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**Mathematical
developments
arising from
HILBERT PROBLEMS**

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**PROCEEDINGS OF SYMPOSIA
IN PURE MATHEMATICS
Volume XXVIII, Part 2**

**MATHEMATICAL DEVELOPMENTS
ARISING FROM
HILBERT PROBLEMS**

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