The Mathematical Heritage of
HENRI POINCARÉ

Volume 39 - Part 1

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

AMERICAN MATHEMATICAL SOCIETY

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AMERICAN MATHEMATICAL SOCIETY
PROVIDENCE, RHODE ISLAND

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Introduction

On April 7–10, 1980, the American Mathematical Society sponsored a week-long Symposium on the Mathematical Heritage of Henri Poincaré, held at Indiana University, Bloomington, Indiana. This volume presents the written versions of all but three of the invited talks presented at this Symposium (those by W. Browder, A. Jaffe, and J. Mather were not written up for publication). In addition, it contains two papers by invited speakers who were not able to attend, S. S. Chern and L. Nirenberg. The Organizing Committee for the Symposium consisted of F. Browder (Chairman), W. Browder, P. Griffiths, J. Moser, S. Smale, and R. O. Wells.

The casual reader may ask: What is the mathematical heritage of Henri Poincaré? How can it be described or delimited? In a certain sense, the essays presented here provide the best answer. To introduce them, let us try to answer the question in a summary form. During the period of his mathematical activity (which as the attached Bibliography of Poincaré's works indicates very sharply, was intense to a remarkable degree), Poincaré worked on a wide variety of mathematical topics stemming both from pure mathematics and from its applications. A central feature of his work was the close relation between his massive involvement in the research activity of his time in celestial mechanics and all the different varieties of physics, both theoretical and experimental, and the very deep and original insights that Poincaré developed in areas today classified under core mathematics. Poincaré made contributions of the most fundamental kind to the study of Riemann surfaces and of discontinuous groups, algebraic geometry, analytic functions of several complex variables, and non-Euclidean geometry. He was for all practical purposes the founder of many major fields of contemporary mathematics, including dynamical systems, algebraic topology, differential topology, ergodic theory, and the study of nonlinear problems using the ideas of topology. As a recent history of functional analysis by Dieudonné testifies, he can also be considered as a major seminal figure in that field as well as in the study of the general theory of partial differential equations.

As Poincaré himself described it, he was a 'pragmatist' in mathematics, both in his practice and in his theoretical self-conception. In the middle of the twentieth century, his pragmatist attitudes toward mathematical practice often were unfashionable in an environment where mathematical abstraction and an emphasis on formal elaboration of mathematical doctrine were a central concern. In recent decades, the tide has turned decisively toward mathematical creativity, as opposed to an emphasis upon rigorization and formalization. Today, even the classical figures of the old Bourbaki claim Poincaré (along with Elie Cartan) as their major precursor. (See the article by Dieudonné, *The work of Bourbaki during the last thirty years* [Notices Amer. Math. Soc. **29** (1982), 618–623].) Poincaré's concept of mathematics stresses intuition (geometric and analytical), creativity, and a strong emphasis upon a major relation of mathematics with the natural sciences.

The contents of this volume speak to this heritage. We regret very much the lack of a contribution by Jurgen Moser, who along with V. I. Arnold, represents in the sharpest and highest form the heritage of Poincaré in the direction of celestial mechanics, a field in which many of Poincaré's most original mathematical inventions were rooted. There are other gaps that one might have wished to fill (asymptotic methods in applied mathematics, or bifurcation theory, for example). One could well produce another volume to supplement the present one, with much more attention to the impact of Poincaré's works and ideas on the development of theoretical physics, or the impact of his views and writings on the foundational controversies of the early part of the twentieth century. In any case, we have before us a very substantial (if not complete) development of some of the most important aspects of the Poincaré tradition as described above, in some of the most active and vital areas of contemporary mathematical research.

Let me close with a remark that needs to be made publicly with respect to the appropriateness of this entreprise as an activity of the American Mathematical Society. If one traces the influence of Poincaré through the major mathematical figures of the early and mid-twentieth century, it is through American mathematicians as well as French that this influence flows, through G. D. Birkhoff, Solomon Lefschetz, and Marston Morse. This continuing tradition represents one of the major strands of American as well as world mathematics, and it is as a testimony to this tradition as an opening to the future creativity of mathematics that this volume is dedicated.

Felix E. Browder

Summary Chronology of the Life of Henri Poincaré

Born: 29 April 1854, in Nancy, France.

Educated in Nancy: (His teacher in Speciale, Elliot à Liard wrote in 1872 to a friend, "J'ai dans ma classe à Nancy, un monstre de mathématiques, c'est Henri Poincaré".)

First mathematical paper: in Nouvelles Annales des Mathématiques, 1873.

Entered: École Polytechnique, Paris, 1873.
Entered: École des Mines, Paris, 1875.
Doctorat d'État: 1879.
Appointed: Maitre des Conferences d'Analyse in Paris, 1881.
Maitre des Conferences, Mathematical physics, 1885.
Chaire de Physique mathématique et Calcul des probabilités at the University of Paris, 1886.
Chaire d'Astronomie mathématique et Mechanique Celeste, in Paris, 1896.
Elected: Membre de la Section de Géometrie de l'Academie des Sciences, 1887. President de l'Academie, 1906.
Elected: to l'Academie Francaise, 1908.

Died: in Paris, July 17, 1912.

Poincaré