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Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot

Analysis, Number Theory, and Dynamical Systems

Michel L. Lapidus (Managing Editor)

Machiel van Frankenhuysen

Editors



American Mathematical Society

Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot

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Providence, Rhode Island

Proceedings of a special session held in January 2002 during the Annual Meeting of the American Mathematical Society in San Diego, California entitled *Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot*. Some of the contributions to this volume are by speakers from a related special session on *Fractal Geometry, Number Theory, and Dynamical Systems* held during the first Joint Meeting of the Société Mathématique de France and the American Mathematical Society at the École Normale Supérieure de Lyon in July 2001.

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Preface

The goal of this volume is to present to interested mathematicians and other scientists a cross-section of recent research in the field of fractal geometry and its applications, either within mathematics or to other sciences. The volume itself arose in part as the proceedings of a special session held in January 2002 during the Annual Meeting of the American Mathematical Society in San Diego, California, and entitled *Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot*. [The award (to MLL) of a grant by the Office for Naval Research (ONR–N00014-02-1-0168) to partially support this conference and its aftermath is hereby gratefully acknowledged.] The purpose of that conference was to bring together leading researchers working in this field as well as to honor Benoît Mandelbrot on the occasion of the jubilee of his thesis. A more detailed description of the goals and contents of this two-part book is provided in the long general introduction to this volume written by one of us (MLL) and placed just after this preface (in Part 1).

We hope that the mixture of tutorial articles, research expository papers and research original articles found in the two parts of this volume will be useful to the experts and the non-experts alike (including graduate students and postdocs). It should, in particular, demonstrate the vitality and diversity of the field of fractal geometry (taken in a broad sense) and hopefully motivate newcomers to further investigate some of the many open problems and potential research directions proposed throughout the volume.

June 2004
Michel L. Lapidus
Machiel van Frankenhuysen

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