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Volume 80, Part 2

Algebraic Geometry
Seattle 2005

2005 Summer Research Institute
July 25–August 12, 2005
University of Washington,
Seattle, Washington

D. Abramovich
A. Bertram
L. Katzarkov
R. Pandharipande
M. Thaddeus
Editors

American Mathematical Society

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Preface

The 2005 AMS Summer Research Institute on Algebraic Geometry was held at the University of Washington, Seattle, from July 25 through August 12, 2005. The advisory committee consisted of the five of us together with V. Alexeev, J. de Jong, J. Li, and K. Smith. The articles in the present volumes represent the plenary lectures, the Clay lectures, and several surveys related to seminar talks.

The first week of the Institute was centered on moduli problems and the interactions between algebraic geometry, symplectic geometry and string theory. The topic of the second week was classical algebraic geometry with a focus on birational geometry. Arithmetic geometry and characteristic p methods were taken up in the third week. The seminar organizers, V. Alexeev, D. Auroux, D. Ben-Zvi, F. Bogomolov, J. Bryan, H. Esnault, W. Fulton, N. Katz, S. Katz, K. Kedlaya, J. Kollár, R. Lazarsfeld, M. Levine, J. Li, E. Miller, Y. Ruan, K. Smith, B. Siebert, Y. Tschinkel, G. Tian, B. Toën, and E. Viehweg, played an important role in structuring the scientific program.

A large fraction of the participants stayed for the entire period. With a total of 513 mathematicians in attendance, Seattle 2005 was perhaps the largest algebraic geometry conference in history.

We would like to thank J. Maxwell, R. Aguiar, and L. Melucci of the AMS for their work in the planning and the day-to-day organization of the Institute. The principal funding for the program came from the NSF. D. Ellwood and the Clay Institute provided crucial additional support as did DARPA and the NSA. S. Kovács, T. Pantev, and R. Vakil put together an outstanding graduate student program that started before and extended for the duration of the Institute.

Finally, we would particularly like to thank all the speakers and participants for their contributions to Seattle 2005. The success of the Institute was due to them.

D. Abramovich, A. Bertram, L. Katzarkov
R. Pandharipande, M. Thaddeus

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Scientific Program [1]

First week

Plenary lectures

K. Hori, Mirror symmetry
I. Madsen, Homotopy theory and the mapping class group: Mumford's conjecture
A. Okounkov, Geometry and physics of localization sums
R. Pandharipande, Gromov-Witten theory in low dimensions
P. Seidel, Geometry and algebra of Lefschetz fibrations

Seminar lectures

D. Arcara, Moduli spaces in the derived category of K3 surfaces
J. Amoros, Mapping tori and homotopy properties of closed symplectic four-manifolds
D. Auroux, Homological mirror symmetry for blowups of $\mathbb{CP}^2$
K. Behrend, Donaldson-Thomas invariants via microlocal geometry
A. Bertram, Relative stable maps and admissible covers
J. Bryan, The local Gromov-Witten theory of curves
A. Caldararu, Duflo, Riemann-Roch, and Cardy — Lie theory, algebraic geometry, and physics: unified
F. Campana, Multiple fibres, orbifolds, and classification theory
L. Caporaso, Néron models over moduli of stable curves
L. Chen, The equivariant cohomology of quot schemes
I. Ciocan-Fontanine, A generalization of the Hori-Vafa conjecture
T. Coates, The Gromov-Witten theory of a point and KdV
H. D'Souza, Automorphism and collineation groups of good curves
R. Donagi, Geometric transitions, Calabi-Yau integrable systems, and open GW invariants
C. Faber, On motives for cusp forms
B. Fantechi, The virtual fundamental class revisited
G. Farkas, Effective divisors on the moduli space of curves
A. Gathmann, Relative Gromov-Witten invariants and tropical geometry
A. Gibney, A higher dimensional analog of the moduli space of stable pointed rational curves
T. Graber, Gromov-Witten theory of orbifolds and their crepant resolutions

[1] A complete record of the scientific program, including abstracts and notes, can be found at `http://www.math.columbia.edu/~thaddeus/seattle/program.html`

L. Göttsche, Instanton counting, Donaldson invariants and line bundles on moduli spaces of sheaves on rational surfaces
M. Gross, Moduli of log Calabi-Yau spaces and mirror symmetry
T. Hausel, Cohomology of hyperkähler moduli spaces via arithmetic harmonic analysis
D. Huybrechts, Derived equivalences of twisted K3 surfaces
T. L. Jarvis, The virtual class in orbifold and stringy cohomology and K-theory
E. Katz, Relative Gromov-Witten invariants and symplectic field theory
S. Katz, Algebraic geometry and string theory
L. Katzarkov, Generalized Hodge structures and homological mirror symmetry
Y.-H. Kiem, Desingularizations of moduli spaces of rank 2 sheaves with trivial determinant
B. Kreussler, Stability and the structure of the
derived category of coherent sheaves on irreducible curves of genus one
A. Ksir, Finite group actions on Riemann Roch spaces and automorphisms of algebraic geometry codes
Y.-P Lee, Invariance of tautological equations: conjectures and applications
J. Li, Dimension zero Donaldson-Thomas invariants
T.-J. Li, Symplectic Calabi-Yau surfaces and equivariant stable homotopy
W.-P. Li, Some computations of Donaldson-Thomas invariants
C.-C. M. Liu, Relative Gromov-Witten theory and Hodge integrals
E. Markman, Moduli spaces of sheaves on K3 and abelian surfaces: their symmetries and monodromy
J. Martens, Instanton counting and non-abelian localization
M. Mulase, Witten-Kontsevich theory revisited: a survey of recent developments
H. Nakajima, Instanton counting: the K-theoretic partition function
I. Nikolaev, Noncommutative geometry of algebraic curves
D. Oprea, On the intersection theory of the moduli space of rank 2 bundles
J. Ross, Stability of polarized varieties
W.-D. Ruan, Homological mirror symmetry for weighted projective spaces
Y. Ruan, Twisted K-theory and its product
B. Siebert, Tropical manifolds – a finite element method in complex and symplectic geometry
H. Tamvakis, Gromov-Witten invariants on isotropic Grassmannians
C. Teleman, Loop groups and moduli of G-bundles on Riemann surfaces

Second week

Plenary lectures

M. Haiman, Macdonald polynomials and Hilbert schemes
J. Harris, Rationality, unirationality, and rational connectivity
J. Kollár, Resolution of singularities in characteristic zero
J. McKernan, On the existence of flips
V. Shokurov, Flips and finitely generated algebras
C. Voisin, Hodge theory and the topology of compact Kähler and complex projective manifolds

Seminar lectures

H. Abo, Construction of rational surfaces in projective fourspace
V. Alexeev, Log canonical pairs and compactified moduli spaces
Y. Amitani, Projective manifolds with hyperplane sections being five-sheeted covers of $\mathbb{P}^n$
S. Billey, Schubert varieties under a microscope
F. Bogomolov, Birational geometry – small fields, finite groups
L. Borisov, Should we teach toric varieties to our students?
A. Buch, Quantum cohomology of homogeneous spaces
A.-M. Castravet, Hilbert's 14th problem and Cox rings
A. Corti, Explicit 3-folds
J.-P. Demailly, Recent results on hyperbolic algebraic varieties
H. Derksen, Quivers and combinatorics
S. Di Rocco, Toric varieties with dual defect and defect polytopes
L. Ein, Multiplier ideals
D. Eisenbud, Varieties, sets, and schemes – "of minimal degree"
P. Eyssidieux, Infinite coverings of complex projective manifolds
S. Grushevsky, Geometry of A_g and its compactifications
C. Hacon, Extension theorems and their applications to birational geometry
T. Holm, Act globally, compute locally: group actions, fixed points, and localization
K. Hulek, Volumes of lattices, the Borcherds modular form, and K3 surfaces
S. Ishii, Irreducible components of contact loci in arc spaces
S. Kaliman, Actions of C^* and C_+ on affine algebraic varieties
M. Kapranov, Infinite-dimensional spaces in algebraic geometry
K. Karu, Intersection cohomology and *cd*-index of fans
Y. Kawamata, Derived categories and birational geometry
A. Knutson, Degenerations to (unions of) toric varieties, old and new
S. Kovács, Subvarieties of moduli stacks
A. Langer, Sheaves and principal G-bundles in positive characteristic
A. Libgober, Topology of the complements to divisors with isolated non normal crossings
R. MacPherson, Schubert varieties in the loop Grassmannian
M. Manetti, Lie cylinders and higher obstructions to deforming submanifolds
G. Mikhalkin, Enumerative geometry and reality
S. Mukai, Hilbert's original fourteenth problem and certain moduli spaces
M. Mustata, Spaces of arcs and singularities in birational geometry
K. O'Grady, Irreducible symplectic 4-folds which look like $Hilb^2(K3)$
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T. Szemberg, Conjectures of Nagata and Hirschowitz and the Zariski decomposition
B. Totaro, Equivariant Chow groups: applications to quadratic forms and algebraic groups
F. Vaccarino, Symmetric products and invariants of matrices
R. Vakil, Geometric positivity in the Schubert calculus

P. Vermeire, The moduli of rank 2 reflexive sheaves on smooth 3-folds
J. Włodarczyk, Factorization of birational maps
A. Yong, On smoothness and Gorensteinness of Schubert varieties
J. Zhang, On the D-dimension of certain types of threefolds

Third week

Plenary lectures

B. Conrad, The role of algebraic geometry in modularity theorems
D. Gaitsgory, Local geometric Langlands correspondence and representations of affine algebras
P. Griffiths, Hodge theoretic invariants of algebraic cycles
P. Griffiths, On the tangent space to the space of algebraic cycles
F. Loeser, Lectures on motivic integration

Seminar lectures

D. Arinkin, Quantum groupoids and completely integrable systems
P. Balmer, Support varieties for triangulated categories
S. Bloch, Motives associated to graphs
A. Bondal, Derived categories of toric varieties
C. Chin, Independence of ℓ of monodromy groups
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