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Ramsey Theory on the Integers

Second Edition

Bruce M. Landman
Aaron Robertson

AMS
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10 9 8 7 6 5 4 3 2 1 19 18 17 16 15 14

To

Eleanor

Emma and Sarah

– Bruce

To

Elisa, Quinn, Ava

Pearl, Doug, Jason

– aaron

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Preface to the Second Edition

As noted in the preface of the first edition, one of the book's primary purposes is to serve as a source of unsolved problems in the flourishing and relatively "wide-open" area of Ramsey theory on the integers. We especially hoped the book would provide beginning researchers, including graduate students and undergraduate students, with a range of accessible topics in which to delve. It seems that we have already been at least somewhat successful because, since the publication of the first edition, a good number of the research problems have been solved or partially solved, and several of the theorems have been improved upon. As a consequence, the new edition includes many substantial revisions and additions.

Various new sections have been added and others have been significantly updated. Among the newly introduced topics are: rainbow Ramsey theory, an "inequality" version of Schur's theorem, monochromatic solutions of recurrence relations, Ramsey results involving both sums and products, monochromatic sets avoiding certain differences, Ramsey properties for polynomial progressions, generalizations of the Erdős-Ginzberg-Ziv theorem, and the number of arithmetic progressions under arbitrary colorings. We also offer many new results and proofs among the topics that are not new to this edition, most of

which were not known when the first edition was published. Furthermore, the book's tables, exercises, lists of open research problems, and bibliography have all been significantly updated. Finally, we have repaired numerous misprints.

Acknowledgements

We wish to thank a number of individuals for finding errors in the first edition and for helping to improve the exposition of the text, including: Charles Baker, Arie Bialostocki, Srashti Dwivedi, Sohail Farhangi, Paul Frigge, Andraes Jönsson, Jonathan Leprince, Ryan Matzke, James Henry Sanders, and Hunter Snevily. Special thanks go to Sarah Landman for her careful proofreading. We are grateful to all the readers of the first edition who contacted us with insightful comments and suggestions. We also express our gratitute to our fellow Ramsey theorists for enlightening us on some recent advances in the field and for pointing us to a number of intriguing problems. We appreciate the kind and professional support given to us throughout the process by Edward Dunne, Christine Thivierge, and the rest of the AMS production staff.

Bruce Landman adds a special debt of gratitude to his wife, Eleanor, for her support, patience, and sense of humor throughout this endeavor, and to his two wonderful daughters, Emma and Sarah, who add so much meaning to everything he does.

Aaron would like to thank: his parents, Pearl and Doug, for providing him with a happy and supportive upbringing; his brother Jason (sorry for waking you with the snare drum, hiding in the closet to scare you, etc.); his advisor, Doron Zeilberger, without whom he

probably would have dropped out of graduate school; and the love-of-his-life, Elisa, for just being fantastic and for giving him two absolutely wonderful children, Quinn and Ava (and one other child ... kidding, readers, we only have two).

Preface to the First Edition

Ramsey Theory on the Integers covers a variety of topics from the field of Ramsey theory, limiting its focus to the set of integers – an area that has seen a remarkable burst of research activity during the past twenty years.

The book has two primary purposes: (1) to provide students with a gentle, but meaningful, introduction to mathematical research – to give them an appreciation for the essence of mathematical research and its inescapable allure and also to get them started on their own research work; (2) to be a resource for all mathematicians who are interested in combinatorial or number theoretical problems, particularly "Erdős-type" problems.

Many results in Ramsey theory sound rather complicated and can be hard to follow; they tend to have a lot of quantifiers and may well involve objects whose elements are sets whose elements are sets (that is not a misprint). However, when the objects under consideration are sets of integers, the situation is much simpler. The student need not be intimidated by the words "Ramsey theory," thinking that the subject matter is too deep or complex – it is not! The material in this book is, in fact, quite accessible. This accessibility, together with the fact that scores of questions in the subject are still to be answered, makes Ramsey theory on the integers an ideal subject for

a student's first research experience. To help students find suitable projects for their own research, every chapter includes a section of "Research Problems," where we present a variety of unsolved problems, along with a list of suggested readings for each problem.

Ramsey Theory on the Integers has several unique features. No other book currently available on Ramsey theory offers a cohesive study of Ramsey theory on the integers. Among several excellent books on Ramsey theory, probably the most well-known, and what may be considered *the* Ramsey theory book, is by Graham, Rothschild, and Spencer (*Ramsey Theory,* 2nd *Edition* [**175**]). Other important books are by Graham (*Rudiments of Ramsey Theory* [**168**]), McCutcheon (*Elemental Methods in Ergodic Ramsey Theory* [**279**]), Nešetřil and Rödl (*Mathematics of Ramsey Theory* [**295**]), Prömel and Voigt (*Aspects of Ramsey Theory* [**305**]), Furstenberg (*Dynamical Methods in Ramsey Theory* [**156**]), and Winn (*Asymptotic Bounds for Classical Ramsey Numbers* [**401**]). These books, however, generally cover a broad range of subject matter of which Ramsey theory on the integers is a relatively small part. Furthermore, the vast majority of the material in the present book is not found in any other book. In addition, to the best of our knowledge, ours is the only Ramsey theory book that is accessible to the typical undergraduate mathematics major. It is structured as a textbook, with numerous (over 150) exercises, and the background needed to read the book is rather minimal: a course in elementary linear algebra and a 1-semester junior-level course in abstract algebra would be sufficient; an undergraduate course in elementary number theory or combinatorics would be helpful, but not necessary. Finally, *Ramsey Theory on the Integers* offers something new in terms of its potential appeal to the research community in general. Books offering a survey of solved and unsolved problems in combinatorics or number theory have been quite popular among researchers; they have also proven beneficial by serving as catalysts for new research in these fields. Examples include *Old and New Problems and Results in Combinatorial Number Theory* [**128**] by Erdős and Graham, *Unsolved Problems in Number Theory* [**190**] by Guy, and *The New Book of Prime Number Records* [**320**] by Ribenboim. With our text we hope to offer mathematicians an additional resource for intriguing unsolved problems. Although not

nearly exhaustive, the present book contains perhaps the most substantial account of solved and unsolved problems in Ramsey theory on the integers.

This text may be used in a variety of ways:

- as an undergraduate or graduate textbook for a second course in combinatorics or number theory;
- in an undergraduate or graduate seminar, a capstone course for undergraduates, or an independent study course;
- by students working under an REU program, or who are engaged in some other type of research experience;
- by graduate students looking for potential thesis topics;
- by the established researcher seeking a worthwhile resource in its material, its list of open research problems, and its somewhat enormous (often a fitting word when discussing Ramsey theory) bibliography.

Chapter 1 provides preliminary material (for example, the pigeonhole principle) and a brief introduction to the subject, including statements of three classical theorems of Ramsey theory: van der Waerden's theorem, Schur's theorem, and Rado's theorem. Chapter 2 covers van der Waerden's theorem; Chapters 3–7 deal with various topics related to van der Waerden's theorem; Chapter 8 is devoted to Schur's theorem and a generalization; Chapter 9 explores Rado's theorem; and Chapter 10 presents several other topics involving Ramsey theory on the integers.

The text provides significant latitude for those designing a syllabus for a course. The only material in the book on which other chapters depend is that through Section 2.2. Thus, other chapters or sections may be included or omitted as desired, since they are essentially independent of one another (except for an occasional reference to a previous definition or theorem). We do, however, recommend that all sections included in a course be studied in the same order in which they appear in the book.

Each chapter concludes with a section of exercises, a section of research problems, and a reference section. Since the questions contained in the Research Problem sections are still open, we cannot say

with certainty how difficult a particular one will be to solve; some may actually be quite simple and inconsequential. The problems that we deem most difficult, however, are labeled with the symbol $*$. The reference section of each chapter is organized by section numbers (including the exercise section). The specifics of each reference are provided in the bibliography at the end of the book.

The material covered in this book represents only a portion of the subject area indicated by the book's title. Many additional topics have been investigated, and we have attempted to include at least references for these in the reference sections. Yet, for every problem that has been thought of in Ramsey theory, there are many more which that problem will generate and, given the great variety of combinatorial structures and patterns that lie in the set of integers, countless new problems wait to be explored.

We would like to thank Dr. Edward Dunne and the members of the AMS production staff for their assistance in producing this book. We also thank Tom Brown, Scott Gordon, Jane Hill, Dan Saracino, Dan Schaal, Ralph Sizer, and the AMS reviewers for their helpful comments and advice, which greatly improved the manuscript. We also express our gratitude to Ron Graham and Doron Zeilberger for their support of this project. We owe a big debt to the pioneers and masters of the field, especially Ron Graham, Jarik Nešetřil, Joel Spencer, Neil Hindman, Tom Brown, Timothy Gowers, Hillel Furstenberg, Vitaly Bergelson, Vojtěch Rödl, Endre Szemerédi, László Lovász (we had to stop somewhere), and of course Bartel van der Waerden, Issai Schur, Richard Rado, and Frank Ramsey. To all of the others who have contributed to the field of Ramsey theory on the integers, we extend our sincere appreciation. Finally, we want to acknowledge that this book would not exist without the essential contributions of the late Paul Erdős. But beyond the content of his achievements, he has personally inspired the authors as mathematicians. Our professional lives would have had far less meaning and fulfillment without his work and his presence in our field. For that pervasive, though perhaps indirect, contribution to this text, we are in his debt.

Notation

	Notation	Description	Page
	$\lceil \cdot \rceil$	Ceiling function	11
	$\lfloor \cdot \rfloor$	Floor function	11
	$\oplus$	Modular addition	117
	$\ominus$	$A \ominus B = \{a - b : a \in A, b \in B\}$	313
	$[a, b]$	$\{a, a+1, \ldots, b\}$	9
	$A - B$	$\{x \in A : x \notin B\}$	9
	$A \prec B$	Nonoverlapping sets: $\max(a \in A) < \min(b \in B)$	342
A	A_D	Family of arithmetic progressions with gaps in D	114
	$ADW(k, \ell)$	Ascending/descending waves numbers	112
	AP	Family of arithmetic progressions	15
	$AP_{a(m)}$	Family of arithmetic progressions with gaps congruent to $a \pmod m$	183
	$AP^*_{a(m)}$	$AP_{a(m)} \cup A_{\{m\}}$	187
	$AP_{(m)}$	Set of arithmetic progressions (mod m)	184
	AUG_b	Family of augmented progressions with tail b	168
	$AW(k; r)$	Ascending waves number	110
B	$B(f; r)$	"Brown" number	320
	$b(m, k; r)$	Ramsey-type number for nonoverlapping m-sets	343
C	$c(R; r)$	Rado-type number for recurrences	286
	cul_j	Culprit of color j	34
	$C(k)$	k-nonrepetitive number	329
D	$\Delta_{\{a,b,c\}}$	Triangle on vertices a, b, c	227
	$\Delta(D, k; r)$	Ramsey-type number for diffsequences	303
	$D_S(k; r)$	Difference S-free number	322
	diam	Diameter of a set	342
	doa	Degree of accessibility	306
	dor	Degree of regularity	150
	dor_k	Degree of regularity for $T_{k-1}(a)$	160
	$DW(k)$	2-color descending wave number	81

	Notation	Description	Page
F	f_n	Fibonacci number	139
	$F(k;r)$	Folkman-Rado-Sanders number	298
	$\mathcal{FS}(T)$	Finite Sums of set T	297
G	$\Gamma_m(k)$	Least s guaranteeing k-term arithmetic progressions in all s-term $[1,m]$-gap sequences	84
	$GQ_\delta(k)$	2-color Ramsey-type function for generalized quasi-progressions	78
	$g(r,k)$	Least integer such that for all S with $\lvert S\rvert = g$ and all $\chi : S \to \{0,\ldots,r-1\}$, whenever$\lvert\chi(S)\rvert = k$ there exist $t_1,t_2,\ldots,t_r \in S$ with $(t_1,t_2,\ldots,t_r)$ zero-sum	338
	gs	Gap size	320
K	(k,n,d)-progression	k-term quasi-progression with diameter n and low-difference d	69
H	$H(s_1,\ldots,s_k)$	2-color Ramsey-type function for homothetic copies of $\{1,1+s_1,\ldots,1+s_1+\cdots+s_k\}$	163
I	$I(k_1,k_2,\ldots,k_r)$	Schur inequality numbers	243
	$\hat{I}(k,k)$	Strict Schur inequality numbers	245
J	$J_k(n;r)$	Asymptotical minimum number of monochromatic arithmetic progressions	52
L	$\lambda(c,k;r)$	Special r-coloring of $[1,cr(k-1)^2]$ that avoids monochromatic k-term c-a.p.'s	122
	$\mathcal{L}(k)$	Equation $x_1+\cdots+x_{k-1}=x_k$	234
M	$\mu(k)$	$\min\{\lvert\mathcal{E}\rvert : \Gamma=(V,\mathcal{E})$ is a hypergraph not satisfying Property B and $\lvert E\rvert = k$ for all $E\in\mathcal{E}\}$	42
	$\mathcal{M}(k)$	Inequality $x_1+\cdots+x_{k-1}<x_k$	243
	$M_\chi(n)$	Number of monochromatic Schur triples in $[1,n]$ under χ	227
N	$\nu(k)$	Erdős and Turán function	47
O	$\Omega_m(k)$	Least n so that every $\{x_1,\ldots,x_n\}$ with $x_i\in[(i-1)m,im-1]$ contains a k-term arithmetic progression	204
P	P_n	Family of sequences generated by iteration of a polynomial of degree n	92
	$P_{n,k}$	Family of k-term members of P_n	92
	$pp(k,n;r)$	Polynomial progression numbers	175
Q	$Q_n(k)$	2-color Ramsey-type function for quasi-progressions	69

	Notation	Description	Page
R	$\mathbb{R}$	The set of real numbers	9
	$R(AP^*_{a(m)}, k, l; r)$	Generalization of $R(AP^*_{a(m)}, k; r)$	188
	$r(\mathcal{E}; s)$	Rado number for equation $\mathcal{E}$	259
	$R(\mathcal{F}, k; r)$	Ramsey-type number for family $\mathcal{F}$	15
	$R(k_1, \ldots, k_r)$	r-color (classical) Ramsey number	9
	$R_r(k)$	$R(\underbrace{k, \ldots, k}_{r})$	9
	$RR(S; r)$	Reverse r-regular number	207
S	$S(k_1, \ldots, k_r)$	Generalized Schur number	235
	$S_r(k)$	Generalized Schur number with all $k_i = k$	235
	$\hat{S}(k_1, \ldots, k_r)$	Strict generalized Schur number	242
	S_n	Family of sequences generated by iteration of a polynomial of degree $\leq n$	93
	$S_{n,k}$	Family of k-term members of S_n	93
	$SP_m(k)$	Ramsey-type number for semi-progressions	83
	$s(r)$	Schur number	223
	$\hat{s}(r)$	Strict Schur number	240
T	$T_{a,b}$	Family of (a, b)-triples	148
	$T(a, b; r)$	Ramsey-type number for (a, b)-triples	148
	$T(a_1, \ldots, a_{k-1})$	Ramsey-type function for generalization of (a, b)-triples	160
	$T_{k-1}(a)$	Ramsey-type function $T(a_1, \ldots, a_{k-1})$ with $a = a_1 = a_2 = \cdots = a_{k-1}$	160
	$\Theta(n)$	Set of permutations of $[1, n]$ with no 3-term arithmetic progression	215
	$\theta(n)$	$\lvert\Theta(n)\rvert$	214
V	V_m	$\{x \in \mathbb{Z}^+ : m \nmid x\}$	312
	$V_{m,n}$	$\{x \in \mathbb{Z}^+ : m \nmid x \text{ and } n \nmid x\}$	345
W	$w(k)$	$w(k; 2)$	15
	$w(k; r)$	van der Waerden number	12
	$w(k_1, \ldots, k_r; r)$	Mixed van der Waerden number	36
	$w_2(k; r)$	Mixed van der Waerden number $w(k, 2, 2, \ldots, 2; r)$	60
	$\widehat{w}(k; r)$	van der Waerden number with d same color	57
	$w'(c, k; r)$	Ramsey-type number for arithmetic progressions with gaps at least c	121
	$w'(f(x), k; r)$	Ramsey-type number for f-a.p.'s	126
	$w^*(k, j)$	Ramsey-type number for arithmetic progressions with color discrepancy at least j	209
	$\overline{w}(k)$	Ramsey-type number for 3-term arithmetic progression of one color or k consecutive integers of the other color	219
Z	$\mathbb{Z}$	Set of integers	9
	$\mathbb{Z}^+$	Set of positive integers	9

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