Proceedings of Symposia in PURE MATHEMATICS

Volume 67

Algebraic K-Theory

AMS-IMS-SIAM Joint Summer Research Conference on Algebraic K-Theory July 13–24, 1997 University of Washington, Seattle

Wayne Raskind Charles Weibel Editors



American Mathematical Society

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American Mathematical Society Providence, Rhode Island

PROCEEDINGS OF AN AMS-IMS-SIAM SUMMER RESEARCH CONFERENCE ON ALGEBRAIC K-THEORY HELD AT UNIVERSITY OF WASHINGTON, SEATTLE, WASHINGTON, JULY 13–24, 1997

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Preface

The American Mathematical Society, the Institute of Mathematical Statistics, and the Society for Industrial and Applied Mathematics held a series of joint Summer Research Conferences in 1997, on the campus of the University of Washington at Seattle. The Summer Research Conference on Algebraic K-theory was held July 13–24, and it was organized by H. Gillet, D. Grayson, W. Raskind (co-chair), J. Rosenberg and C.Weibel (co-chair). It was attended by about 90 mathematicians from all over the world.

The main ideas of K-theory were introduced in the 1950's by Whitehead and Grothendieck. Whitehead was interested in creating invariants to distinguish between homotopy equivalences in topology, while Grothendieck was interested in using invariants of vector bundles to formulate a very general Riemann-Roch theorem in algebraic geometry. Applying these ideas to topological vector bundles quickly led to solutions of many outstanding problems, and that circle of ideas became known as topological K-theory. In algebra, the applications to projective modules and general linear groups developed into algebraic K-theory in the 1960's. Driven by applications to number theory, and then algebraic geometry, higher algebraic K-theory was defined around 1970 by Quillen and others. The subject developed steadily in the 1970's, balanced between its intrinsic structure and its applications. In the 1980's, the introduction of arithmetic methods and cyclic homology made for even more rapid development, and the subject received its own Mathematics Subject Classification Number (19).

Algebraic K-theory has always been an interdisciplinary subject. During the 1990's, it has continued to develop, in many directions. Many of the recent developments related to arithmetic and algebraic geometry have involved motivic cohomology, especially the approach of Suslin and Voevodsky. It now seems that the full "Quillen-Lichtenbaum conjectures," which would provide a beautiful conceptual framework for the subject, are finally within reach. Many of the new developments in topology have centered around topological cyclic homology (which also seems to have arithmetic applications) and controlled topology. K-theory has played an increasingly important role in the study of operator algebras, both for applications to topology (such as new attacks on the Novikov conjectures) and to provide invariants which are being used to classify important classes of nuclear C^* -algebras. Because of the synthesis now going on, we believe that it is actually easier to see the state of the field and where it might be going now than it was ten years ago.

PREFACE

This conference provided a setting for bringing together workers in all these diverse branches of the field, and the schedule was designed to let them interact. The scientific program consisted of a series of main lectures in the mornings and some early afternoons, with shorter seminar talks in the middle and late afternoons. The list of speakers is appended. The talks covered most of the subject, and the speakers did an excellent job of presenting the field to the current and next generations.

The conference was dedicated to the memory of Robert Thomason, who died in late 1995. The influence of his work in algebraic K-theory could be seen in many of the talks. In addition, there was a special program on the mathematics of Robert Thomason at the end of the first week, with talks by Gillet, Mitchell, Soulé and Weibel.

The collection of papers in this volume represents the proceedings of the conference. It consists of high-level work by many of the leading workers in algebraic K-theory. They represent many but of course not all of the myriad aspects of this diffuse subject. We hope that this volume will be of great utility to current and future workers in and near the subject.

The editors would like to thank Romy Cascella for her organizational help before, during, and after the conference. We are very grateful to the National Science Foundation and the National Security Agency for funding. The first named editor would like to thank C. Bloomquist for his help in the preparation of the manuscripts for publication.

Wayne Raskind Charles Weibel August 2, 1999

Program

1st Week

Main Lectures:

J.-L. Colliot-Thélène H. Esnault

R. McCarthy V. Voevodsky

Seminar Speakers:

L.	Barbieri-Viale	
H.	Gillet	
S.	Mitchell	
E.	Peyre	
С.	Soulé	
A.	Wadsworth	

Main Lectures:

G. Carlsson

- N. Higson
- J. Rognes

Seminar Speakers:

- P. Adhikari J. Cuntz K. Knudson A. Lindenstrauss
- D. Pineda
- B. Williams

T. GeisserB. KahnJ. RosenbergC. Weibel

D. Edidin M. Hanamura S. Müller-Stach W. Raskind M. Spiess M. Walker

2nd Week

J. Cuntz R. Jardine

S. Mitchell

S. Betley B. Dundas A. Kuku J. Lodder C. Schochet D. GraysonS. LichtenbaunC. Soulé

H. GanglY. KoyaA. NenashevA. RosenschonV. VoevodskyC. Weibel

L. Hesselholt M. Karoubi M. Weiss

J. Block S. Jackowski F. Lecomte I. Madsen B. Totaro

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- 31 Joseph L. Doob, Editor, Probability (University of Illinois at Urbana-Champaign, Urbana, March 1976)
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- 28 Felix E. Browder, Editor, Mathematical developments arising from Hilbert problems (Northern Illinois University, Dekalb, May 1974)
- 27 S. S. Chern and R. Osserman, Editors, Differential geometry (Stanford University, Stanford, California, July/August 1973)

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