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Stochastic Analysis

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July 11–30, 1993
Cornell University
Ithaca, New York

Michael C. Cranston
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Editors

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Providence, Rhode Island

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Preface

The present volume is an outgrowth of the AMS Summer Institute on Stochastic Analysis, which was held on the campus of Cornell University during the period of July 11-30, 1993. This effort included 69 one-hour lectures which are reflected in the 46 papers in the current volume.

In a broad way, we have made some effort to achieve both depth and diversity. The classical connections betweeen stochastic analysis and differential geometry, partial differential equations and mathematical physics should be well apparent in many of the papers of the current volume. More recent trends in infinite-dimensional analysis and stochastic partial differential equations are also well represented.

Although somewhat tentative, we have made an attempt to organize the table of contents with respect to several main viewpoints: i) problems in analysis, ii) problems in geometry, iii) infinite-dimensional problems and iv) stochastic PDE/stochastic flows. Of course many papers overlap these somewhat provisional labels which we have constructed. The effort is meant to give some coherence rather than erect artificial boundaries.

The AMS conference was organized by a scientific committee chaired by Rick Durrett, whose tireless efforts were apparent throughout. Further valuable assistance was provided by the AMS staff and the Cornell Conference Services. The financial support was principally through a National Science Foundation grant made to the AMS and supplemented by smaller grants from the National Security Agency and the ARO-funded Mathematical Sciences Institute of Cornell University. We are extremely grateful for these efforts in bringing our participants to the Cornell campus for this unprecedented effort in stochastic analysis.

All papers in the present volume have been refereed prior to publication.

Michael C. Cranston
Mark A. Pinsky

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