Proceedings of Symposia in
PURE MATHEMATICS

Volume 72, Part 1

Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot

Analysis, Number Theory, and Dynamical Systems

Michel L. Lapidus (Managing Editor)
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http://dx.doi.org/10.1090/pspum/072.1

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Proceedings of a special session held in January 2002 during the Annual Meeting of the American Mathematical Society in San Diego, California entitled *Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot.* Some of the contributions to this volume are by speakers from a related special session on *Fractal Geometry, Number Theory, and Dynamical Systems* held during the first Joint Meeting of the Société Mathématique de France and the American Mathematical Society at the École Normale Supérieure de Lyon in July 2001.

2000 *Mathematics Subject Classification.* Primary 28A12, 28A80, 11K60, 11M41, 37A45, 37F45, 60G57, 60J45, 68U05, 82C41.

Library of Congress Cataloging-in-Publication Data

Fractal geometry and applications : a jubilee of Benoît Mandelbrot / Michel L. Lapidus (managing editor), Machiel van Frankenhuijsen, editors.

p. cm. — (Proceedings of symposia in pure mathematics ; v. 72)

Includes bibliiographical references.

Contents: pt. 1. Analysis, number theory, and dynamical systems – pt. 2. Multifractals, probability and statistical mechanics, applications.

ISBN 0-8218-3292-1 (set : acid-free paper) – ISBN 0-8218-3637-4 (pt. 1 : acid-free paper) – ISBN 0-8218-3638-2 (pt. 2 : acid-free paper)

1. Measure theory–Congresses. 2. Fractals–Congresses. 3. Ergodic theory–Congresses. I. Mandelbrot, Benoit B. II. Lapidus, Michel L. (Michel Laurent), 1956– III. van Frankenhuysen, Machiel, 1967– IV. Series.

QA325.F73 2004
515′.42–dc22 2004057084

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10 9 8 7 6 5 4 3 2 1 09 08 07 06 05 04

Contents

Part 1. Analysis, Number Theory, and Dynamical Systems

ANALYSIS

NUMBER THEORY

DYNAMICAL SYSTEMS

Contents

Part 2. Multifractals, Probability and Statistical Mechanics, Applications

MULTIFRACTALS

PROBABILITY AND STATISTICAL MECHANICS

APPLICATIONS

Preface

The goal of this volume is to present to interested mathematicians and other scientists a cross-section of recent research in the field of fractal geometry and its applications, either within mathematics or to other sciences. The volume itself arose in part as the proceedings of a special session held in January 2002 during the Annual Meeting of the American Mathematical Society in San Diego, California, and entitled *Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot.* [The award (to MLL) of a grant by the Office for Naval Research (ONR–N00014-02-1-0168) to partially support this conference and its aftermath is hereby gratefully acknowledged.] The purpose of that conference was to bring together leading researchers working in this field as well as to honor Benoît Mandelbrot on the occasion of the jubilee of his thesis. A more detailed description of the goals and contents of this two-part book is provided in the long general introduction to this volume written by one of us (MLL) and placed just after this preface (in Part 1).

We hope that the mixture of tutorial articles, research expository papers and research original articles found in the two parts of this volume will be useful to the experts and the non-experts alike (including graduate students and postdocs). It should, in particular, demonstrate the vitality and diversity of the field of fractal geometry (taken in a broad sense) and hopefully motivate newcomers to further investigate some of the many open problems and potential research directions proposed throughout the volume.

June 2004
Michel L. Lapidus
Machiel van Frankenhuijsen

List of Tributes

Titles in This Series

72 **Michel L. Lapidus and Machiel van Frankenhuijsen, Editors,** Fractal geometry and applications: A jubilee of Benoît Mandelbrot, Parts 1 and 2 (San Diego, California, 2002 and École Normale Supérieure de Lyon, 2001)

71 **Gordana Matić and Clint McCrory, Editors,** Topology and Geometry of Manifolds (University of Georgia, Athens, Georgia, 2001)

70 **Michael D. Fried and Yasutaka Ihara, Editors,** Arithmetic fundamental groups and noncommutative algebra (Mathematical Sciences Research Institute, Berkeley, California, 1999)

69 **Anatole Katok, Rafael de la Llave, Yakov Pesin, and Howard Weiss, Editors,** Smooth ergodic theory and its applications (University of Washington, Seattle, 1999)

68 **Robert S. Doran and V. S. Varadarajan, Editors,** The mathematical legacy of Harish-Chandra: A celebration of representation theory and harmonic analysis (Baltimore, Maryland, 1998)

67 **Wayne Raskind and Charles Weibel, Editors,** Algebraic K-theory (University of Washington, Seattle, 1997)

66 **Robert S. Doran, Ze-Li Dou, and George T. Gilbert, Editors,** Automorphic forms, automorphic representations, and arithmetic (Texas Christian University, Fort Worth, 1996)

65 **M. Giaquinta, J. Shatah, and S. R. S. Varadhan, Editors,** Differential equations: La Pietra 1996 (Villa La Pietra, Florence, Italy, 1996)

64 **G. Ferreyra, R. Gardner, H. Hermes, and H. Sussmann, Editors,** Differential geometry and control (University of Colorado, Boulder, 1997)

63 **Alejandro Adem, Jon Carlson, Stewart Priddy, and Peter Webb, Editors,** Group representations: Cohomology, group actions and topology (University of Washington, Seattle, 1996)

62 **János Kollár, Robert Lazarsfeld, and David R. Morrison, Editors,** Algebraic geometry—Santa Cruz 1995 (University of California, Santa Cruz, July 1995)

61 **T. N. Bailey and A. W. Knapp, Editors,** Representation theory and automorphic forms (International Centre for Mathematical Sciences, Edinburgh, Scotland, March 1996)

60 **David Jerison, I. M. Singer, and Daniel W. Stroock, Editors,** The legacy of Norbert Wiener: A centennial symposium (Massachusetts Institute of Technology, Cambridge, October 1994)

59 **William Arveson, Thomas Branson, and Irving Segal, Editors,** Quantization, nonlinear partial differential equations, and operator algebra (Massachusetts Institute of Technology, Cambridge, June 1994)

58 **Bill Jacob and Alex Rosenberg, Editors,** K-theory and algebraic geometry: Connections with quadratic forms and division algebras (University of California, Santa Barbara, July 1992)

57 **Michael C. Cranston and Mark A. Pinsky, Editors,** Stochastic analysis (Cornell University, Ithaca, July 1993)

56 **William J. Haboush and Brian J. Parshall, Editors,** Algebraic groups and their generalizations (Pennsylvania State University, University Park, July 1991)

55 **Uwe Jannsen, Steven L. Kleiman, and Jean-Pierre Serre, Editors,** Motives (University of Washington, Seattle, July/August 1991)

54 **Robert Greene and S. T. Yau, Editors,** Differential geometry (University of California, Los Angeles, July 1990)

53 **James A. Carlson, C. Herbert Clemens, and David R. Morrison, Editors,** Complex geometry and Lie theory (Sundance, Utah, May 1989)

52 **Eric Bedford, John P. D'Angelo, Robert E. Greene, and Steven G. Krantz, Editors,** Several complex variables and complex geometry (University of California, Santa Cruz, July 1989)