

Proceedings of Symposia in PURE MATHEMATICS

Volume 76, Part 1

Spectral Theory and Mathematical Physics: A Festschrift in Honor of Barry Simon's 60th Birthday

Quantum Field Theory,
Statistical Mechanics, and
Nonrelativistic Quantum Systems

Fritz Gesztesy (Managing Editor)

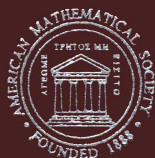
Percy Deift

Cherie Galvez

Peter Perry

Wilhelm Schlag

Editors



American Mathematical Society

**Spectral Theory and
Mathematical Physics:
A Festschrift in Honor of
Barry Simon's 60th Birthday**

**Quantum Field Theory,
Statistical Mechanics, and
Nonrelativistic Quantum Systems**

SPECTRAL THEORY & MATHEMATICAL PHYSICS

A Conference in Honor of
**Barry Simon's
60th Birthday**

Monday, March 27 - Friday, March 31, 2006 • Ramo Auditorium • California Institute of Technology

Lectures at the conference will present mainly surveys of areas reflecting Barry Simon's scientific interests over the years in Mathematical Physics and Analysis.

Confirmed Speakers.

Michael Aizenman (Princeton)
Yosi Avron (Technion, Haifa)
Alexei Borodin (Caltech, Pasadena)
Jean Bourgain (IAS, Princeton)
David Damanik (Caltech, Pasadena)
E. Brian Davies (King's College, London)
Percy Deift (Courant Institute, New York)
Laszlo Erdős (U. Munich)
Charles Fefferman (Princeton)
Richard Froese (UBC, Vancouver)
Jürg Fröhlich (ETH-Zürich)
Christian Gerard (U. Paris Sud, Orsay)
Leonid Golinskii (ILT Kharkov, Ukraine)
Gian Michele Graf (ETH-Zürich)
George Hagedorn (Virginia Tech, Blacksburg)
Evans Harrell (Georgia Tech, Atlanta)
Ira Herbst (U. Virginia, Charlottesville)
Dirk Hundertmark (U. Illinois, Urbana-Champaign)
Vojkan Jaksic (McGill U., Montreal)
Svetlana Jitomirskaya (UC Irvine)
Sergey Khrushchev (Atılım U., Ankara)
Rowan Killip (UCLA)
Alexander Kiselev (U. Wisconsin, Madison)
Yoram Last (Hebrew U., Jerusalem)
Peter Perry (U. Kentucky, Lexington)
William Reinhardt (U. Washington, Seattle)
Lon Rosen (UBC, Vancouver)
Wilhelm Schlag (Caltech, Pasadena)
Vilmos Totik (U. South Florida, Tampa; U. Szeged)

Topics.

Monday: Quantum Field Theory, Statistical Mechanics and Random Matrix Theory
Tuesday: General Nonrelativistic QM, including N-Body Systems and Resonances
Wednesday: Nonrelativistic QM in Electric and Magnetic Fields; Semi-Classical Limit
Thursday: Singular Continuous Spectrum, Random and Ergodic Schrödinger Operators
Friday: Orthogonal Polynomials; Non-Selfadjoint Spectral Theory

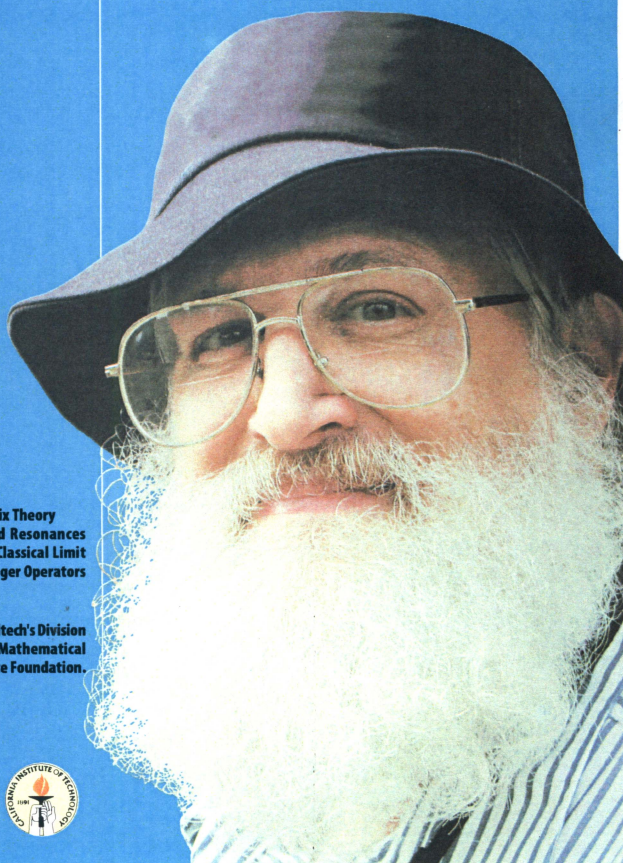
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Registration and further information.

www.math.caltech.edu/mp2006conf.html

Contacts.

Cherie Galvez (SimonFest@caltech.edu)
Fritz Gesztesy (fritz@math.missouri.edu)



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California Institute of Technology
Pasadena, California

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Preface

This Festschrift is dedicated to Barry Simon on the occasion of his sixtieth birthday.

Barry Simon was born on April 16, 1946, to parents Minnie and Hy Simon. Together with his older brother Rick, he grew up in a one-bedroom apartment on Kings Highway in Brooklyn, New York. Barry attended James Madison High School in Brooklyn. He obtained a perfect score on the American High School Mathematics Examination (AHSME) in 1962.

Barry's decision to go into physics was influenced by Sam Marantz, an inspired physics teacher he had in high school. Mr. Marantz was also the one who suggested he apply to Harvard; before that Barry's first choice for undergraduate studies was Caltech.

Barry received his undergraduate education at Harvard. He was elected to Phi Beta Kappa in his junior year and graduated with a B.A. *summa cum laude* from Harvard College in 1966. He was a Putnam Competition Winner (top six) in 1965. He then entered Princeton University and received his Ph.D. in Physics in 1970 under the direction of Arthur Wightman. Barry would not have thought of Princeton for graduate school except that when he asked George Mackey if there was anyone who combined physics with mathematical proof, Mackey told him about Wightman.

In 1968, Martha Katzin entered Princeton Graduate School in Mathematics where she eventually got her Ph.D. under the direction of Robert Gunning. In January 1971, shortly after Barry became an Assistant Professor at Princeton, they were married. Martha has taught in a variety of schools in the New York/New Jersey area and then California and is currently a Lecturer in Mathematics at California State University at Northridge. Barry and Martha have five children (Rivka, Benjamin Pesach, Zvi, Aryeh and Chana) and, so far, seven grandchildren.

Barry's subsequent career at the Departments of Mathematics and Physics at Princeton University, after being an instructor in Mathematics there in 1969/70, reads as follows: Assistant Professor (1970–72), Associate Professor (1972–76), and Professor (1976–81). He was on leave from Princeton as a Visiting Professor at IHES, Bures-sur-Yvette, France; the CNRS, Marseille, France; and the ETH, Zürich, Switzerland, in 1972/73; and at Yeshiva University, New York City, in 1976/77.

During the academic year 1980/81, he was again on leave as a Sherman Fairchild Distinguished Visiting Scholar at the California Institute of Technology, Pasadena,

California. He joined Caltech as Professor of Mathematics and Theoretical Physics in 1981. Since 1984 he has held the position of IBM Professor of Mathematics and Theoretical Physics at Caltech. He was a Distinguished Visitor at the University of California at Irvine in 2002 and the Lady Davis Visiting Professor at Hebrew University, Jerusalem, in 2004/05.

Barry has received numerous honors, among which we mention the following: Sloan Foundation Fellowship, 1972–76; Invited Speaker, International Congress of Mathematicians (ICM), 1974 (Vancouver); Invited Speaker, American Physical Society (APS), 1975 (Anaheim) and 1980 (Los Angeles); Invited Speaker, American Mathematical Society (AMS), 1977 (New York) and 1990 (Louisville); Invited Speaker, International Association of Mathematical Physics (IAMP), 1981 (Berlin); the Medal of the International Academy of Atomic and Molecular Science, 1981; the Stampacchia Prize, 1982; a Guggenheim Fellowship, 1988/89; Corresponding Member of the Austrian Academy of Sciences, 1990; D.Sc.h.c., Technion–Israel Institute of Technology, Haifa, Israel, 1999; Fellow of the American Academy of Arts & Sciences, 2005; Honorary Fellowship of the University of Wales Swansea, 2006.

From Barry’s vast administrative experience we mention, in particular, his position as Chair of the Scientific Organizing Committee for the International Congress of Mathematical Physics at Swansea, 1988; Vice President of the AMS, 1988/89; Executive Officer (Department Chair) for Mathematics, Caltech, August 1997–July 2003.

Barry has served as an Associate Editor or Editor of a variety of journals and book series. Currently, he serves on the editorial board of the *Journal of Operator Theory* (since 1979), *Communications in Mathematical Physics* (since 1979), Cambridge University Press (since 2002), *Journal of Approximation Theory* (since 2005), and *Inverse Problems and Imaging* (since 2006).

Barry supervised eleven Ph.D. students at Princeton University and fifteen Ph.D. students at Caltech so far. Throughout his career, Barry has mentored many scientists. His webpage of “mentees” lists 43 names. According to the Mathematics Genealogy Project, he has 73 mathematical descendants.

At this point, Barry has about 108 coauthors and he lists over 350 papers and proceedings contributions. Thus far he has published fourteen books in science (plus four computer books) and co-edited one Festschrift volume.

With so many co-authors, Barry has had a tremendous impact on people and their careers, but also on science itself, both through his vast research accomplishments and through his highly influential books. In particular, his four-volume book series, *Methods of Modern Mathematical Physics, I–IV*, written jointly with Mike Reed in the period 1972–79, played the role that Courant–Hilbert I, II did for the preceding generation. It has educated and profoundly influenced a whole generation of mathematical physicists, and will continue to play that distinguished role in the foreseeable future. His other books, most notably *Functional Integration and Quantum Physics* and *Trace Ideals and Their Applications*, are standard references that have influenced research in many areas.

As a sign of Barry's impact, we note that the last time we checked, the author citation database of MathSciNet (based on citations in bibliographies of papers reviewed in the last five years or so) said that Barry is cited 5,217 times by 2,430 authors.

A gifted and lucid expositor of science, encyclopedic in his knowledge, and a grand master of mathematical structure and abstract analysis, Barry has been a phenomenal force in mathematical physics and applied mathematics in the broadest sense. As the acknowledged and undisputed authority on spectral theory for Schrödinger operators, he created a school, and much of what we know today about spectral phenomena, including exotic aspects in connection with singular continuous spectra, is due to Barry and coworkers around him. While in recent years Barry's focus has been primarily on problems in nonrelativistic quantum mechanics and orthogonal polynomials, his research has had great breadth, with significant contributions to quantum field theory, statistical mechanics, and abstract functional analysis.

This Festschrift had its origins in a conference called SimonFest held at Caltech, March 27–31, 2006, but it is not a Proceedings volume in the usual sense. Barry requested that both his birthday conference and this Festschrift concentrate not so much on what the speaker or writer has done recently, but instead on reviews of the state of the art, with a focus on recent developments and open problems. While the number of speakers at Simonfest was originally limited by the number of hour slots in a full week, the contributions to this Festschrift contain a few additionally selected reviews. In the end, the bulk of the articles in this Festschrift are of this state of the art survey form with a few that instead review Barry's contributions to a particular area.

Here in Part 1, the focus is on the areas of Quantum Field Theory, Statistical Mechanics, Nonrelativistic Two-Body and N -Body Quantum Systems, Resonances, Electric and Magnetic Fields, and Semiclassical Limit. In Part 2, the focus is on the areas of Random and Ergodic Schrödinger Operators, Singular Continuous Spectrum, Orthogonal Polynomials, and Inverse Spectral Theory.

Those who know Barry are aware of the fact that one of his secret weapons to be so productive is Cherie Galvez, his superb secretary (aka Administrative Assistant). We borrowed her services for this Festschrift. She not only was her usual efficient self, but was a wonderful editor and TeXpert. She also did a great job of dealing with author corrections and making sure that the styles of the different manuscripts meshed. We appreciate her hard work and thank her for her efforts. Since we wanted to be able to thank her, Cherie is not a signer of this Preface, although she is rightly listed as one of this Festschrift's editors.

At SimonFest, Barry told the story that one day, shortly after Dirk Hundertmark (who is German) started as a postdoc, Barry overheard him saying on the phone "Cherie is her weight in gold worth." He then said to Dirk: "You made two mistakes. First, you used German grammar. Second, Cherie is not worth her weight in gold; she is worth my weight in gold."

We are grateful to Sergei Gelfand, Christine Thivierge, and the staff at AMS for their support throughout the preparations of this volume. We also thank all the authors for their contributions and the referees for their invaluable assistance.

We sincerely thank the following sponsors of SimonFest for their financial support: Caltech's Center for the Mathematics of Information (CMI); Caltech's Division of Physics, Mathematics, and Astronomy; International Association of Mathematical Physics (IAMP); International Union of Pure and Applied Physics (IUPAP); U.S. National Science Foundation (NSF).

Happy Birthday, Barry!

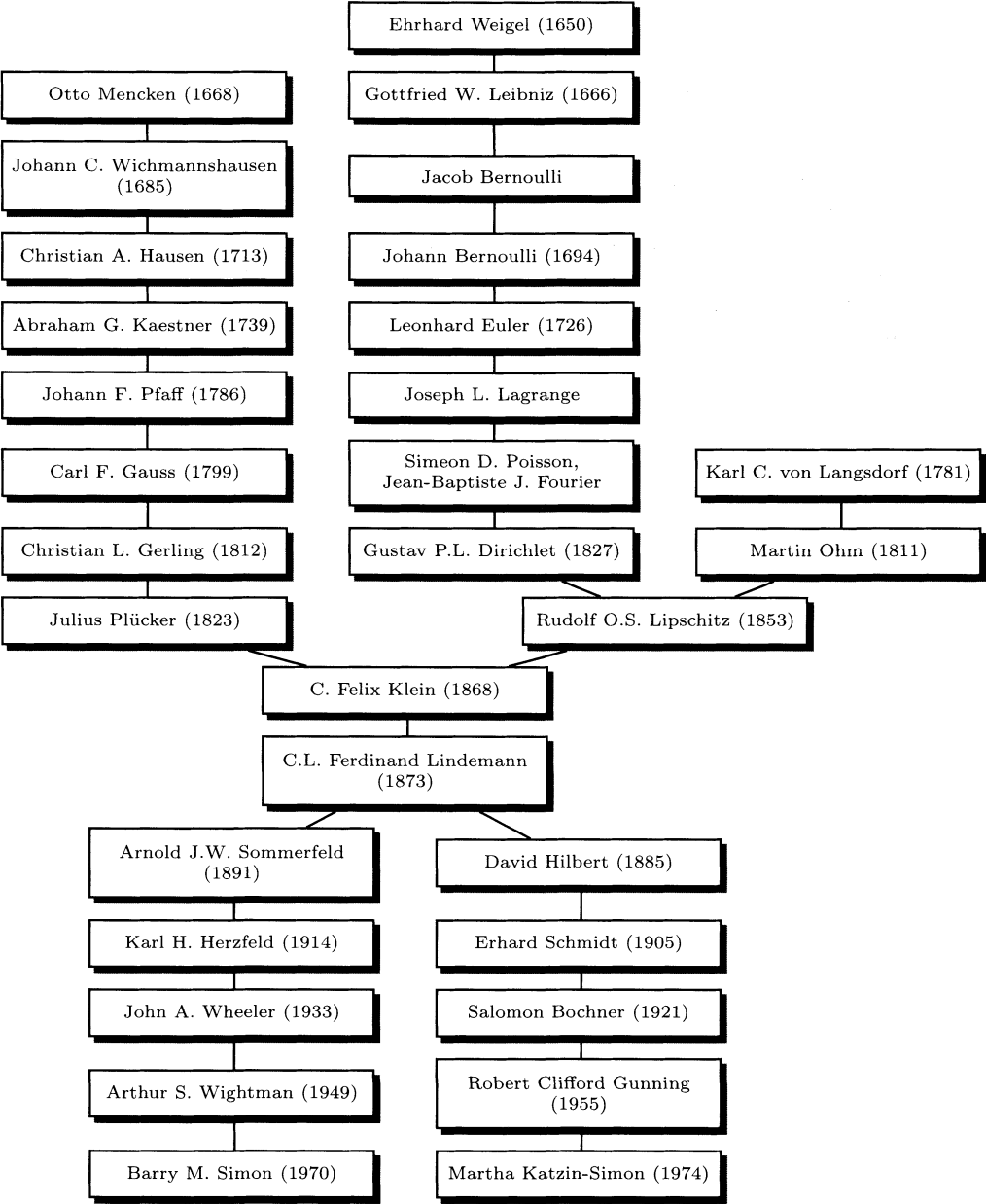
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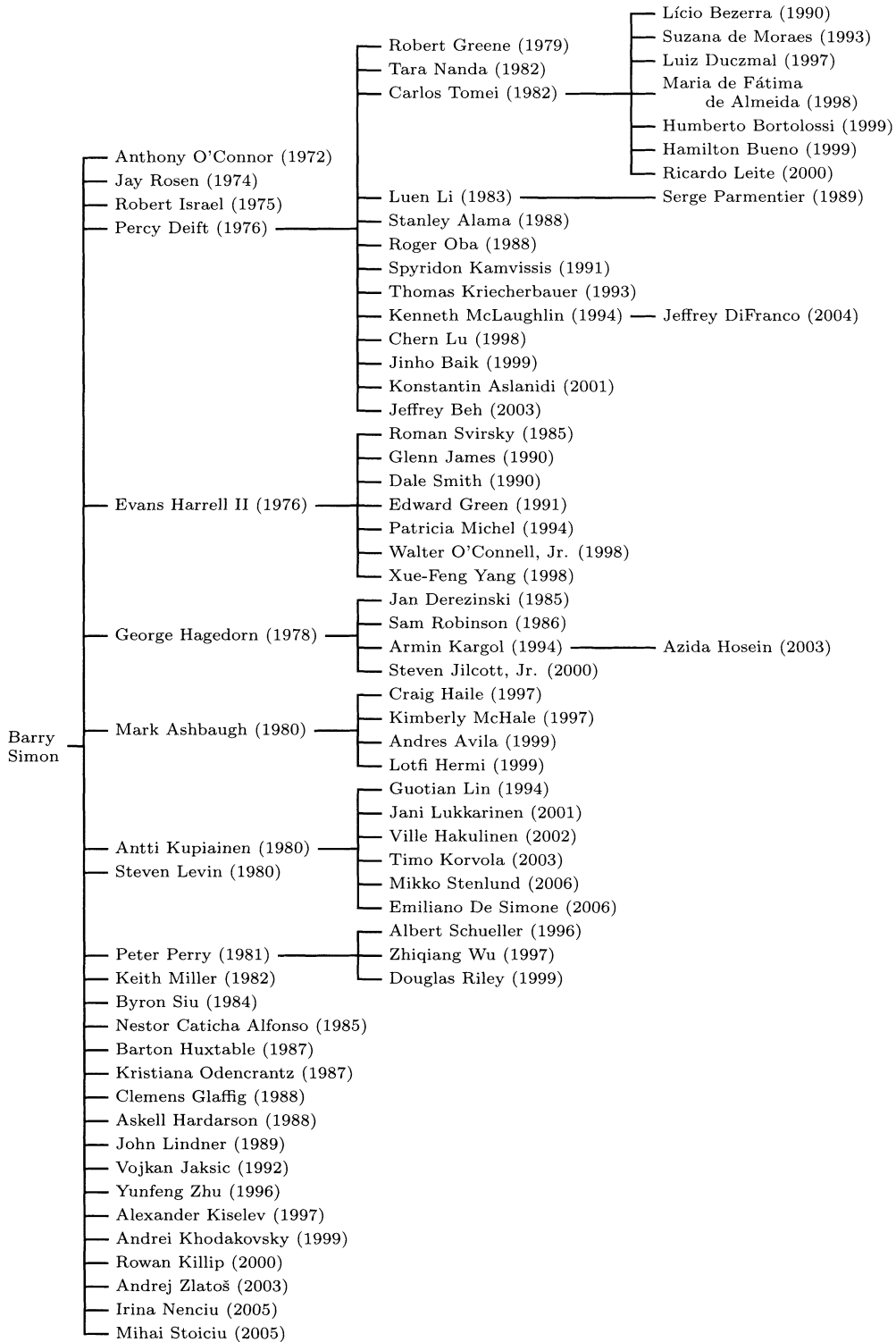
October 2006

A Mathematical Genealogy

The following pages include the mathematical ancestries of Barry Simon and Martha Katzin-Simon as well as the mathematical descendants, to date, of Barry Simon.* As the formal advisor system is about 150 years old, relationships older than this must be regarded with caution. Doctorates were awarded in years shown in parentheses. Information for this genealogy was obtained from the Mathematics Genealogy Project: <http://genealogy.math.ndsu.nodak.edu/>.

*The editors are grateful to Stephen L. Clark for compiling and formatting these mathematical family trees.





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A Selection of Barry Stories

The organizers of SimonFest invited Barry's friends, colleagues, and potential conference attendees to submit "Barry Stories" to a webpage <http://www.math.caltech.edu/SimonFest/stories.html>. The response was so extensive that we can't include them all here (but they will remain posted at the website for some time) so we asked Barry to pick his favorites.

– *The Editors*

In the early eighties, Murph Goldberger was the president of Caltech. Murph was a great fan of Barry since their Princeton days and concocted a scheme to try and lure Barry from Princeton to Caltech. The scheme started innocently enough by inviting Barry to spend a year in sunny California as a Fairchild Scholar. Murph's plan indeed worked like a charm and a year of leave turned out to be a permanent move.

Princeton, at the time, was the Mecca of mathematical physics. It probably had the largest math physics group. Caltech, at the time, and as far as mathematical physics was concerned, was like the rest of Saudi Arabia: a desert (with a lot of oil). As you probably know, Barry thrives in company. So he looked for someone to accompany him to the desert. After a lot of fruitless searching, Barry must have remembered that I had some practical experience living in a real desert and asked me to join him. The arrangement was curious. I must have been the only Princeton Assistant Professor who had his paycheck sent to California. And, I remember that the bank teller once asked me if I could swing such a trick for her. She preferred the Bahamas, however. Anyway, since I had no official standing at Caltech, I decided to decorate myself as the Fairbaby Scholar.

The Fairkid year we spent together was a remarkable year, for I had Barry almost all to myself. In Princeton, where I first met Barry as his postdoc, I would get as an appointment the walks to the parking lot, and occasionally a ride to Edison spending a pleasant Shabat with Barry and Martha. How much leisure we had in this Fairkid year is evidenced by the fact that, hold on to your chairs, Barry and I used to go swimming regularly at lunch time at the Caltech swimming pool!

Can you imagine how many papers did not get written because of my bad influence on Barry! Swimming at the Caltech pool was very good for me: Here was a place where I could beat Barry fair and square.

Barry, on the other hand, felt very guilty about this waste of time. To comfort himself he told me that the lost hours at the pool would be paid up by gained

years in better health. Well, what could be more appropriate than to celebrate this wisdom twenty years later, at his 60th anniversary party.

Let me now tell you a story involving Ira Herbst and Barry. I was an overwhelmed Wigner fellow at Princeton, working with Ira and Barry and we were quite close. Ira came back startled from one job interview, I forgot where. The chairman had asked him if he ever wrote a wrong paper. Ira was shocked and said “Of course not!” The reaction of the chairman was unexpected. He said: “Then you do not write enough.” Now, nobody could ever accuse Barry of not writing enough. But I take the credit for writing a wrong paper with Barry. Here is how this came to be: There was a lot of excitement in Caltech when Voyager first sent those spectacular pictures of the rings of Saturn. I had learnt that Saturn appears to have infinitely many rings from a math grad student at Caltech. Barry and I were working at the time on almost-periodic Schrödinger equations and were fascinated with fractal spectra. I knew about Hill lunar theory and Barry had just written a review of Arnold’s book on Mechanics. This led us to make a theory where the near incommensuration of the periods of the moon of Saturn (and also involved the Sun) gave rise to rings with fractal structure. Peter Goldreich, the czar of planetary physics in Caltech, did not like the theory because it was linear stability. But Feynman did because it was simple. The theory could not account for the order of magnitude for the observed gaps and so turned out to be wrong.

Barry’s time axis is divided into BC and AC: Before Caltech and After Caltech. In the BC era Barry had difficulties deciding if his heart lay with constructive field theory, statistical mechanics or quantum mechanics. In the AC period the die was cast in favor of quantum mechanics and spectral theory. Barry also contracted a chronic strain of PC flu.

Since I am an old-timer let me reminisce about the prehistoric BC era.

At Princeton there were the lunch time seminars and the Math-Phys seminars. In the lunch time seminar, Barry was the prima ballerina. (Can you imagine Barry on his tiptoes?) He would normally either tell a new result of his or a new result of someone else. With Dyson, Lieb and Wightman in the audience, most grad students and postdocs were too terrified to expose their slowness if they were to ask an innocent question. Most of the time, nobody dared open his mouth. The notable exception was the fresh grad student from Harvard, Alan Sokal, who never had a fear of authority and was sufficiently smart and self-confident to argue with Barry.

The math phys seminars were a different business. There was an outside speaker most of the time. Wigner would usually show up and ask his typical Wignerian questions. Barry would sit in the audience and write a paper. From time to time he would look up from his notes and ask a question that would unsettle most speakers: Someone in the audience seemed to know more about what he was talking about than himself. Sometimes, at the end of the talk, Barry would go to the board and give his version of the proof, which was always slick.

Barry, you are now 60. Most of us probably do not enjoy being reminded about our advanced age, but I think that one of the nice things about you, Barry, is your optimism. You probably enjoy being 60! I wish you fun with math and good health in the next sixty years.

– Yosi Avron

In 1983 Derek Robinson invited me to visit him at the Mathematical Institute in Canberra for a month. As one of the inducements, he mentioned that he had also invited Barry Simon for the same period. The prospect of seeing Barry, Derek and kangaroos was enough to make my decision, and in July 1983 I set off on the gruelling journey.

I was very surprised shortly before my departure to hear from Derek that he was unable to avoid a commitment to go to a conference in Japan, and would not return until ten days after my arrival. Barry arrived about the same time as myself, and I asked him a problem about heat kernels of Schrödinger operators which I had solved in one dimension by a method that could not be extended to higher dimensions. At that time I had read several of the papers on hypercontractivity, a concept that was invented specifically to solve problems in quantum field theory, in which there are an infinite number of degrees of freedom.

Barry listened to my question carefully and agreed that some progress should be possible. The very next time I saw him he told me that he had solved the problem by an improvement of the standard hypercontractive estimates that made use of the finite dimensionality. He then proceeded, without notes, to give me a lecture on the subject, explaining every step in detail, including the infinite summation procedure that allowed one to pass from $L^2 \rightarrow L^p$ bounds to $L^2 \rightarrow L^\infty$ bounds by controlling the constants involved.

Although very impressed, I had the temerity to suggest at the end of his lecture that although he had clearly done what was needed, I did not like his solution aesthetically, and would prefer an account that depended on differential inequalities. I had the feeling that this would yield better constants and be in some sense more natural. On the next occasion Barry rewrote the entire account in this language, and we realized that this was going to have enough ramifications to occupy the entire month. By the time Derek got back we were fully committed to the project, and I now feel embarrassed that I did not spend nearly enough time talking to him. Derek was, however, the person who coined the term ultra-contractivity, which was the focus of almost all of my research over the next ten years.

A research relationship is, of course, not symmetrical, and for Barry it was just one of many different projects he pursued in that decade. Indeed I wonder whether his main memory of the month was quite different. He had gone to Canberra with Martha and his four children; together they might have comprised the entire Orthodox Jewish community there. With typical thoroughness he arranged for kosher meat to be delivered to him from Sydney every few weeks. The Jewish butcher had agreed to prepare a parcel and then put it in a deep freeze before sending it by rail. On one occasion it did not arrive, and I was involved in a very strange mathematical discussion, which was punctuated every hour or so by long conversations between Barry and the delivery service, in which he explained that if his parcel did not arrive within a few hours he would have to throw it away. I believe that eventually everything was okay. Surviving in Canberra for a month with his family while observing Orthodox rules was just another challenge of the type that Barry always seemed to relish. Perhaps the absence of such problems in London, which has a very large kosher Jewish community, was one of the reasons why he has visited me on a number of occasions there.

– E. B. Davies

Close interactions with Barry have their great rewards—and perils!

◇ While Barry’s lightning mental agility, his extraordinary talent to strip the unnecessary clutter surrounding an argument, getting straight to its core, and his remarkable ability to see connections to related topics other than the obvious ones in question, are legendary, the following observations will sound familiar to many of us:

Scenario #1: You joyously walk into Barry’s office to present him with a new idea, just to exit a few minutes later, your idea having been shred to pieces. “Back to the drawing board” is sometimes his comment, with a broad grin on his face. All this sounds more cruel than it is: After all, you have just been saved from going down a cul-de-sac and you can start regrouping!

Scenario #2: You proudly walk into Barry’s office to show him something new. Barry thinks for a second, then jumps up to the blackboard and explains to you in no uncertain terms what you “really had in mind.” That’s great, because at this point you realize a joint paper will eventually be written.

Scenario #3: You march into Barry’s office and this time you’re convinced you have a blockbuster at your fingertips. You start to explain to Barry, and then he says “time out” and silence fills the room. After a bit of eerie silence you realize this time you’re going to write a very nice joint paper with Barry. Of course, after a few more moments scenario #2 will be repeated, but that’s quite alright!

Scenario #4: Barry asks you into his office and explains what he was struggling with lately. (He likes to put it this way, though: “I was banging my head against the wall about this...”) After he suggests jointly working on this you return to your office with a big puzzle in your hands. Those rare instances in which you can actually do the job asked of you and complete the argument are priceless.

◇ Barry likes to pick on me since I’m usually not afraid of computing anything, well, almost anything (while he doesn’t have the patience to do so!). So once in July of 1997, he confessed to me that he had a terrible contradiction in his long manuscript on “The classical moment problem as a self-adjoint finite difference operator” (it later appeared in *Advances in Math.* **137**, 82–203 (1998)), but he just couldn’t find the error. So I was supposed to look at this. It was an intricate puzzle! I spent a day on it and well after midnight was sure I had found the error. So I e-mailed him what I thought was the culprit and slumped home to the apartment. Next morning I opened my e-mail and there was Barry pointing out that I was dead wrong. It was “back to the drawing board” as he still grinned during our brown bag lunch meeting that day. I was dejected! Well, I had another day before going away with my wife to Hawaii, and I was not about to let this ruin our vacation! So I frantically computed like a dog and finally saw the light: This was it! I decided to treat myself to dinner and left after e-mailing him my second attempt to find the culprit. After returning from dinner late that night, I had received quite a different message from Barry. It started out: THANK YOU, THANK YOU, ... and went on like this for half a page.

– Fritz Gesztesy

Below is an amusing little story that shows Barry's good sense of humor and how quickly he could think of a clever comment.

Barry had many graduate students at Princeton. Although he treated us well, he used to kid around about abusing us or treating us as some sort of subhumans. It was all in good fun, and I always found it amusing.

In November 1976, the night that Jimmy Carter was elected president, Barry invited some of us to to have dinner and watch the election results on the television at the Simons' house in Edison, New Jersey. Shortly after I arrived, I was hanging my coat up in the closet when Barry's daughter Rivka pointed at me and said, "Daddy, what's that man's name?" Without the slightest hesitation, Barry replied, "Rivka, that's not a man, that's a graduate student!"

– *George Hagedorn*

* * *

◊ One of the rare categories in which I can compete pretty well with Barry is in the execrability of our handwriting. I was at MIT as a postdoc when we wrote our long paper on the Stark effect. Back in those days, one actually wrote articles by hand and a secretary, using a device called a "typewriter," turned them into manuscripts, leaving blanks for formulae to be inserted. As a lowly postdoc I didn't get first pick of the secretarial staff, and the manuscript ended up in the hands of a well-intentioned but struggling secretary who would produce about one page per day, which was usually sent back multiple times with corrections, often amusing. One day a favorite adjective of Barry's, "operator-theoretic," came back as "operator neurotic," and I knew the manuscript was taking its toll on her. With lots of encouragement and little gifts she finished the manuscript after months of work, as the term ended. But she didn't return the next term—it was doubtless the last mathematical manuscript she ever typed.

◊ Barry may or may not have said "This is my cross to bear" according to Ed Nelson, but might he ever have been a cross bear? The first time Barry's bearlikeness crossed my mind was at a party at his home in New Jersey, where even graduate students were invited, and little Rivka appeared looking for one of her toys. I think it may have been a stuffed animal, a "big horse" or something like that. Finding the toy was way beyond the powers of a graduate student, but Barry walked by in the next room so I sent Rivka in that direction, telling her "Just follow the big bear, and he'll find your toy." Bears have acute hearing, and I remember him growling, "I heard that!" but Rivka didn't hesitate at my description of her father.

Many years later Barry gave a seminar at Georgia Tech. In the audience was a colleague from another university, whose rather nice early work had been discovered and promoted by Barry, but who had been inactive for some time while struggling with mental illness. At the end of the seminar the visitor asked some rather peculiar questions, which went on and on. Barry handled them with his usual aplomb, but clearly something was amiss, so after everyone left I took my friend off to find his medication. When he was again rational, he told me that half way through the seminar, Barry had turned into an enormous grizzly bear before his eyes, and that

he needed to distract the bear with mathematical questions, or the audience would be devoured!

◇ Barry has always been remarkable for his vast knowledge of mathematics, so it was many years before I can recall ever telling him a published theorem he didn't already know. One day I saw Barry in Princeton shortly after a meeting and told him about an old inequality for PDEs, which, as I could tell from his intent look, was new to him. I said, "It seems to be useful. Do you want to see the proof?" His response was "No, that's OK." Then he went to the board and wrote down a flawless proof on the spot.

– *Evans Harrell*

* * *

◇ Soon after I arrived in Princeton as a postdoc, Barry suggested that we work on proving Borel summability of the perturbation series for the Schwinger functions in two-dimensional ϕ^4 quantum field theory. He had an idea which he showed me and after a couple of days I had another idea. But I soon realized that these two ideas were not enough. Up to this time I was a very hard worker but having thought about how ideas percolate up through the subconscious, I decided to relax and let them do just that. WRONG! A couple of weeks later a preprint arrived from Geneva with a proof.

◇ Barry, Yosi Avron and I were working on magnetic fields. As everyone knows Barry is a very fast worker and he writes up his work even faster. Barry and Yosi felt we should write something and as usual I wanted to get more done first. One day the two of them arrived in my office and began trying to convince me again that we should write something up. I protested, at which point Barry took his hand from behind his back and with a smile produced a manuscript which he had presumably written the night before.

– *Ira Herbst*

* * *

Many of you know Barry from his academic work and community achievements. I have a rather, uh, different perspective. I had the distinct honor and privilege of co-writing a handful of computer books with Barry, including several *Mother of All Windows* books, and *The Mother of All PC Books*.

I'll never forget Barry's squeals of delight when he found foolish inconsistencies in Windows, the way his voice would drop low—and he'd talk fast—when he was working through a particularly snarly problem, and the way he'd rub his hands with glee when a solution suddenly appeared.

Barry wrote about PCs with extraordinary clarity and wit. The Mother books became (in)famous for their casts of characters—no dry technical mumbo-jumbo here. My favorite character from the early Mother books was the eight-legged cockroach (and bug expert) known as Erwin. We gave Erwin the enviable assignment of pinpointing and explaining bugs in Windows, a task for which he was eminently qualified.

Barry describes Erwin's birth this way in *The Mother of All Windows 95* book, from the perspective of uber-iconic Mom, the Mother of the Mother books, as it were:

Erwin has been with me since my first book, pointing out bugs and warning folks about the unthinkable. He's a dashing eight-legged refugee from the 1930s. The physicists in the audience will no doubt recall Erwin Schrödinger, one of the founders of quantum mechanics, who invented a famous "thought experiment" in which a vial of poison gas might (or might not) kill a cat. Schrödinger's cat became justly famous among the psi-squared crowd. A few years ago, a computer book writer had the temerity to refer to Schrödinger's cat in a book submitted to IBM. The IBM Thought Police wouldn't put up with such an offensive allusion to a cuddly animal, so they changed the manuscript, exorcising Schrödinger's cat and introducing in his stead Schrödinger's cockroach, an animal that could be (presumably) sometimes dead without offending the more delicate readers of IBM manuals.

Barry is one of the most intensely intelligent people I've ever met—and delightful, in every sense of the term. Except for the puns. The puns were really, really bad.

Hey, Barry! Wanna write another Windows book? NO! Put DOWN that brickbat!

– Woody Leonhard

* * *

◇ In the late 1960s, Barry was a graduate student in physics at Princeton and attended some courses I taught. I soon learned that I did not need to prepare with great thoroughness; it was enough to get things approximately right and Barry from where he was sitting would tell us how to get them precisely right. I miss Barry.

◇ Once Barry was engaged in an acerbic priority dispute with someone at another institution. I offered to intervene, but Barry said, verbatim, "This is my cross to bear." (I told this story at a meeting in June 2004 and Barry said mildly, "I would never have said such a thing!")

◇ Once Barry wrote a paper on hypercontractive semigroups and when he got it back from the typist, every instance of "hypercontractive" was rendered as "hyper-contraceptive."

◇ Barry received an ugly, uncivil letter from a mathematician complaining that he had not been given sufficient credit in a volume of Reed–Simon for his work on a certain topic. Barry responded with a dispassionate two-page letter calmly reviewing the entire history of the topic and the contribution of each person to it. He concluded the letter with a one-sentence paragraph: "I hope that you will receive this letter in the same spirit in which you sent yours."

◇ Shortly after moving to Caltech, Barry came east for a visit. He said that someone had stolen his attaché case. When we asked whether he had lost anything of importance, he replied, "Only the paper I wrote on the flight."

◇ One Saturday afternoon I saw Barry and Martha strolling with the Cappells. It was a different Barry. Gone were the intense energy and concentration, and in

their place were peace, calm, and repose. I had a strong and somewhat wistful sense of the Sabbath as a gift.

– *Ed Nelson*

* * *

In his third year at Caltech, Barry taught a graduate class in group representations. There were two undergraduates in the class, Zinovy Reichstein and me. I couldn't get up at 9 am for lectures and Zinovy took beautiful notes. I typically photocopied Zinovy's notes every week prior to doing the homework. One week Zinovy had to be away. Forewarned I showed up in class. Barry walked into class, did a double-take when he spotted me and announced "It's undergraduate number conservation!" It never bothered Barry that I did not come to lectures, even though I was one of his undergraduate advisees. When I went to graduate school and expressed some dismay at the less than supportive or encouraging attitudes of the faculty, he wrote me long encouraging letters. I don't think I ever understood how someone as busy as he was, with every minute of his time scheduled, could find the time to write such letters.

Many years later, I was an Assistant Professor at Princeton when Barry came to give a talk. I saw him in the corridor as I was on my way to an undergraduate Senior Thesis oral examination. I greeted him and asked him if he recognized me. He remembered me after a little while, and then was amused (I think) when I told him that I was on my way to an undergraduate oral examination, and reminded him of my first oral examination at Caltech in rigorous statistical mechanics, when he asked me to outline Onsager's solution. He started that particular exam by informing me that he had wanted to put a sophomore through a really rigorous oral ever since his own sophomore year.

– *Vipul Periwal*

* * *

As an undergraduate I took Quantum Mechanics from Barry, little knowing that I would later wind up a student. At the same time, I was taking Functional Analysis from Ira Herbst. The Quantum Mechanics course met on Tuesdays and Thursdays and there was a break in the middle of the lecture. One day I had my copy of *Methods of Modern Mathematical Physics, Vol. I, Functional Analysis* with me. Barry walked up to me during the break and broke into a big smile followed by mock indignation when he saw that I had a copy of Reed–Simon. "Don't read that stuff!" he admonished me. "It'll pollute your mind! It's worse than comic books!" Long before Barry became a department chair he was already a master recruiter. Several years later, I had the good fortune to begin thesis work with Barry as a graduate student.

– *Peter Perry*

* * *

In the mid-1970s, Barry visited Moscow. One day he went into a store to buy some eggs. He handed over a 10-ruble bill to the storekeeper and said “Eggs” in Russian; it was the only word in Russian which he knew. She asked him whether he wanted to spend all 10 rubles (a considerable amount in those days) on eggs. But this was a different phrase which Barry didn’t understand, and in reply he just smiled his charming smile. She then gave him a check for a hundred eggs.

The following day, Barry gave a seminar at the university. It was his last day since he was leaving Moscow the next day. After his talk, he distributed the eggs among the participants. Following the American tradition, undergrads and graduate students received the largest number of eggs and professors received almost nothing.

– *Yakov Sinai*

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