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The Mathematical Heritage of Hermann Weyl

Proceedings of a Symposium
on the Mathematical Heritage of
Hermann Weyl
May 12–16, 1987
Duke University
Durham, North Carolina

R. O. Wells, Jr.
Editor

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Preface

This volume celebrates the rich legacy we have inherited from Hermann Weyl, one of the great mathematicians of this century. It represents the proceedings of a conference held in his honor at Duke University, May 12-16, 1987, two years after the 1985 centennial of his birth. This conference lasted 5 days and featured 23 speakers, almost all of whom have contributed articles to this volume.

The organizing committee consisted of Michael Atiyah (Oxford), Lipman Bers (Columbia), Felix Browder (Chicago), S. S. Chern (Berkeley), G. D. Mostow (Yale), and myself as chairman. We decided to have a wide spectrum of speakers representing many of the diverse areas in which Weyl made significant contributions. We intended from the beginning to have a conference of sufficient size so that mathematicians, graduate students, and others interested in Hermann Weyl's mathematics would be able to attend such a conference from all over the country and from abroad. There were other celebrations of Hermann Weyl's 100th birthday, most notably the lectures by Armand Borel, Roger Penrose, and C. N. Yang in Zürich in October of 1985. The conference at Duke allowed the North American mathematical community to participate in the celebration, noting that the last part of Weyl's career was spent at the Institute for Advanced Study in Princeton.

The speakers at the conference and the titles of their talks were:

Raoul Bott (Harvard), Induced representations

Felix E. Browder (Rutgers), Hermann Weyl as a philosopher, and the difference it made to his mathematics and physics

Dennis P. Sullivan (CUNY and IHES), Riemann surfaces applied to one-dimensional dynamical systems

Robert P. Langlands (IAS), Representation theory and arithmetic

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Roger E. Howe (Yale), The oscillator semigroup

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Clifford Taubes (Harvard), The stable topology of self-moduli spaces: A nonlinear Hodge theory

Roger Penrose (Oxford), Fundamental asymmetry in physical laws

Edward Witten (Princeton), Some mathematical applications of quantum field theory

There were some 300 participants who attended the conference, and the high level of exposition and inspiring lectures had a magnetic effect on the entire conference. Both C. N. Yang (SUNY, Stony Brook) and Harry Furstenberg (Hebrew University) were scheduled to speak, but were unable to attend for personal reasons, and their presence was missed.

Hermann Weyl was one of the most successful mathematicians of all time to tackle successfully serious problems in physics as well as philosophy. Currently we are witnessing a tremendous interaction between mathematics and physics, on a scale and in a fashion not seen in recent times since the Göttingen days of the invention of quantum mechanics in the early decades of this century. A significant number of the papers in this volume bear vivid testimony to this time-honored tradition of the mutual influence of these two, in principle quite different, intellectual activities.

The eighteen papers in this proceedings are presented in the order given, which was partly topical. The opening contribution is by Raoul Bott who spent some time with Hermann Weyl at the Institute for Advanced Study, and the closing two papers by Roger Penrose and Edward Witten represent contributions in the area of physics, a subject which was very important to Weyl. Included in the collection is a second paper by Roger Howe (which was not presented at the conference) on the classical groups, a subject in which Weyl made major contributions, and for which Howe's survey is most appropriate.

This conference was supported financially by the National Science Foundation, whose generous help is much appreciated. It was sponsored by the American Mathematical Society whose centennial is in 1988. I'm happy that the volume can help celebrate both the centennial of Hermann Weyl and make a contribution to the celebration of the mathematical heritage of the 100-year-old American Mathematical Society at the same time. The dedicated staff of the AMS, in particular Dottie Smith and John Balletto, took care of numerous logistical details and organized a splendid conference setting for the participants. I want to thank them for doing a splendid job. Duke University was very generous in its support, and I want to thank, in particular, Michael C. Reed, Chairman of the Mathematics Department, and Ann Tustall, Administrative Assistant, for their generous help and assistance after the Duke site was chosen. Phillip Griffiths, Provost of

Duke University, as well as a mathematical protégé of the heritage of Hermann Weyl, helped insure that everything went very smoothly. I'd like to thank him for both his assistance and for his fine lecture at the conference. Finally, I want to thank all of the other speakers for contributing beautiful lectures, and for writing up their ideas in a fashion which gives the contemporary reader some idea of the breadth and beauty of the ideas of Hermann Weyl.

R. O. Wells, Jr.
Rice University
May 1988

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