Proceedings of Symposia in
PURE MATHEMATICS

Volume 54, Part 1

Differential Geometry: Partial Differential Equations on Manifolds

American Mathematical Society

http://dx.doi.org/10.1090/pspum/054.1

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Robert Greene
S. T. Yau
Editors

American Mathematical Society
Providence, Rhode Island

PROCEEDINGS OF THE SUMMER RESEARCH INSTITUTE
ON DIFFERENTIAL GEOMETRY
HELD AT THE UNIVERSITY OF CALIFORNIA, LOS ANGELES
LOS ANGELES, CALIFORNIA
JULY 8–28, 1990

with the support from the National Science Foundation
Grant DMS–8913610.

1991 *Mathematics Subject Classification.*
Primary 53A10, 49F20, 58E20, 58E12, 58G11 (Part 1)
81E13, 58C80, 53B50, 32C10, 53C15, 58F05, 83C75 (Part 2)
53C20, 53C40, 58C40, 58G25, 58F07, 58F11, 58F17 (Part 3).

Library of Congress Cataloging-in-Publication Data

Partial differential equations on manifolds/Robert Greene and S. T. Yau, editors.
p. cm.—(Proceedings of symposia in pure mathematics; v. 54, pt. 1) (Differential geometry; pt. 1)
"Proceedings of a Summer Research Institute on Differential Geometry, held at the University of California, Los Angeles, July 8–28, 1990"—T.p. verso.
ISBN 0-8218-1494-X (Part 1)
ISBN 0-8218-1495-8 (Part 2)
ISBN 0-8218-1496-6 (Part 3)
ISBN 0-8218-1493-1 (set) (alk. paper)
1. Differential equations, Partial—Congresses. 2. Complex manifolds—Congresses. 3. Harmonic maps—Congresses. I. Greene, Robert Everist, 1943– . II. Yau, Shing-Tung, 1949– . III. Summer Research Institute on Differential Geometry (1990: University of California, Los Angeles) IV. Series. V. Series: Differential geometry; pt. 1.
QA641.D3833 1993 pt. 1
[QA614.9]
516.3′6 s—dc20 92-32827
[515′.353] CIP

This publication was typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX,
the American Mathematical Society's TEX macro system.

10 9 8 7 6 5 4 3 2 1 98 97 96 95 94 93

Dedication.

To Professor S.-S. Chern,

in appreciation of his formative influence on

modern differential geometry.

Contents

(* denotes one-hour survey lectures)

PART 3

Preface

The 1990 American Mathematical Society Summer Institute on Differential Geometry took place at University of California, Los Angeles from July 9 to July 27, 1990. This was the largest AMS Summer Institute ever. There were 426 registered participants and 270 lectures. The organizing committee for the Institute consisted of Robert Bryant, Duke University; Eugenio Calabi, University of Pennsylvania; S. Y. Cheng, University of California, Los Angeles; H. Blaine Lawson, State University of New York, Stony Brook; H. Wu, University of California, Berkeley; and as co-chairmen, Robert E. Greene, University of California, Los Angeles, and S. T. Yau, Harvard University.

In the years since the previous AMS Summer Institute on Differential Geometry in 1973, the field has undergone a remarkable expansion, both in the number of people working in the area and in the number and scope of the topics under investigation. Even in the context of the rapid growth of mathematics as a whole during this period, the growth of geometry is striking.

It is our hope that the three volumes of these proceedings, taken as a whole, will provide a broad overview of geometry and its relationship to mathematics in toto, with one obvious exception; the geometry of complex manifolds and the relationship of complex geometry to complex analysis were the subject of one week of the (three-week) 1989 AMS Summer Institute on Several Complex Variables and Complex Geometry. While some topics in complex geometry arose naturally in the 1990 Summer Institute and are covered in these Proceedings, the coverage of this subject in 1989 justified a reduced emphasis in 1990.

Thus the reader seeking a complete view of geometry would do well to add the second volume on complex geometry from the 1989 Proceedings to the present three volumes.

Each week of the 1990 Summer Institute was given a general emphasis as to subject and the Proceedings volumes are organized in the same way. While overlap is natural, and indeed inevitable, the subjects of the volumes are as follows:

I. Partial differential equations on manifolds: harmonic functions and mapping, Monge-Ampre equation, differential systems, minimal submanifolds;

II. Geometry in mathematical physics and related topics: gauge theory, symplectic geometry, complex geometry, L^2 cohomology, Lorentzian geometry;

III. Riemannian geometry: curvature and topology, groups and manifolds, dynamical systems in geometry, spectral theory of Riemannian manifolds.

The articles in these Proceedings are also of several types. We requested broad-ranging surveys from the people who had given one-hour survey lectures at the Institute. These articles are marked with an asterisk in the table of contents. Such surveys were also encouraged from other participants. The remaining articles are either research papers in the usual sense or relatively brief announcements of results. But in these categories as well, we encouraged authors to provide more background information and references to related work than usual. Thus we hope that readers will find the volumes a source of broad perspectives on the rapidly expanding literature of geometry.

The editors themselves essayed two efforts in this direction. Volume I begins with a problem list by S. T. Yau, successor to his 1980 list [Seminar in Differential Geometry, Annals of Math. Studies, no. 102, Princeton University Press]. Volume III begins with an overview by R. E. Greene of some recent trends in Riemannian geometry, in the interests of identifying themes common to the remaining papers in that volume and in outlining certain topics that, as it happened, would not otherwise have been included.

An event such as the 1990 AMS Summer Institute involves the efforts of a great many people. We would like to thank all the participants, both for their participation in the Institute and for their prompt and abundant response to our call for papers for these Proceedings. We are indebted to the staff of the UCLA Mathematics Department for their cooperation during the Summer Institute and to the American Mathematical Society in general and in particular to Mr. Wayne Drady and Ms. Susan Blyth for their invaluable on-the-spot assistance during the Institute. Without the financial support of the National Science Foundation, the Summer Institute could not have had anything like the scope it in fact attained; we are particularly grateful for their willingness to support the participation of advanced graduate students, a willingness that we believe was a valuable investment in the future of geometry. Finally, we are indebted to the publication staff of the American Mathematical Society and particularly to Ms. Alison Buckser, Ms. Donna Harmon, and Ms. Christine Thivierge, whose unfailing patience and helpfulness made the preparation of these volumes a far easier task than it would have been otherwise.

Robert E. Greene
S. T. Yau

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