reductive Lie group, 305, 313
reductive $p$-adic group, 377
regular character, 404
regular element, 404
regular representation, 10, 36, 236, 373
relative discrete series, 305, 306, 307, 313
representation, 107, 355
admissible, 169, 378-380
coadjoint, 20
cuspidal, 380, 402
discrete series, 40, 281, 285, 320, 345
induced, 309
character of, 237
irreducible, 377-380
metaplectic, 162, 252, 347, 348
parabolic, 402
primary, 35
projective, 236
regular, 10, 36, 236
smooth, 425
spherical, 110
square-integrable, 239
restricted root, 104
essential, 266
inessential, 266
root, 14, 17
compact, 141
imaginary, 282
noncompact, 141
real, 282
restricted, 104
simple, 105
rotation method (for singular integrals), 370
Schur orthogonality relations, 172
Schwartz space, 67, 185
Selberg trace formula, 432
semiregular point, 53
series of representations, 305, 320
$S(F)$, 321
Siegel domain, 255, 256, 259
$\sigma$-spherical function, 389
simple root, 105
singular integral operator, 360, 363, 370
smooth function, 168
smooth representation, 425
solvable Lie group, 14, 249-252
solvmanifold, 249
$SO(n)$, 343
special representation, 380, 423
special torus, 176
spectra of discrete subgroups, 431
spectral density, 447
spectral function of $\Gamma$, 431
spectrum, 249, 305
continuous, 358
discrete, 358
spherical function, 108, 318, 336, 339, 341, 351, 352, 353, 390, 420, 442
bounded, 109, 422
<table>
<thead>
<tr>
<th>Subjects</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>spherical harmonics, 344, 352, 353, 357</td>
<td></td>
</tr>
<tr>
<td>spherical principal series, 275, 389</td>
<td></td>
</tr>
<tr>
<td>spherical representation, 110</td>
<td></td>
</tr>
<tr>
<td>spherical series</td>
<td></td>
</tr>
<tr>
<td>principal, 422</td>
<td></td>
</tr>
<tr>
<td>supplementary, 422</td>
<td></td>
</tr>
<tr>
<td>spherical transform, 111</td>
<td></td>
</tr>
<tr>
<td>$Sp(m, \mathbb{R})$, 320, 343, 344, 345</td>
<td></td>
</tr>
<tr>
<td>square-integrable, 172, 276, 278, 279</td>
<td></td>
</tr>
<tr>
<td>square-integrable partially harmonic form, 315</td>
<td></td>
</tr>
<tr>
<td>square-integrable representation, 239</td>
<td></td>
</tr>
<tr>
<td>standard Lipschitz space, 363</td>
<td></td>
</tr>
<tr>
<td>standard torus, 178</td>
<td></td>
</tr>
<tr>
<td>stationary phase, method of, 155</td>
<td></td>
</tr>
<tr>
<td>stationary process, 207</td>
<td></td>
</tr>
<tr>
<td>Steinberg character, 187, 413</td>
<td></td>
</tr>
<tr>
<td>subelliptic operator, 359</td>
<td></td>
</tr>
<tr>
<td>subordinate subalgebra, 5</td>
<td></td>
</tr>
<tr>
<td>complex, 19</td>
<td></td>
</tr>
<tr>
<td>subquotient, 290, 293</td>
<td></td>
</tr>
<tr>
<td>subquotient theorem, 289, 290</td>
<td></td>
</tr>
<tr>
<td>supercuspidal, 174, 383, 407, 410</td>
<td></td>
</tr>
<tr>
<td>supercuspidal representation, 280, 395</td>
<td></td>
</tr>
<tr>
<td>supplementary spherical series, 422</td>
<td></td>
</tr>
<tr>
<td>symplectic form, 252</td>
<td></td>
</tr>
<tr>
<td>symplectic group, 252</td>
<td></td>
</tr>
<tr>
<td>abstract, 344</td>
<td></td>
</tr>
<tr>
<td>real, 320, 343</td>
<td></td>
</tr>
<tr>
<td>symplectic manifold, 9, 20, 148</td>
<td></td>
</tr>
<tr>
<td>$\tau$-spherical, 295</td>
<td></td>
</tr>
<tr>
<td>elementary, 297, 318</td>
<td></td>
</tr>
<tr>
<td>$\tau$-spherical function, 51</td>
<td></td>
</tr>
<tr>
<td>tempered, 285</td>
<td></td>
</tr>
<tr>
<td>tempered distribution</td>
<td></td>
</tr>
<tr>
<td>on a semisimple Lie group, 70</td>
<td></td>
</tr>
<tr>
<td>on a vector space, 63</td>
<td></td>
</tr>
<tr>
<td>Titchmarsh-Kodaira, formula of, 447</td>
<td></td>
</tr>
<tr>
<td>transferred operator, 369</td>
<td></td>
</tr>
<tr>
<td>tree, 419</td>
<td></td>
</tr>
<tr>
<td>type I, 15, 29</td>
<td></td>
</tr>
<tr>
<td>type $\delta$ spectrum, 434</td>
<td></td>
</tr>
<tr>
<td>type 1 spectrum, 434</td>
<td></td>
</tr>
<tr>
<td>unipotent class, 378, 380</td>
<td></td>
</tr>
<tr>
<td>unipotent radical, 309</td>
<td></td>
</tr>
<tr>
<td>unitarily related polarizations, 152</td>
<td></td>
</tr>
<tr>
<td>unitary character, 305</td>
<td></td>
</tr>
<tr>
<td>universal boundary, 198</td>
<td></td>
</tr>
<tr>
<td>$U(n,n)$, 347</td>
<td></td>
</tr>
<tr>
<td>unramified, 176</td>
<td></td>
</tr>
<tr>
<td>unrestricted admissible convergence, 331, 333</td>
<td></td>
</tr>
<tr>
<td>upper half-plane, generalized</td>
<td></td>
</tr>
<tr>
<td>$\mathfrak{w}(A)$, 176</td>
<td></td>
</tr>
<tr>
<td>wave packet, 189, 300</td>
<td></td>
</tr>
<tr>
<td>weak containment, 373</td>
<td></td>
</tr>
<tr>
<td>weak equivalence, 111</td>
<td></td>
</tr>
<tr>
<td>Weil representation, 250, 252, 437</td>
<td></td>
</tr>
<tr>
<td>Weisner's principle, 355</td>
<td></td>
</tr>
<tr>
<td>Weyl chamber, 105</td>
<td></td>
</tr>
<tr>
<td>Weyl function, 435</td>
<td></td>
</tr>
<tr>
<td>Weyl group, 105, 320</td>
<td></td>
</tr>
<tr>
<td>Weyl's formula, 281, 285</td>
<td></td>
</tr>
<tr>
<td>Whittaker model, 403, 426</td>
<td></td>
</tr>
<tr>
<td>$X/F$, 151</td>
<td></td>
</tr>
<tr>
<td>$\xi$-discrete series, 306</td>
<td></td>
</tr>
<tr>
<td>zeta function, 381</td>
<td></td>
</tr>
<tr>
<td>$\exists$-finite distributions, 52</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>^{\wedge^n}</td>
</tr>
<tr>
<td>$(^{\wedge^n})^{1/2}T^*Y$, 157</td>
<td></td>
</tr>
</tbody>
</table>