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Multiple Dirichlet Series, Automorphic Forms, and Analytic Number Theory

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on Multiple Dirichlet Series
Bretton Woods, New Hampshire
July 11–14, 2005

Solomon Friedberg (Managing Editor)
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Bretton Woods Group photo.

Contents

Preface

This volume represents the proceedings of the Bretton Woods Workshop on Multiple Dirichlet Series which took place at the Mount Washington Hotel in Bretton Woods, New Hampshire during the period July 11–14, 2005. The workshop was organized by Daniel Bump, Solomon Friedberg, Dorian Goldfeld, and Jeffrey Hoffstein, and was funded by an NSF Focussed Research Group grant[1].

Multiple Dirichlet series are Dirichlet series in several complex variables. A multiple Dirichlet series is said to be perfect if it satisfies a finite group of functional equations and has meromorphic continuation everywhere. The earliest examples came from Mellin transforms of metaplectic Eisenstein series and were intensively studied over the last twenty years by the organizers above and their students. More recently, many other examples have been discovered and it appears that all the classical theorems on moments of L-functions as well as the conjectures (such as those predicted by random matrix theory) can now be obtained via the theory of multiple Dirichlet series. Furthermore, new results, not obtainable by other methods, are just coming to light. It was felt that the subject had sufficiently developed that an account of some of the major results to date and the opportunities for the future should be recorded at this time. The pristine environment of the White Mountains and the hospitality of the Mount Washington Hotel provided an ideal venue to bring together researchers from around the world working in multiple Dirichlet series and allied fields.

The workshop was centered around the following four themes:

- An exposition of the main results in the theory of multiple Dirichlet series,
- Moments of zeta and L-functions,
- New examples of multiple Dirichlet series,
- Connections with allied fields.

These themes appear in the papers of this volume in different mixes. The contributions of Brubaker-Bump, Brubaker-Bump-Chinta-Friedberg-Hoffstein, Chinta-Friedberg-Hoffstein, Deitmar, Diaconu-Goldfeld, Masri, Murty-Sinha, and Zhang offer overviews of or new developments concerning multiple Dirichlet series. These papers are presented in the first part of this volume, which is arranged thematically, so that one can obtain an overview of the field by reading the papers consecutively. Almost all of these papers describe connections to related fields as well. The papers of Choie-Diamantis, Ginzburg, Huxley, Ivić, Jutila-Motohashi, Motohashi, Mozzochi, and Rudnick-Soundararajan concern the allied fields of automorphic forms

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and analytic number theory. These papers are presented in the second part of the volume, and are arranged alphabetically. The theme of moments of zeta and L-functions appears in papers in both parts of the volume, providing one indication of the connection between the theory of multiple Dirichlet series and allied fields.

We would like to thank the National Science Foundation for funding the workshop on multiple Dirichlet series and the Mount Washington Hotel for hosting it. We express our deep appreciation to the AMS, in particular to Christine Thivierge, for making it possible to publish this proceedings. Also, we would like to thank C. J. Mozzochi for all the wonderful conference pictures and Steven J. Miller for preparing TEX files of many of the conference talks as they were being delivered. Finally, special thanks go to Doreen Pappus of Brown University, without whose help the running of this conference would not have been possible.

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75 **Solomon Friedberg (Managing editor), Daniel Bump, Dorian Goldfeld, and Jeffrey Hoffstein, Editors,** Multiple Dirichlet series, automorphic forms, and analytic number theory (Bretton Woods, New Hampshire, July 11–14, 2005)

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TITLES IN THIS SERIES

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