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Volume 56, Part 2

Algebraic Groups and Their Generalizations: Quantum and Infinite- Dimensional Methods

Summer Research Institute on
Algebraic Groups and Their Generalizations
July 6–26, 1991
Pennsylvania State University
University Park, Pennsylvania

William J. Haboush
Brian J. Parshall
Editors



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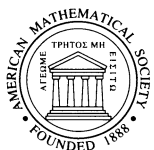
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Brian J. Parshall
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Preface

These volumes contain papers based on lectures presented at the conference on “Algebraic Groups and Their Generalizations” held at Pennsylvania State University from July 6–26, 1991 and organized by Igor Frenkel, William Haboush (Chair), Jens Jantzen, and Brian Parshall. A response to the remarkable proliferation of Lie theory in the last decade and a half, this conference reflected both the diversification of technique in the classical theory and the beginnings of the study of new objects such as quantum groups, vertex operator algebras, and various kinds of infinite-dimensional groups and algebras, inspired by new work in mathematical physics and quantum field theory. It is hoped that the papers in these volumes will provide the interested reader with an introduction to these astonishing new mathematical worlds.

The first volume focuses on classical methods, while the second centers on quantum and infinite-dimensional methods. Each section begins with various expositions and then turns to papers on previously unknown results.

As construed by the editors, classical methods include the geometry of Schubert varieties and homogeneous spaces, Frobenius splitting techniques, the apparatus of Kazhdan-Lusztig theory including uses of intersection homology, the study of certain categories of representations based on highest weight modules, and invariant theory.

Likewise, the notions of quantum and infinite-dimensional theory have been broadly defined to include various notions of quantum groups and quantized enveloping algebras, crystal bases, vertex operator algebras, and infinite-dimensional objects and topics such as braidings and the equations of Yang-Baxter type and their generalizations. We hope that these volumes represent a useful addition to the basic literature in these areas.

The editors particularly wish to acknowledge the generous assistance of Catherine Chevalley who made possible the publication of the article by Claude Chevalley in this volume. The belated appearance of this manuscript, which has influenced so many mathematicians, makes its special insights, both historical and mathematical, available to the mathematics community. For this we thank Ms. Chevalley.

William J. Haboush
Brian J. Parshall

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