Proceedings of Symposia in 
PURE MATHEMATICS 

Volume 67

Algebraic $K$-Theory

AMS-IMS-SIAM Joint Summer Research 
Conference on Algebraic $K$-Theory 
July 13–24, 1997 
University of Washington, Seattle

Wayne Raskind 
Charles Weibel 
Editors 

American Mathematical Society
Selected Titles in This Series

67 Wayne Raskin and Charles Weibel, Editors, Algebraic K-theory (University of Washington, Seattle, 1997)

66 Robert S. Doran, Ze-Li Dou, and George T. Gilbert, Editors, Automorphic forms, automorphic representations, and arithmetic (Texas Christian University, Fort Worth, 1996)


63 Alejandro Adem, Jon Carlson, Stewart Priddy, and Peter Webb, Editors, Group representations: Cohomology, group actions and topology (University of Washington, Seattle, 1996)

62 János Kollár, Robert Lazarsfeld, and David R. Morrison, Editors, Algebraic geometry—Santa Cruz 1995 (University of California, Santa Cruz, July 1995)


59 William Arveson, Thomas Branson, and Irving Segal, Editors, Quantization, nonlinear partial differential equations, and operator algebra (Massachusetts Institute of Technology, Cambridge, June 1994)

58 Bill Jacob and Alex Rosenberg, Editors, K-theory and algebraic geometry: Connections with quadratic forms and division algebras (University of California, Santa Barbara, July 1992)

57 Michael C. Cranston and Mark A. Pinsky, Editors, Stochastic analysis (Cornell University, Ithaca, July 1993)

56 William J. Haboush and Brian J. Parshall, Editors, Algebraic groups and their generalizations (Pennsylvania State University, University Park, July 1991)

55 Uwe Jannsen, Steven L. Kleiman, and Jean-Pierre Serre, Editors, Motives (University of Washington, Seattle, July/August 1991)

54 Robert Greene and S. T. Yau, Editors, Differential geometry (University of California, Los Angeles, July 1990)

53 James A. Carlson, C. Herbert Clemens, and David R. Morrison, Editors, Complex geometry and Lie theory (Sundance, Utah, May 1989)

52 Eric Bedford, John P. D’Angelo, Robert E. Greene, and Steven G. Krantz, Editors, Several complex variables and complex geometry (University of California, Santa Cruz, July 1989)

51 William B. Arveson and Ronald G. Douglas, Editors, Operator theory/operator algebras and applications (University of New Hampshire, July 1988)

50 James Glimm, John Impagliazzo, and Isadore Singer, Editors, The legacy of John von Neumann (Hofstra University, Hempstead, New York, May/June 1988)

49 Robert C. Gunning and Leon Ehrenpreis, Editors, Theta functions – Bowdoin 1987 (Bowdoin College, Brunswick, Maine, July 1987)

48 R. O. Wells, Jr., Editor, The mathematical heritage of Hermann Weyl (Duke University, Durham, May 1987)

47 Paul Fong, Editor, The Arcata conference on representations of finite groups (Humboldt State University, Arcata, California, July 1986)

46 Spencer J. Bloch, Editor, Algebraic geometry – Bowdoin 1985 (Bowdoin College, Brunswick, Maine, July 1985)

45 Felix E. Browder, Editor, Nonlinear functional analysis and its applications (University of California, Berkeley, July 1983)
Algebraic $K$-Theory
Proceedings of Symposia in
Pure Mathematics

Volume 67

Algebraic $K$-Theory

AMS-IMS-SIAM Joint Summer Research
Conference on Algebraic $K$-Theory
July 13–24, 1997
University of Washington, Seattle

Wayne Raskind
Charles Weibel
Editors

American Mathematical Society
Providence, Rhode Island
The 1997 AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences on “Algebraic K-theory” was held at the University of Washington on July 13-24, 1997, with support from the National Science Foundation, Grant DMS-9618514, and the National Security Agency, Grant MDA904-97-1-0071.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the view of the National Science Foundation.


Library of Congress Cataloging-in-Publication Data
   p. cm. — (Proceedings of symposia in pure mathematics, ISSN 0082-0717 ; v. 67)
   Includes bibliographical references.
   ISBN 0-8218-0927-X (alk. paper)
QA612.33.A47 1997
512.55—dc21 199-37722
CIP

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Assistant to the Publisher, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 1999 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights except those granted to the United States Government.
Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.
Visit the AMS home page at URL: http://www.ams.org/

10 9 8 7 6 5 4 3 2 1 04 03 02 01 00 99
## Contents

Preface ix  
List of speakers xi  
Conjectures de type local-global sur l’image des groupes de Chow dans la cohomologie étale  
   J.-L. Colliot-Thélène 1  
Algebraic theory of characteristic classes of bundles with connection  
   H. Esnault 13  
Polylogarithmic identities in cubical higher Chow groups  
   H. Gangl and S. Müller-Stach 25  
Topological cyclic homology of schemes  
   T. Geisser and L. Hesselholt 41  
Filtrations on higher algebraic $K$-theory  
   H. Gillet and C. Soulé 89  
Motivic cohomology of smooth geometrically cellular varieties  
   B. Kahn 149  
Integral homology of $PGL_2$ over elliptic curves  
   K. Knudsen 175  
Application of motivic complexes to negligible classes  
   E. Peyre 181  
Two-primary algebraic $K$-theory of spaces and related spaces of symmetries  
   of manifolds  
   J. Rognes 213  
A mini-course on recent progress in algebraic $K$-theory and its relationship  
   with topology and analysis  
   J. Rosenberg 231  
The Chow ring of a classifying space  
   B. Totaro 249  
Voevodsky’s Seattle lectures: $K$-theory and motivic cohomology  
   V. Voevodsky (Notes by C. Weibel) 283  
Products in higher Chow groups and motivic cohomology  
   C. Weibel 305
Preface

The American Mathematical Society, the Institute of Mathematical Statistics, and the Society for Industrial and Applied Mathematics held a series of joint Summer Research Conferences in 1997, on the campus of the University of Washington at Seattle. The Summer Research Conference on Algebraic $K$-theory was held July 13–24, and it was organized by H. Gillet, D. Grayson, W. Raskind (co-chair), J. Rosenberg and C. Weibel (co-chair). It was attended by about 90 mathematicians from all over the world.

The main ideas of $K$-theory were introduced in the 1950’s by Whitehead and Grothendieck. Whitehead was interested in creating invariants to distinguish between homotopy equivalences in topology, while Grothendieck was interested in using invariants of vector bundles to formulate a very general Riemann-Roch theorem in algebraic geometry. Applying these ideas to topological vector bundles quickly led to solutions of many outstanding problems, and that circle of ideas became known as topological $K$-theory. In algebra, the applications to projective modules and general linear groups developed into algebraic $K$-theory in the 1960’s. Driven by applications to number theory, and then algebraic geometry, higher algebraic $K$-theory was defined around 1970 by Quillen and others. The subject developed steadily in the 1970’s, balanced between its intrinsic structure and its applications. In the 1980’s, the introduction of arithmetic methods and cyclic homology made for even more rapid development, and the subject received its own Mathematics Subject Classification Number (19).

Algebraic $K$-theory has always been an interdisciplinary subject. During the 1990’s, it has continued to develop, in many directions. Many of the recent developments related to arithmetic and algebraic geometry have involved motivic cohomology, especially the approach of Suslin and Voevodsky. It now seems that the full “Quillen-Lichtenbaum conjectures,” which would provide a beautiful conceptual framework for the subject, are finally within reach. Many of the new developments in topology have centered around topological cyclic homology (which also seems to have arithmetic applications) and controlled topology. $K$-theory has played an increasingly important role in the study of operator algebras, both for applications to topology (such as new attacks on the Novikov conjectures) and to provide invariants which are being used to classify important classes of nuclear $C^*$-algebras. Because of the synthesis now going on, we believe that it is actually easier to see the state of the field and where it might be going now than it was ten years ago.
This conference provided a setting for bringing together workers in all these diverse branches of the field, and the schedule was designed to let them interact. The scientific program consisted of a series of main lectures in the mornings and some early afternoons, with shorter seminar talks in the middle and late afternoons. The list of speakers is appended. The talks covered most of the subject, and the speakers did an excellent job of presenting the field to the current and next generations.

The conference was dedicated to the memory of Robert Thomason, who died in late 1995. The influence of his work in algebraic $K$-theory could be seen in many of the talks. In addition, there was a special program on the mathematics of Robert Thomason at the end of the first week, with talks by Gillet, Mitchell, Soulé and Weibel.

The collection of papers in this volume represents the proceedings of the conference. It consists of high-level work by many of the leading workers in algebraic $K$-theory. They represent many but of course not all of the myriad aspects of this diffuse subject. We hope that this volume will be of great utility to current and future workers in and near the subject.

The editors would like to thank Romy Cascella for her organizational help before, during, and after the conference. We are very grateful to the National Science Foundation and the National Security Agency for funding. The first named editor would like to thank C. Bloomquist for his help in the preparation of the manuscripts for publication.

Wayne Raskind
Charles Weibel
August 2, 1999
# Program

<table>
<thead>
<tr>
<th>1st Week</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main Lectures:</strong></td>
</tr>
<tr>
<td>J.-L. Colliot-Thélène</td>
</tr>
<tr>
<td>H. Esnault</td>
</tr>
<tr>
<td>R. McCarthy</td>
</tr>
<tr>
<td>V. Voevodsky</td>
</tr>
<tr>
<td><strong>Seminar Speakers:</strong></td>
</tr>
<tr>
<td>L. Barbieri-Viale</td>
</tr>
<tr>
<td>H. Gillet</td>
</tr>
<tr>
<td>S. Mitchell</td>
</tr>
<tr>
<td>E. Peyre</td>
</tr>
<tr>
<td>C. Soulé</td>
</tr>
<tr>
<td>A. Wadsworth</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2nd Week</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main Lectures:</strong></td>
</tr>
<tr>
<td>G. Carlsson</td>
</tr>
<tr>
<td>N. Higson</td>
</tr>
<tr>
<td>J. Rognes</td>
</tr>
<tr>
<td><strong>Seminar Speakers:</strong></td>
</tr>
<tr>
<td>P. Adhikari</td>
</tr>
<tr>
<td>J. Cuntz</td>
</tr>
<tr>
<td>K. Knudson</td>
</tr>
<tr>
<td>A. Lindenstrauss</td>
</tr>
<tr>
<td>D. Pineda</td>
</tr>
<tr>
<td>B. Williams</td>
</tr>
</tbody>
</table>


Selected Titles in This Series

(Continued from the front of this publication)

44 William K. Allard and Frederick J. Almgren, Jr., Editors, Geometric measure
theory and the calculus of variations (Humboldt State University, Arcata, California,
July/August 1984)
43 François Trèves, Editor, Pseudodifferential operators and applications (University of
Notre Dame, Notre Dame, Indiana, April 1984)
42 Anil Nerode and Richard A. Shore, Editors, Recursion theory (Cornell University,
Ithaca, New York, June/July 1982)
41 Yum-Tong Siu, Editor, Complex analysis of several variables (Madison, Wisconsin,
April 1982)
40 Peter Orlik, Editor, Singularities (Humboldt State University, Arcata, California,
July/August 1981)
39 Felix E. Browder, Editor, The mathematical heritage of Henri Poincaré (Indiana
University, Bloomington, April 1980)
38 Richard V. Kadison, Editor, Operator algebras and applications (Queens University,
Kingston, Ontario, July/August 1980)
37 Bruce Cooperstein and Geoffrey Mason, Editors, The Santa Cruz conference on
finite groups (University of California, Santa Cruz, June/July 1979)
36 Robert Osserman and Alan Weinstein, Editors, Geometry of the Laplace operator
(University of Hawaii, Honolulu, March 1979)
35 Guido Weiss and Stephen Wainger, Editors, Harmonic analysis in Euclidean spaces
(Williams College, Williamstown, Massachusetts, July 1978)
34 D. K. Ray-Chaudhuri, Editors, Connections between combinatorics and other parts of
mathematics (Ohio State University, Columbus, March 1978)
33 A. Borel and W. Casselman, Editors, Automorphic forms, representations and
L-functions (Oregon State University, Corvallis, July/August 1977)
32 R. James Milgram, Editor, Algebraic and geometric topology (Stanford University,
Stanford, California, August 1976)
31 Joseph L. Doob, Editor, Probability (University of Illinois at Urbana-Champaign,
Urbana, March 1976)
30 R. O. Wells, Jr., Editor, Several complex variables (Williams College, Williamstown,
Massachusetts, July/August 1975)
29 Robin Hartshorne, Editor, Algebraic geometry – Arcata 1974 (Humboldt State
University, Arcata, California, July/August 1974)
28 Felix E. Browder, Editor, Mathematical developments arising from Hilbert problems
(Northern Illinois University, Dekalb, May 1974)
27 S. S. Chern and R. Osserman, Editors, Differential geometry (Stanford University,
Stanford, California, July/August 1973)

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/.
