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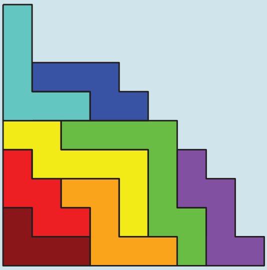
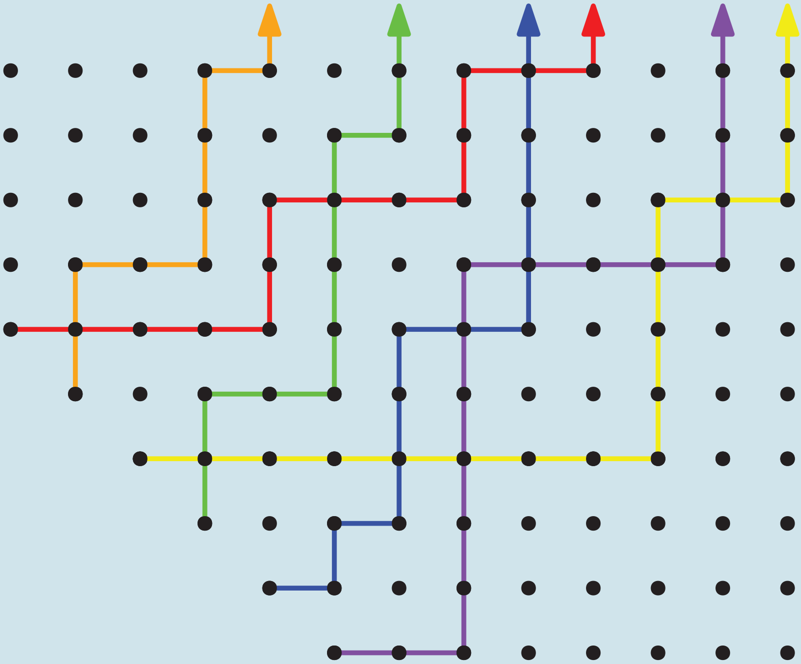
An Introduction to Symmetric Functions and Their Combinatorics

Eric S. Egge





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Preface

Many excellent books are devoted entirely or in part to the foundations of the theory of symmetric functions, including the books of Loehr [**Loe17**], Macdonald [**Mac15**], Mendes and Remmel [**MR15**], Sagan [**Sag01**], and Stanley [**Sta99**]. These books approach symmetric functions from several different directions, assume various amounts of preparation on the parts of their readers, and are written for a variety of audiences. This book's primary aim is to make the theory of symmetric functions more accessible to undergraduates, taking a combinatorial approach to the subject wherever feasible, and to show the reader both the core of the subject and some of the areas that are active today.

We assume students reading this book have taken an introductory course in linear algebra, where they will have seen bases of vector spaces, transition matrices between bases, linear transformations, and determinants. We also assume readers have taken an introductory course in combinatorics, where they will have seen (integer) partitions and their Ferrers diagrams, binomial coefficients, and ordinary generating functions. For those who would like to refresh their memories of some of these ideas or who think there might be gaps in their linear algebraic or combinatorial background, we have included summaries of the ideas from these areas we will use most often in the appendices. In particular, we have included explanations of dual bases and the relationship between determinants and permutations, since these will play important roles at key moments in our study of symmetric functions, and they may not appear in a first course in linear algebra.

Symmetric functions have deep connections with abstract algebra and, in particular, with the representation theory of the symmetric group, so it is notable that we do not assume the reader has any familiarity with groups. Indeed, we develop the one algebraic fact we need—that each permutation is a product of adjacent transpositions—from scratch in Appendix C. Leaving out the relationship between symmetric functions and representations of the symmetric group makes the book accessible to a broader audience. But to see the subject whole, one must also explore the relationship between symmetric functions and representation theory. So we encourage interested students, after reading this book, to learn about representations of the symmetric group, and how they are related to symmetric functions. Two sources for this material are the books of James [**Jam09**] and Sagan [**Sag01**].

As in other areas of mathematics, over the past several decades the study of symmetric functions has benefited from the use of computers and from the dramatic increase in the amount of available computing power. Indeed, many contemporary symmetric functions researchers use software, such as Maple, Mathematica, and SageMath, for large symmetric functions computations. These computations, in turn, often lead to new conjectures, new ideas, and new directions for exploration.

We take a dual approach to the use of technology in the study of symmetric functions. On the one hand, we do not assume the reader has any computer technology available at all as they read: a patient reader will be able to work out all of the examples, solve all of the problems, and follow all of the proofs without computer assistance. On the other hand, we encourage readers to become proficient with some kind of symmetric functions software. Specific programs and platforms come and go, so we do not recommend any platform or software in particular. But we do recommend that you find a way to have your computer do your symmetric functions computations for you and to use it to explore the subject.

Many of the main results in the theory of symmetric functions can be understood in several ways, and they have a variety of proofs. Whenever possible, we have given a proof using combinatorial ideas. In particular, we use families of lattice paths and a tail-swapping involution to prove the Jacobi–Trudi identities, we use the Robinson–Schensted–Knuth (RSK) correspondence to prove Cauchy's formula, and we use RSK insertion, *jeu de taquin*, and Knuth equivalence to prove the Littlewood–Richardson rule. The study of symmetric functions can motivate these ideas and constructions, but we find (and we think the reader will find) they are rich and elegant, and they will reward study for its own sake.

The study of symmetric functions is old, dating back at least to the study of certain determinants in the mid- to late-nineteenth century, but it remains an active area of research today. While our primary goal is to introduce readers to the core results in the field, we also want to convey a sense of some of the more recent activity in the area. To do this, we spend Chapter 5 looking at three areas of contemporary research interest. In Section 5.1 we introduce skew Schur functions, and we discuss the problem of finding pairs of skew tableaux with the same skew Schur function. In Sections 5.2 and 5.3 we introduce the stable and dual stable Grothendieck functions, which are analogues of the Schur functions. Finding analogues of results about Schur functions for these symmetric functions is a broad and lively area of current research. Finally, in Section 5.4 we discuss Stanley's chromatic symmetric functions. These symmetric functions, which are defined for graphs, are the subject of at least two longstanding open questions first raised by Stanley. We introduce one of these questions, which is whether there are two nonisomorphic trees with the same chromatic symmetric function.

One of the best ways to learn mathematics is to do mathematics, so in many cases we have tried to describe not only what a result says and how we prove it, but also how we might find it. In particular, we introduce several topics by raising a natural question, looking at some small examples, and then using the results of those examples to formulate a conjecture. This process often starts with questions arising from linear algebra, which we then use combinatorial ideas to answer, highlighting the way the two subjects interact to produce new mathematics. We could use a different expository approach to cover the material more efficiently, but we hope this approach brings the subject to life in a way a more concise treatment might not.

To get the most out of this book, we suggest reading actively, with a pen and paper at hand. When we generate data to answer a question, try to guess the answer yourself before reading ours. Generate additional data of your own to support (or refute) your conjecture, and to verify patterns you've observed. Similarly, we intend the examples to be practice problems. Trying to solve them yourself before reading our solutions will strengthen your grasp of the core ideas, and prepare you for the ideas to come.

Speaking of doing mathematics, we have also included a variety of problems at the end of each chapter. Some of these problems are designed to test and deepen the reader's understanding of the ideas, objects, and methods introduced in the chapter. Others give the reader a chance to explore subjects related to those in the chapter, that we didn't have enough space to cover in detail. A few of the problems of these types ask the reader to prove results which will be used later in the book. Finally, some of the problems are there to tell the reader about bigger results and ideas related to those in the chapter. A creative and persistent reader will be able to solve many of the problems, but those of this last type might require inventing or reproducing entirely new methods and approaches.

This book has benefitted throughout its development from the thoughtful and careful attention, ideas, and suggestions of a variety of readers. First among these are the Carleton students who have used versions of this book as part of a course or a senior capstone project. The first of these students were Amy Becker '11, Lilly Betke-Brunswick '11, Mary Bushman '11, Gabe Davis '11, Alex Evangelides '11, Nate King '11, Aaron Maurer '11, Julie Michelman '11, Sam Tucker '11, and Anna Zink '11, who used this book as part of a senior capstone project in 2010–11. Back then it wasn't really a book; it was just a skeletal set of lecture notes. Based on their feedback I wrote an updated and more detailed version, which I used as a text for a seminar in the combinatorics of symmetric functions in the fall of 2013. The students in this seminar were Leo Betthauser '14, Ben Breen '14, Cora Brown '14, Greg Michel '14, Dylan Peifer '14, Kailee Rubin '14, Alissa Severson '14, Aaron Suiter '15, Jon Ver Steegh

’14, and Tessa Whalen-Wagner ’15. Their interest and enthusiasm encouraged me to add even more material and detail, which led to a nearly complete version of the book in late 2018. In the winter and spring of early 2019, Patty Commins ’19, Josh Gerstein ’19, Kiran Tomlinson ’19, and Nick Vetterli ’19 used this version as the basis for a senior capstone project. All three groups of students pointed out various typographical errors, generously shared their comments, criticisms, corrections, and suggestions, and suggested several ways in which the material could be presented more clearly and efficiently. I am grateful to all of them for their care, interest, and enthusiasm.

Also crucial in the development of this book were several other readers who shared their ideas, corrections, and suggestions. Jeff Remmel showed me the combinatorial approach I use to prove the Murnaghan–Nakayama rule. Becky Patrias suggested including the results involving stable and dual stable Grothendieck polynomials and elegant tableaux. And several anonymous reviewers provided very thorough and detailed comments, corrections, and suggestions. I am grateful for the time and effort all of these people put in to improve this book.

Finally, Eko Hironaka, senior editor of the AMS book program, has been patiently but persistently encouraging me to finish this book for longer than I care to admit. Thank you, Eko, for not giving up on it.

In spite of everyone’s best efforts, it is likely some errors remain. These are all mine. There are also undoubtedly still many ways this book could be improved. You can find more information related to the book, along with a list of known errors, at www.ericegge.net/cofsf. I would very much like to hear from you: if you have comments or suggestions, or find an error which does not already appear on the list, please email me at eegge@carleton.edu. And thanks for reading!

Eric S. Egge
July 2019

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Sara Rubinstein, Rubinstein Photo

This book is a reader-friendly introduction to the theory of symmetric functions, and it includes fundamental topics, such as the monomial, elementary, homogeneous, and Schur function bases; the skew Schur functions; the Jacobi–Trudi identities; the involution Ω; the Hall inner product; Cauchy's formula; the RSK correspondence and how to implement it with both insertion and growth diagrams; the Pieri rules; the Murnaghan–Nakayama rule; Knuth equivalence; *jeu de taquin*; and the Littlewood–Richardson rule. The book also includes glimpses of recent developments and active areas of research, including Grothendieck polynomials, dual stable Grothendieck polynomials, Stanley's chromatic symmetric function, and Stanley's chromatic tree conjecture. Written in a conversational style, the book contains many motivating and illustrative examples. Whenever possible it takes a combinatorial approach, using bijections, involutions, and combinatorial ideas to prove algebraic results.

The prerequisites for this book are minimal—familiarity with linear algebra, partitions, and generating functions is all one needs to get started. This makes the book accessible to a wide array of undergraduates interested in combinatorics.

$$\begin{array}{ll} 4 & 5 \\ 3 & 3 \\ 1 & 2 \end{array} \;*\; \begin{array}{ll} 5 & \\ 2 & 4 \\ 1 & 3 \end{array} \;=\; \begin{array}{llll} 4 & & & \\ 3 & 3 & 5 & \\ 2 & 3 & 4 & \\ 1 & 1 & 2 & 3 \end{array}$$

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