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Smooth Ergodic Theory and Its Applications

Proceedings of the AMS Summer Research Institute
on Smooth Ergodic Theory and Its Applications
July 26–August 13, 1999
University of Washington, Seattle

Anatole Katok
Rafael de la Llave
Yakov Pesin
Howard Weiss
Editors

American Mathematical Society

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Preface

This volume is primarily based on the mini-courses and lectures presented at the American Mathematical Society Summer Research Institute (SRI) on Smooth Ergodic Theory and Its Applications in Seattle, Washington, on July 26–August 13, 1999. Smooth Ergodic Theory studies the statistical properties of differentiable dynamical systems. The origin of this theory can be traced to the seminal works of H. Poincaré, and later many great mathematicians contributed to its development.

One of the most systematically developed areas is the dynamics of systems exhibiting hyperbolic behavior and preserving a natural invariant measure (e.g., absolutely continuous measure, SRB measure, or a more general Gibbs measure). Other major parts of smooth ergodic theory are the study of persistence of stable behavior in certain systems (e.g., KAM-theory) and various rigidity phenomena for dynamical systems and smooth actions of more general groups (e.g., invariant measures for unipotent homogeneous systems, entropies of geodesic flows on negatively curved manifolds, and smooth conjugacy for Anosov systems). Recently, there has been significant progress in a number of other areas including conformal dynamics, coupled map lattices, dimension theory of dynamical systems, and stable ergodicity theory.

Smooth ergodic theory, especially the theory of nonuniformly hyperbolic systems, provides the principle paradigm for the rigorous study of complicated or *chaotic* behavior in deterministic systems. This paradigm asserts that if a non–linear dynamical system exhibits sufficiently pronounced exponential behavior, then global properties of the system can be deduced from studying the linearized system. One can then obtain detailed information on topological properties (such as the growth of periodic orbits, topological entropy, and dimension of invariant sets including attractors), as well as statistical properties (such as the existence of invariant measures, asymptotic behavior of typical orbits, ergodicity, mixing, decay of correlation, and measure-theoretic entropy). Smooth ergodic theory also provides a foundation for numerous applications throughout mathematics (e.g., to Riemannian geometry, number theory, Lie groups, and partial differential equations) as well as other sciences.

During the past decade there have been several major new developments in smooth ergodic theory which have attracted substantial new interest to the field from mathematicians as well as scientists using dynamics in their work. In spite of an impressive literature, it has been extremely difficult for a student, or even an established mathematician who is not an expert in the area, to acquire a working knowledge of smooth ergodic theory and to learn how to use its tools. Accordingly, the Summer Research Institute had a strong educational component including ten mini-courses on various aspects of smooth ergodic theory and its applications which were presented by experts in their fields.

The purpose of this volume is two-fold: first, to serve as a useful gateway to smooth ergodic theory for students and nonspecialists, and, second, to provide a state-of-the-art report on important aspects of the subject at the turn of the millennium. It is a testimony to the vitality of the field that, in spite of the size of the volume, some very important areas had to be omitted from the Summer Research Institute in order to achieve the goal of a sufficiently comprehensive coverage of the core areas.

The structure of the volume reflects this pedagogical aspect and purpose. For reader's convenience the volume is divided into three parts: lecture notes, survey–expository articles, and original research articles.

The first part consists of three long expositions with proofs, each aimed to serve as a comprehensive and largely self-contained introduction to a particular area of smooth ergodic theory. Two of these expositions (by Barreira and Pesin and by de la Llave) represent greatly expanded notes of mini-courses given in Seattle while the third (by Katok and Robinson) is an updated and expanded version of an earlier unpublished manuscript.

The second part is in turn subdivided into thematic sections. All but two of the articles in this part are based either on mini-courses or on survey lectures given in Seattle. The exceptions are the article by Kalinin and Katok written specially for the volume, and by Pöschel which is an update of an earlier unpublished manuscript.

The third part contains a number of original contributions either presented at the research sessions during the Seattle meeting or closely related to the topics discussed there.

All papers in this volume have been refereed by experts whose identities were not disclosed to the authors, and various improvements were made following suggestions of the referees. On behalf of all authors we would like to express our gratitude to the referees for their valuable efforts.

The Institute was funded by the National Science Foundation and sponsored by the American Mathematical Society. The editors/scientific organizers thank Dr. James Maxwell, Associate Executive Director of the AMS, for his considerable efforts in organizing the conference. We also thank the local AMS conference organizer Mr. Wayne Drady who did a superb job. Finally, we would like to thank Ilie Ugarcovici, who very quickly and efficiently prepared files of the papers for publication.

Editors and Scientific Organizers of the Summer Research Institute,
A. Katok, R. de la Llave, Ya. Pesin (managing editor), H. Weiss

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