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Quantization, Nonlinear Partial Differential Equations, and Operator Algebra

1994 John von Neumann Symposium on
Quantization and Nonlinear Wave Equations
June 7-11, 1994
Massachusetts Institute of Technology,
Cambridge, Massachusetts

William Arveson
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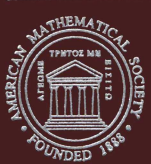
Editors



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Preface

John von Neumann was one of the greatest mathematical talents of the 20th Century, and a worthy successor to Poincare, Hilbert, and Weyl, in both intensity and breadth. His achievements are all the more impressive if one takes into consideration the short span of his life, and his very active national defense role during World War II and thereafter.

In addition to his personal research contributions he was immensely influential in general scientific terms, in a range extending from the abstraction of projective-like but continuous geometries to the practical application of high-speed computers. No single theme can adequately represent his broad scientific thrust. But his scientific impact as well as personal involvement was greatest in the work that provided mathematical coherence for quantum mechanics, and established the algebraic paradigm in modern analysis. The problem of developing an appropriate and rigorous formalism for quantum field theory was the chief motivation for his late monumental work on operator algebra.

This work remains unfinished, a long-standing challenge to advanced mathematical ideas and techniques, as well as fundamental for theoretical physics. In the meantime, in the past several decades, the global theory of nonlinear partial differential equations, the understanding and application of which was one of von Neumann's principal motivations in the development of computers, has made great strides. Nonlinear wave equations are what quantum field theory is all about, and thus a symposium bringing together these and related areas was a very fitting memorial to von Neumann's work and spirit.

Recent inroads in higher-dimensional nonlinear quantum field theory and in the global theory of relevant nonlinear wave equations have been accompanied by very interesting cognate developments. The latter include symplectic quantization theory on manifolds and in group representations, the operator algebraic implementation of quantum dynamics, as well as differential geometric, general relativistic, and purely algebraic aspects. "Quantization and Nonlinear Wave Equations" thus appeared highly appropriate as the theme of the first John von Neumann Symposium, a new series made possible by the establishment of a fund by Dr. and Mrs. Caroll V. Newsom in honor of his memory, intended to treat topics of emerging significance that are likely to underlie future mathematical developments.

The present volume is a microcosm of the recent seminal progress in the entire area. The work on quantization exemplifies both the Hilbert space and symplectic manifold approaches. That on nonlinear wave equations runs a gamut from

microlocal theory through the Boltzmann equation to General Relativity. A variety of operator algebraic approaches to quantum dynamics, describing current developments and indicating key problems, are presented. Evolutionary aspects of what is essentially quantum probability, a new line of research branching off from the quantization problem, is exposed from different vantage points. Geometric aspects of partial differential operators are brought into relation to these matters. These topics are moreover interwoven in a coherent way based on the theme of the Symposium.

The Organizing committee for the Symposium, whose topic was selected by the AMS Committee on Summer Institutes and Special Symposia, consisted of Haim Brezis and Irving Segal, co-chairs, in addition to William Arveson, Robert Blattner, and Thomas Branson. The Symposium was supported in part by the National Science Foundation, and took place at the Massachusetts Institute of Technology from June 6–12, 1994. In addition to the papers presented here, invited lectures were given by Robert Blattner, Leonard Gross, Victor Guillemin, Roger Howe, Victor Kac, J. T. Stafford, and Zhengfang Zhou. Additional lectures were contributed by participants in the Symposium.

Irving Segal

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