

# Nonlinear Functional Analysis and Its Applications

**Volume 45-Part 1**

**PROCEEDINGS OF  
SYMPOSIA IN  
PURE MATHEMATICS**

**AMERICAN MATHEMATICAL SOCIETY**

NONLINEAR FUNCTIONAL ANALYSIS  
AND ITS APPLICATIONS

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PROCEEDINGS OF SYMPOSIA  
IN PURE MATHEMATICS  
Volume 45, Part 1

# Nonlinear Functional Analysis and Its Applications

Felix E. Browder, Editor

AMERICAN MATHEMATICAL SOCIETY  
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## Foreword

The two volumes *Nonlinear Functional Analysis and Its Applications*, published in the series *Proceedings of Symposia in Pure Mathematics* (vol. 45, parts 1 and 2), are the result of the thirty-first Summer Research Institute of the American Mathematical Society held at the University of California at Berkeley from July 11 to July 29, 1983. This institute was partially supported by a grant from the National Science Foundation, and organized by an Organizing Committee consisting of Haim Brezis, Felix Browder (Chairman), Tosio Kato, J.-L. Lions, Louis Nirenberg, and Paul Rabinowitz.

The purpose of the institute was to present and develop research on an international basis in nonlinear functional analysis and its applications, especially in the study of boundary value problems for nonlinear partial differential equations and corresponding problems in geometry and mathematical physics. Major topics which were covered in a series of expository lectures as well as research talks included: Minimax methods in the calculus of variations, existence theory for variational problems without compactness, theories of degree of mapping, inverse function theorems of Nash–Moser type, nonlinear semigroup theory, nonlinear equations of evolution, nonlinear problems of control theory, periodic solutions of Hamiltonian systems, generalizations of the Morse theory, nonlinear partial differential equations in gauge field theory, the theory of Feigenbaum cascades, the study of the Navier–Stokes equations, nonlinear elliptic equations in differential geometry, and a variety of topics concerning nonlinear elliptic boundary value and eigenvalue problems, bifurcation theory, nonlinear hyperbolic equations, nonlinear conservation laws, nonlinear Hamiltonian–Jacobi equations, and an even wider variety of physical applications.

There were 13 series of expository lectures totaling 39 hours of lectures which summarized main directions and methods in current research. In addition, there were 115 one-hour research lectures.

A total of 203 mathematicians registered for the Institute, twenty of whom were students. The international character of the Institute is reflected in the national origins of the participants. Twenty-two countries not in North America were represented by the following numbers of participants: Africa (1), Australia (3), Belgium (1), Brazil (1), China (4), Czechoslovakia (1), England (4), France (22),

Israel (2), Italy (14), Japan (8), Korea (1), Netherlands (2), New Zealand (1), Poland (1), Rumania (1), Scotland (1), Spain (4), Sweden (2), Switzerland (3), and West Germany (12).

One final comment may be in order. As compared to an earlier volume, *Nonlinear Functional Analysis*, which appeared in the Proc. Sympos. Pure Math. series 15 years ago, the present volume as well as the Institute itself represent a much more forceful emphasis upon applications as opposed to general theory. This reflects, in my view, a major shifting of focal emphasis in the field as well as in the tastes of different organizing committees. Though new conceptual advances are being made and new general methods are being developed, they tend on the whole to be much more closely linked with particular domains of application. In part, this represents the process of assimilation of the general theories developed in previous decades, as well as a mood of distrust of general theories which move too far from the context of applications. This probably reflects a more general process going on in the mathematical world at large, but it can be seen in a transparent way in the present context. No one can predict with greater security than their own self-confidence whether this tendency is irreversible or part of a broad pendular swing, and no one can determine to anyone else's satisfaction whether it is due to the internal processes of mathematical development or to the pressures arising from the external institutional context in which mathematics is being done today. Suffice it to record the facts in their broad outline so that we can look at them in the classical spirit of non-attachment.

FELIX E. BROWDER

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