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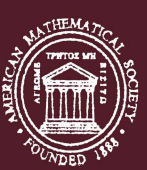
The Mathematical Legacy of Harish-Chandra

A Celebration of Representation Theory and Harmonic Analysis

An AMS Special Session
Honoring the Memory of Harish-Chandra
January 9–10, 1998
Baltimore, Maryland

Robert S. Doran
V. S. Varadarajan
Editors



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The Mathematical Legacy of Harish-Chandra

A Celebration of Representation Theory and Harmonic Analysis

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Harish-Chandra (1923–1983)

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Dedicated to the memory
of Harish-Chandra

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Preface

Harish-Chandra was a mathematician of great power, vision, and remarkable ingenuity. His profound contributions to the representation theory of Lie groups, harmonic analysis, and related areas of mathematics left a rich legacy for researchers that continues today. This volume contains the proceedings of an AMS Special Session entitled *Representation Theory and Noncommutative Harmonic Analysis: A Special Session Honoring the Memory of Harish-Chandra.* This session, held in Baltimore, Maryland, January 9-10, 1998, marked the seventy-fifth anniversary of Harish-Chandra's birth in 1923 and the fifteenth year after his untimely death in 1983 at the age of sixty.

The Table of Contents reveals contributions by an outstanding group of internationally known mathematicians. The papers range from expository and historical surveys to original research articles. The influence of Harish-Chandra's fundamental ideas form a backdrop throughout these papers. All of these articles, which span a rather broad range of mathematics, have been carefully refereed and will not appear elsewhere. Four Memorial Talks were given at Harish-Chandra's 1983 Memorial Service held at the Institute for Advanced Study by mathematicians who knew Harish-Chandra and his work well. These talks contain personal recollections of Harish-Chandra and were collected in a small memorial pamphlet by the Institute. They are published more widely here for the first time with the permission of the authors and the Institute. Two additional articles, *Some Recollections of Harish-Chandra* by Armand Borel, and *Harish-Chandra's c-Function: A Mathematical Jewel* by Sigurdur Helgason have been reprinted in this collection with permission of the authors and the publishers.

Most of the papers that appear herein are expanded versions of the authors' talks in Baltimore and were written especially for this volume. The lead-off paper, *Harish-Chandra, His Work, and its Legacy* by the second named co-editor, was also written especially for this volume at the request of the first named co-editor. The paper is based on his one hour opening address of the Special Session. The editors thank Rebecca Herb for her expository paper, *An Elementary Introduction to Harish-Chandra's Work*, which provides newcomers with a brief point of entry to some of Harish-Chandra's most basic mathematical ideas together with references to explore his work further.

The editors wish to express their sincere gratitude and thanks to all who made this volume possible. Special thanks go to the speakers for their beautiful talks and their willingness to spend many hours writing them up so that the results would be available to the larger mathematical community. We also acknowledge the hard work and help of the referees. We are indebted to Shirley Doran, our technical secretary, for a thoroughly professional job. Finally, we wish to thank Sergei Gelfand, Christine Thivierge, and the dedicated staff at the American Mathematical Society for their efforts in publishing these proceedings.

Robert S. Doran
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