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K-Theory and Algebraic Geometry: Connections with Quadratic Forms and Division Algebras

Summer Research Institute on
Quadratic Forms and Division Algebras
July 6–24, 1992
University of California, Santa Barbara

Bill Jacob
Alex Rosenberg
Editors



American Mathematical Society

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Preface

During the decade of the 1980s profound connections were discovered relating modern algebraic geometry and algebraic K -theory to arithmetic problems. Indeed, the phrase “arithmetic algebraic geometry” was popularized during that time and is now used by many to denote an entire branch of 20th century number theory. In addition, these same developments in algebraic geometry and K -theory greatly influenced research into the arithmetic of fields in general, in particular the algebraic theory of quadratic forms and the theory of finite-dimensional division algebras. The purpose of the 1992 AMS Summer Research Institute was to provide the research community with both a broad overview of the tools from algebraic geometry and K -theory that proved to be the most powerful in solving problems in the theory of quadratic forms and division algebras, as well as provide a forum for exposition of recent research.

The three week institute had six week-long speakers: three in K -theory, R. Swan, A. A. Suslin, and A. S. Merkurjev; and three in algebraic geometry, J.-L. Colliot-Thélène, W. Raskind, and D. Saltman. A substantial portion of their lectures are reproduced in their expository articles in this volume. The editors hope these articles will help introduce young researchers to these important tools. In addition the institute had a series of individual research lectures, many of which can be found here. The organizers would like to thank the NSF for financial support, as well as the AMS and conference coordinator Wayne Drady for their efforts in administering the institute.

Bill Jacob
Alex Rosenberg
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