

# Proceedings of Symposia in PURE MATHEMATICS

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Volume 55.2

## Motives

Summer Research Conference on Motives  
July 20–August 2, 1991  
University of Washington  
Seattle, Washington

Uwe Jannsen  
Steven Kleiman  
Jean-Pierre Serre  
Editors



**American Mathematical Society**

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**Motives**

# Proceedings of Symposia in PURE MATHEMATICS

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Volume 55, Part 2

## Motives

Uwe Jannsen  
Steven Kleiman  
Jean-Pierre Serre  
Editors



**American Mathematical Society**  
Providence, Rhode Island

PROCEEDINGS OF THE SUMMER RESEARCH CONFERENCE  
ON MOTIVES

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# Contents

Preface	xi
Program	xiii

## PART 1

The Standard Conjectures	
STEVEN L. KLEIMAN	3
Review of $\ell$ -adic Cohomology	
NICHOLAS M. KATZ	21
A Summary of Mixed Hodge Theory	
J. H. M. STEENBRINK	31
Crystalline Cohomology	
LUC ILLUSIE	43
Conjectures on Algebraic Cycles in $\ell$ -adic Cohomology	
JOHN TATE	71
Some Remarks on the Hodge Type Conjecture	
MORIHIKO SAITO	85
Independence of $\ell$ and Weak Lefschetz	
NICHOLAS M. KATZ	101
Décompositions dans la catégorie dérivée	
PIERRE DELIGNE	115
Arithmetic Analogs of the Standard Conjectures	
H. GILLET AND C. SOULÉ	129
A quoi servent les motifs?	
PIERRE DELIGNE	143
Classical Motives	
A. J. SCHOLL	163

On the Chow Motive of an Abelian Scheme KLAUS KÜNNEMANN	189
Weight Filtrations in Algebraic $K$ -Theory DANIEL R. GRAYSON	207
An Elementary Presentation for $K$ -Groups and Motivic Cohomology SPENCER BLOCH	239
Motivic Sheaves and Filtrations on Chow Groups UWE JANNSSEN	245
Motivic Complexes STEPHEN LICHTENBAUM	303
On the Bijectivity of Some Cycle Maps MORIIHIKO SAITO	315
Tannakian Categories LAWRENCE BREEN	337
Propriétés conjecturales des groupes de Galois motiviques et des représentations $\ell$ -adiques JEAN-PIERRE SERRE	377
Motives over Finite Fields J. S. MILNE	401
Motives for Absolute Hodge Cycles A. A. PANCHISHKIN	461
CM Motives and the Taniyama Group NORBERT SCHAPPACHER	485
Structures de Hodge mixtes réelles PIERRE DELIGNE	509
$L$ -Functions of Mixed Motives CHRISTOPHER DENINGER	517
$L$ -Functions at the Central Critical Point BENEDICT H. GROSS	527
Beilinson's Conjectures JAN NEKOVÁŘ	537
Height Pairings and Special Values of $L$ -Functions A. J. SCHOLL	571

<b>Autour des conjectures de Bloch et Kato: cohomologie galoisienne et valeurs de fonctions <math>L</math></b>	
<b>JEAN-MARC FONTAINE ET BERNADETTE PERRIN-RIOU</b>	<b>599</b>
<b>Motivic <math>L</math>-Functions and Regularized Determinants</b>	
<b>CHRISTOPHER DENINGER</b>	<b>707</b>
<b>On a Result of Deninger Concerning Riemann's Zeta Function</b>	
<b>M. SCHRÖTER AND C. SOULÉ</b>	<b>745</b>

PART 2

<b>Classical Polylogarithms</b>	
<b>RICHARD M. HAIN</b>	<b>3</b>
<b>Polylogarithms and Motivic Galois Groups</b>	
<b>A. B. GONCHAROV</b>	<b>43</b>
<b>Interprétation motivique de la conjecture de Zagier reliant polylogarithmes et régulateurs</b>	
<b>A. BEILINSON ET P. DELIGNE</b>	<b>97</b>
<b>The Elliptic Polylogarithm</b>	
<b>A. BEILINSON AND A. LEVIN</b>	<b>123</b>
<b>Iwasawa Theory and <math>p</math>-adic Deformations of Motives</b>	
<b>RALPH GREENBERG</b>	<b>193</b>
<b><math>p</math>-adic Points of Motives</b>	
<b>PETER SCHNEIDER</b>	<b>225</b>
<b>Admissible Non-Archimedean Standard Zeta Functions Associated with Siegel Modular Forms</b>	
<b>A. A. PANCHISHKIN</b>	<b>251</b>
<b>A <math>p</math>-adic Property of Hodge Classes on Abelian Varieties</b>	
<b>DON BLASIUS</b>	<b>293</b>
<b>Drinfeld Modules: Cohomology and Special Functions</b>	
<b>DAVID GOSS</b>	<b>309</b>
<b>The Local Langlands Correspondence: The Non-Archimedean Case</b>	
<b>STEPHEN S. KUDLA</b>	<b>365</b>
<b>Local Langlands Correspondence: The Archimedean Case</b>	
<b>A. W. KNAPP</b>	<b>393</b>
<b>Pure Motives and Automorphic Forms</b>	
<b>DINAKAR RAMAKRISHNAN</b>	<b>411</b>



Shimura Varieties and Motives	
J. S. MILNE	447
Zeta Functions of Shimura Varieties	
DON BLASIUS AND JONATHAN D. ROGAWSKI	525
Hodge-de Rham Structures and Periods of Automorphic Forms	
MICHAEL HARRIS	573
Galois Representations Congruent to Those Coming from Shimura Varieties	
J. TILOUINE	625
Report on mod $\ell$ Representations of $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$	
KENNETH A. RIBET	639

## Preface

The American Mathematical Society, the Institute of Mathematical Statistics, and the Society for Industrial and Applied Mathematics held a joint summer research conference at the University of Washington at Seattle from July 20 to August 2, 1991 on the topic of motives. The conference was organized by Alexander Beilinson (MIT and Moscow), Pierre Deligne (IAS), Uwe Jannsen (Köln), Steven Kleiman (MIT, co-chair), Robert MacPherson (MIT), Jean-Pierre Serre (Collège de France), and Kari Vilonen (Brandeis, co-chair).

The theory of motives was introduced in the middle 1960s by Alexander Grothendieck to explain the analogies among the various cohomology theories for algebraic varieties, to play the role of the missing rational cohomology, and to provide a blueprint for proving Weil's conjectures about the zeta function of a variety over a finite field. Remarkably, over the last ten years or so, researchers in various areas—Hodge theory, algebraic  $K$ -theory, polylogarithms, automorphic forms,  $L$ -functions,  $\ell$ -adic representations, trigonometric sums, and algebraic cycles—have discovered that an enlarged (and in part conjectural) theory of “mixed” motives indicates and explains phenomena appearing in each area. Thus the theory holds the potential of enriching each area and of unifying them all.

The Seattle conference was the first symposium ever held on motives. It presented a unique opportunity to bring together researchers and students in these diverse areas to exchange ideas and discover common themes. Everyone who applied was invited to attend, and about 140 people from all over the world registered and participated. About a third of the participants were students.

The scientific program ran eleven days. Each day, there were four one-hour lectures; the number was limited to encourage informal discussion. The first lectures introduced and surveyed the entire field; subsequent lectures elaborated on the individual areas. On the last day there was a single one-hour main lecture, followed by six half-hour subsidiary lectures. The lecturers

were assigned topics, and were asked to paint panoramic views from their vantage points. A copy of the program is appended.

These volumes contain the proceedings of the conference. They include the revised texts of nearly all the lectures and a number of related works, forty-seven papers in all. There are general introductions, specialized surveys, and research papers. Each paper was refereed and is in final form.

The University of Washington provided a convenient, comfortable, and attractive site, which was conducive to the success of the conference. The AMS did a superb job of administration, freeing the organizing committee to concentrate on the scientific program. In particular, Carole Kohanski, the AMS Conference Coordinator, went far beyond the call of duty. On behalf of the entire organizing committee and all of the participants, the editors wish to express their gratitude to everyone who contributed to the success of the conference and to the production of these proceedings.

Uwe Jannsen

Steven Kleiman

Jean-Pierre Serre

# Program

## First Week

**SUNDAY** (Classical motives):

1. Historical introduction (Serre)
2. Standard conjectures (Kleiman)
3. Examples (Scholl)
4. An overview (Deligne)

**MONDAY** (Cohomology theories):

1. Étale cohomology (Katz)
2. Hodge theory (Steenbrink)
3. Crystalline cohomology (Illusie)
4. The Tate conjectures (Tate)

**TUESDAY** (Tannakian categories):

1. Tannakian categories and the motivic Galois group (Breen)
2. Motives for absolute Hodge cycles (Panchishkin)
3. CM-motives and the Taniyama group (Schappacher)
4. Motives over finite fields (Milne)

**WEDNESDAY** ( $L$ -functions):

1. Motivic Galois groups (Serre)
2.  $L$ -functions (Deninger)
3. The conjectures of Deligne and of Birch/Swinnerton-Dyer (Gross)
4.  $K$ -theoretic background (Grayson)

**THURSDAY** (Beilinson conjectures):

1. Beilinson conjectures I (Soulé)
2. Beilinson conjectures II (Nekovář)
3. Beilinson conjectures III: Reformulation in terms of mixed motives (Scholl)

4. Mixed motives and motivic sheaves (Jannsen)

**FRIDAY** (Bloch-Kato conjectures, Beilinson-Lichtenbaum complexes):

1. Bloch-Kato conjectures I (Perrin-Riou)
2. Bloch-Kato conjectures II (Fontaine)
3. Beilinson-Lichtenbaum complexes (Lichtenbaum)
4. Higher Chow groups (Bloch)

**Second Week****SUNDAY (Mixed Tate motives):**

1. Polylogarithms and the line minus three points (Hain)
2. Mixed Tate motives I (MacPherson)
3. Mixed Tate motives II: Zagier's conjecture (Goncharov)
4. Beilinson's work on the Zagier conjecture (Deligne)

**MONDAY (Automorphic forms I):**

1. The local Langlands conjecture (Kudla)
2. Pure motives and automorphic forms (Ramakrishnan)
3. Shimura varieties and motives (Milne)
4.  $L$ -functions of Shimura varieties (Rogawski)

**TUESDAY (Automorphic forms II):**

1. Hodge-de Rham structures and periods (M. Harris)
2. Mixed motives coming from Shimura varieties (Harder)
3. Galois representations congruent to those arising from Shimura varieties (Tilouine)
4. mod- $p$  Galois representations and Serre's conjectures (Ribet)

**WEDNESDAY ( $p$ -adic theory, function fields):**

1.  $p$ -adic  $L$ -functions (Coates)
2. Iwasawa theory for motives (Greenberg)
3.  $p$ -adic motives (Schneider)
4. Function fields (Goss)

**THURSDAY (Miscellaneous topics):**

1. Exponential sums (Katz)
2.  $\ell$ -adic representations associated to abelian varieties (Serre)
3.  $p$ -adic properties of absolute Hodge cycles (Wintenberger)
4. The motive of an abelian variety (Künnemann)
5. Parshin-Beilinson adèles for schemes (Huber)
6. Hodge modules, questions (M. Saito)
7.  $F_q$ -points of a variety and a Hodge-theoretic analogue (Esnault)

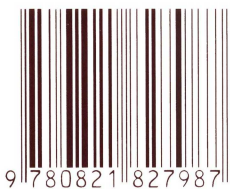
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